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The Design of Learning Trajectory for Parabola Equation in Geometry STEM-Based Learning for Flexibility Skills

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ABSTRACT

The flexibility of mathematical thinking among prospective mathematics teachers remains relatively low, primarily due to learning approaches that do not meaningfully connect mathematical concepts with real-world contexts. This study aims to design and implement a STEM-based learning trajectory on the topic of parabolic equations to foster the development of mathematical thinking flexibility in prospective teachers. The research employs a design research approach consisting of two stages: a pilot experiment to test and refine the learning trajectory, and a teaching experiment to implement and evaluate its effectiveness. The participants were students who had completed an Analytical Geometry course. Data were collected through activity sheets, Desmos documentation, video recordings, and interviews. Data were analyzed using a retrospective analysis method, which involved three main steps: (1) organizing data from various sources, (2) conducting within-case analysis to trace students' thought processes throughout each activity, and (3) synthesizing patterns across cases to identify the development of mathematical flexibility. The results show that the learning trajectory consisting of four main activities: video analysis, elevation angle experiments, graphing parabolas using Desmos, and determining parabolic equations effectively facilitated the development of mathematical flexibility in three aspects: representational, conceptual, and procedural. Students demonstrated the ability to shift between different representations, understand the interconnections among mathematical concepts, and adapt problem-solving strategies to contextual situations. The teaching experiment also revealed increased student engagement, higher quality of discussion, and a greater diversity of strategies employed. This study recommends the integration of real-world contexts, such as football throw-ins, to support STEM-based mathematics instruction aimed at developing flexible mathematical thinking in prospective teachers.

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A. INTRODUCTION

Understanding geometric concepts particularly parabolas is a critical foundation in mathematics education, both for school students and prospective mathematics teachers. Geometry supports the development of spatial reasoning, visualization, and logical thinking key skills for constructing meaningful mathematical understanding (Dintarini et al., 2024; Fitriyani et al., 2023). For future teachers, mastery of parabolas strengthens not only their grasp of quadratic functions but also their ability to design contextual and applicative learning strategies. Meanwhile, for students, connecting the parabolic shape to real-world phenomena such as the trajectory of a thrown object helps to bridge abstract concepts with lived experiences, increasing both engagement and comprehension (Arcavi, 2003; Battista M.T, 2007). Geometric reasoning further reinforces the connection between spatial and algebraic representations, which is essential in developing pedagogical content knowledge (Jones, 2001; Wahyudin et al., 2022).

Despite its importance, parabola instruction in higher education often remains abstract and overly procedural. Research has shown that many students only understand superficial features such as the direction of a parabola's opening without being able to model its structure mathematically or apply it contextually (Meinarni et al., 2020). While some studies have emphasized the importance of understanding key elements like the focus and directrix for enhancing proof skills (Hidayati, 2020), and linking visual and symbolic representations (Pasandaran & Mufidah, 2020), there is limited emphasis on developing mathematical thinking flexibility the ability to move between representations, adopt diverse strategies, and adapt to contextual challenges (Rittle-Johnson et al., 2017; Star & Rittle-Johnson, 2008).

One promising direction is the integration of STEM (Science, Technology, Engineering, and Mathematics) in mathematics education. STEM-based learning promotes interdisciplinary connections, real-world problem-solving, and hands-on exploration, while also enhancing critical thinking, creativity, and adaptability-key competencies in 21st century learning (Bybee, 2013; National Research, 2014a; Nopriyanti et al., 2024). For example, Risnawati et al. (2024) found that using tools like Math City Map within STEM frameworks improves students' creative and critical mathematical thinking. Roberts et al. (2022) similarly emphasize the value of integrated STEM practices in deepening understanding and promoting flexible problem-solving.

In the context of parabola learning, using sports-based scenarios such as soccer throw-ins provides a rich opportunity to model quadratic functions authentically. Studies by Queiruga-Dios et al. (2025) and Schubert & Ludwig (2020) demonstrate that such contexts enhance students' grasp of mathematical and physical principles. James Strickland (1996) supports the use of football to bridge classroom mathematics with real-life application, while Jazuli (2021) argues that STEM learning grounded in physical activities strengthens the integration of conceptual and procedural knowledge. However, there remains a significant research gap: few studies have systematically designed STEM-based learning trajectories that integrate sports contexts, technological tools like Desmos, and a focus on mathematical flexibility particularly for preservice teachers.

Although previous studies underscore the value of phased learning designs to support conceptual understanding (Fonger et al., 2020; Jaramilo, 2021; Technology, 2024), most have not explored how such designs can be combined with digital platforms like Desmos or augmented reality in sports contexts to foster flexible thinking. Robiah & Peni (2023), for instance, found that Desmos improves flexibility in solving quadratic equations, yet their research did not integrate physical exploration or contextual modeling in a structured trajectory. Thus, current literature lacks a comprehensive instructional model that connects real-world experiences, digital technology, and progressive thinking development in the study of parabolas.

This study addresses that gap by designing a STEM-based learning trajectory using the context of football throw-ins to enhance the mathematical thinking flexibility of prospective mathematics teachers. The model systematically incorporates Desmos at various stages from initial visual representations and physical experiments to algebraic modeling supporting learners' ability to interpret and transition across multiple forms of representation. The trajectory is grounded in the Hypothetical Learning Trajectory (HLT**)** framework (Clements & Sarama, 2011; Simon, 2020), complemented by Realistic Mathematics Education (Freudenthal, 1991), van Hiele's geometric thinking levels (Van Hiele, 1986) and Vygotsky's social constructivism (Vygotsky, 1978), emphasizing interaction, context, and cognitive progression.

The novelty of this research lies in three contributions: (1) the development of a structured STEM-based learning trajectory anchored in football context; (2) the systematic integration of Desmos technology to support modeling and visualization; and (3) the explicit focus on representational, conceptual, and procedural flexibility as key learning outcomes. This approach aims to provide an adaptive instructional model that aligns with the goals of 21st-century mathematics education and addresses current gaps in both theory and practice.

B. METHODS

This research uses a Design Research approach based on Gravemeijer and Cobb (2004), with the aim of developing a STEM-based learning trajectory in the context of sports (throw-ins in soccer) to enhance the mathematical thinking flexibility of prospective teachers. This design was chosen because it can bridge the real needs in the field with a dynamic learning process. The research subjects consisted of 22 students from the mathematics education study program for the pilot experiment and 27 students for the teaching experiment phase. Participants are in their fifth semester and have completed the courses of Analytic Geometry and Mathematics Learning Strategies. The selection was conducted through purposive sampling, as the subjects were at the stage of preparing for teaching practice and relevant to the research objectives.

The research was conducted in two main stages, namely the pilot experiment and the teaching experiment. In the pilot experiment stage, an initial trial of the learning design was conducted using worksheets, learning activity videos, and semi-structured interviews to evaluate participants' conceptual understanding and strategy flexibility. Data from this stage were used to revise the content and sequence of learning activities.

The teaching experiment phase applies the revised design in a more comprehensive learning situation. Additional instruments such as observation sheets, Desmos screen recordings, and documents of group discussion results are used to trace the development of students' representations and thinking strategies in depth. Data analysis techniques were conducted qualitatively through content analysis, including video transcripts, interviews, and worksheets based on flexibility indicators. Each stage of the research is designed to support the main objective, which is to produce a valid learning design for the flexibility of mathematical thinking skills of prospective teachers on the topic of parabolas.

C. RESULT AND DISCUSSION

1. Research Preparation

The preparation stages carried out by the researcher before conducting the research were: (1) analyzing literature reviews, (2) Interviews and discussions with lecturers who teach Geometry courses, and (3) Hypotenical Learning Trajectory (HLT) design. At the stage of analyzing literature reviews, the researcher examined those related to learning trajectories in parabolic equation material in STEM-based learning, flexibility skills for prospective teacher students, previous relevant studies that used the context of football.

After the researcher conducted a literature review, the researcher conducted interviews with lecturers regarding the importance of students' flexibility skills for prospective teacher students and about contextual learning that can be applied in parabola material. After the interview and discussion stages, the results of the selection of research samples were obtained, namely students who had taken the Analytical Geometry of Planes and Space courses. In addition, the flexibility skills focused on in this study are students' ability to see the solution to a problem in various ways to find solutions to problems related to the parabola equation in the context of football. The learning that will be carried out uses STEM-based learning with the use of the Desmos application.

The results of the literature review stage and interviews and discussions with course lecturers are used as the basis for designing the Hypothetical Learning Trajectory (HLT). The designed HLT can be seen in table 1 which consists of Learning Objectives, Designing Learning Activities, and predicting the learning process of prospective teacher students. The technology used is the Desmos application to facilitate or bridge the real context with mathematics. The Hypothetical Learning Trajectory (HLT) design developed in this study consists of four learning activities that are designed progressively to facilitate students' understanding of the concept of parabolic equations in the context of STEM-based learning. The structure of this HLT refers to the framework proposed by Simon (2020), which states that an HLT consists of three main components, namely learning goals, learning activities, and hypotheses about the student learning process (hypothetical learning process) which in this context is represented in the form of conjectures or student conjectures, as shown in Table 1.

Table 1. HLT For Parabola Material in STEM-Based Learning

Learning Goal	Learning Activities	Conjectures of students
Activity 1 In this activity 1 students can: 1. Analyze videos to understand about throw-ins 2. Understand the factors that influence throw-ins	1. Using digital video as a source of observation and analysis. (technology) 2. Identify physical factors such as angle, force, and direction of throw. (Science) 3. Analyze throwing techniques to evaluate their effectiveness. (Engineering) 4. Developing a conceptual understanding of the shape of a parabola. (Mathematic)	The success of Arhan's throw-in was influenced by the correct technique and way of throwing so as to produce a long throwing distance.

<table>
<tr><th>Learning Goal</th><th>Learning Activities</th><th>Conjectures of students</th></tr>
<tr><td>Activity 2
In this activity 2, students are expected to be able to find the relationship between the initial position, elevation angle and throwing accuracy.</td><td>1. 1 student from the group will play the role of Arhan and throw 5 times and fill in the table in the activity.
<table>
<tr><th rowspan="2">Nama Mahasiswa</th><th colspan="2">Posisi Awal</th><th rowspan="2">Sudut Lemparan</th><th rowspan="2">Waktu</th><th rowspan="2">Jarak</th></tr>
<tr><th>Stay</th><th>Lari Kecil</th></tr>
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2. Students discuss the relationship between the initial position, throwing angle and distance based on the results and graphs created.</td><td>1. Students do the throw according to the video that has been given 5 times with a starting position of jogging and standing still, measuring the distance, time and elevation angle.
2. The smaller the elevation angle, the further the horizontal distance between the thrower and the point where the ball falls.</td></tr>
<tr><td>Activity 3
In activity 3, students can:
1. Analyzing parabolic graphs in solving problems and their applications.
2. Using the Desmos application to solve problems related to parabolic equations</td><td>1. Students individually analyze videos using the Desmos application by predicting the accuracy of the ball throw based on predictions and using parabolic graphs. Students can measure the distance of the player from videos and photos and predict the height of the ball.</td><td>The students thought they could graph a parabola based on the points they took from the video and adjust the coefficients a , b , c to match the trajectory of the ball.
Students estimate that the quadratic function $y=ax^2+bx+c$ can be used to estimate the accuracy of the target if the start, peak, and end points of the ball's trajectory are known.</td></tr>
<tr><td>Activity 4
In this activity 4, students can predict the equation of a parabola from a known parabola graph.</td><td>Students find the parabola equation from the given ball trajectory image and prove it in the Desmos application and understand the relationship between coefficients a, b and c and the shape of the curve.</td><td>The student assumes that three coordinate points taken from the ball's trajectory are sufficient to determine one parabola equation (general form: $y=ax^2+bx+c$), if the equation is correct when proven in Desmos the graph formed will pass through the ball's points.
The students estimated that the value of a must be negative for the parabola to open downwards according to the trajectory of the ball formed.</td></tr>
</table>

The learning trajectory structured in this HLT shows a progressive flow from concrete contexts to formal representations. The four activities gradually build students' ability to think flexibly, from understanding real phenomena, exploring experiments, to mathematical modelling. The STEM-based approach used in this design supports the development of 21st-century skills such as problem solving, critical thinking, and interdisciplinary integration (Bybee, 2013; English, 2016).

Based on Table 1 above is a design of STEM-based learning activities for parabolic equation material using the context of throw-ins using the Desmos application. This learning consists of 4 activities that begin with analyzing throw-in videos and seeing the elements that influence a throw. The trajectory begins with a real context in the form of a throw-in, applying the concept of a parabolic equation as in a ball throw such as a ball trajectory that forms a parabolic curve and can obtain the maximum height of the ball, determining the parabolic equation from the ball trajectory that is formed. The learning approach used is the STEM approach to see that there is a mathematical concept in football. This trajectory design directs prospective teacher students to develop their *flexibility skills* to find many ways or solutions to existing problems. The HLT designed in activities 3 and 4 has been implemented into the Desmos application. So that prospective teacher students can interact and solve them directly. The following is the design of prospective teacher student activities in Desmos, as shown in Figure 1.

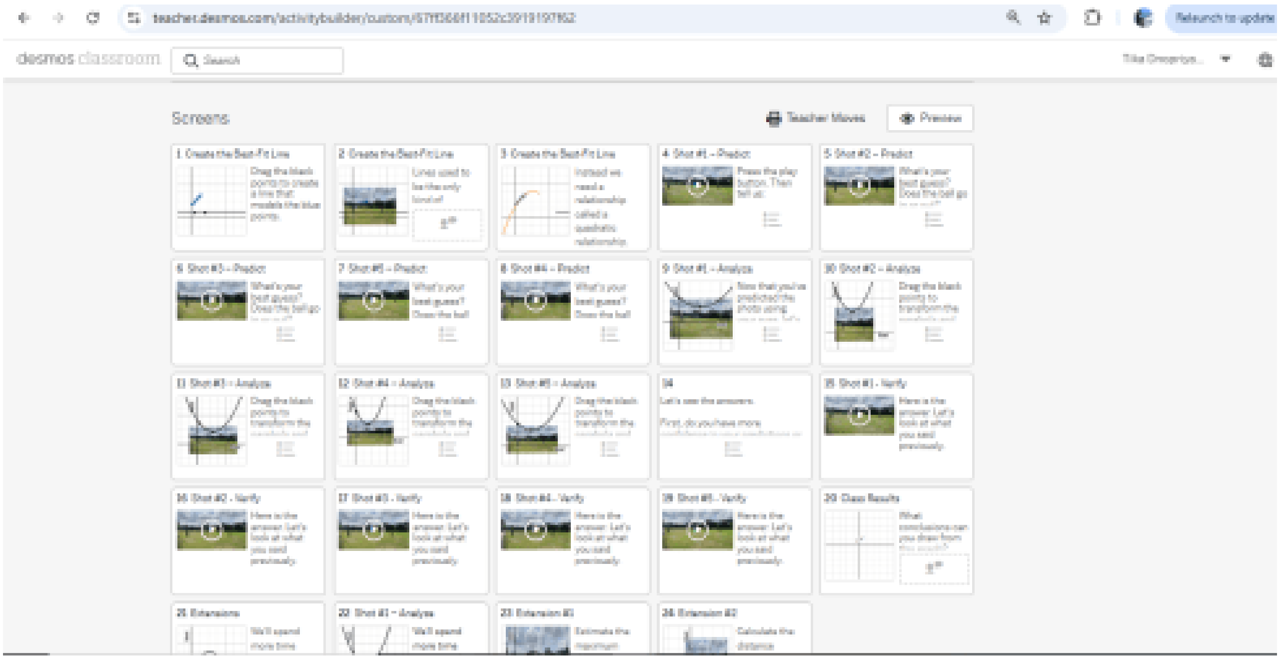


Figure 1. Student Activity Design for Prospective Teachers on Desmos

2. Research Desain

At this stage there are two cycles, namely the pilot experiment stage and the teaching experiment stage. The pilot experiment stage involved 22 prospective mathematics teacher students and the teaching experiment stage involved 27 prospective mathematics teacher students. Each cycle consists of 4 stages, namely: (1) Arhan Video Analysis, (2) Arhan Throw Experiment, (3) Parabola Graphic Representation Using Desmos, and (4) Determining the Parabola Equation.

Teaching experiment stage begins with learning objectives, apperception and motivation. In the apperception the lecturer will ask "*Do you know a football player Arhan? What makes Arhan's throw-in phenomenal?*" and the student teachers will answer the lecturer's questions. After the students have answered the lecturer will distribute activity sheets that will be discussed by the student teachers.

a. Arhan Video Analysis

Activity 1 aims to analyze videos to understand about throw- ins and understand the factors that influence throw-ins. In this activity 1, student teachers build a learning context through an authentic phenomenon, namely a throw-in by Pratama Arhan.

Students observe and analyze videos in groups to identify key factors that influence the success of the throw. Video-based approaches and motion analysis are also recommended in STEM learning to increase engagement and relevance (Oktaviyanthi & Agus, 2023). In activity 1, the lecturer will ask questions to stimulate students' analytical and thinking skills:

Lecturer: *"After watching the two videos of Arhan's throw-ins, do you think the two throws had the same result?"*

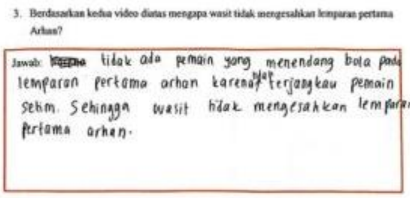
Student: *"Both scored goals, ma'am, but one was invalid and the other was valid."*

Lecturer: *"Okay, now you can discuss with your respective groups why that 1 throw is considered invalid!"*

In addition to building meaningful contexts, this activity also plays an important role in developing students' flexibility skills, especially in the aspects of representational flexibility and conceptual flexibility. In this activity, students begin to observe physical phenomena through visual representations (videos), then identify elements that affect the ball's trajectory such as elevation angle, speed, and body position. This activity is the starting point for students to connect the shape of the trajectory with the basic idea of a parabola, without having to go directly into formal mathematical symbols. This transition from visual experience to mathematical ideas is a key feature of representational flexibility, namely the ability to move between forms of representation meaningfully (Lesh et al., 2020; Star & Rittle-Johnson, 2008).

In addition, the process of group discussion and interpretation in this activity also encourages the growth of conceptual flexibility, namely the ability to understand the concept of trajectory and motion variables in various situations. Students realize that the trajectory of the ball does not only depend on a single factor, but is the result of the interaction of several parameters. This kind of understanding is important as a basis for building more flexible problem-solving strategies in the next stage. A study by Rittle-Johnson et al. (2017) showed that early exposure to rich contextual situations before the introduction of symbolic procedures contributed significantly to students' conceptual flexibility and generalization abilities.

Thus, activity 1 not only serves as a contextual introduction to parabola learning but also as an important foundation for developing students' conceptual and representational thinking flexibility. This activity supports the view that STEM-based mathematics learning should integrate real-world contexts, exploratory experiences, and reflective discussions to build deeper and more adaptive understanding (Lee et al., 2021), as shown in Figure 2.


3. Berdasarkan kedua video diatas mengapa wasit tidak mengesahkan lemparan pertama Arhan?

Jawab: tidak ada pemain yang menendang bola pada lemparan pertama arhan karena tak terjangkau pemain sekim. Sehingga wasit tidak mengesahkan lemparan pertama arhan.

Based on the two videos above, why did the referee not validate Arhan's first throw?
Answer : No player kicked the ball on Arhan's first throw because it was out of reach of any team player so the referee did not validate Arhan's first throw.

Figure 2. Student Answers at the Pilot Experiment Stage

b. Experiment to Become an Arhan

Students conduct direct experiments by throwing five times under varying conditions, then measuring the elevation angle, horizontal distance, and travel time of the ball. The results of the observations are analyzed to find patterns of relationships between variables. This activity demonstrates the application of the principle of guided reinvention, where students gradually reconstruct the concept of a parabola from empirical experience (Gravemeijer, 2004). This experimental activity also fosters conceptual flexibility, namely the ability to understand the relationship between parameters in the context of parabolic motion (Rittle-Johnson et al., 2017).

In the pilot experiment stage, students were given the freedom to measure the distance between the thrower and the point where the ball fell, so that there were students who measured with fathoms, meters and steps that they had previously measured 1 fathom and 1 step represented how many meters. However, this has limitations in terms of consistency and accuracy. Based on the analysis of the results of the pilot experiment, at the teaching experiment stage, changes were made by providing additional measuring tools, namely a rope that is 3 meters long with knots for each meter, as shown in Figure 3 and Figure 4.

Figure 3. Students in the pilot phase of the experiment measure distance

Figure 4. Students at the teaching stage Experiment measuring distance

c. Parabola Graphical Representation Using Desmos

In this stage, students use the Desmos application to model the trajectory of the ball into a quadratic function graph. Students manipulate the parameters a, b, and c in a form $y = ax^2 + bx + c$ to match the graph to the data obtained from the video. This activity develops representational flexibility , which is the ability to connect and move between mathematical representations from visual (video), numeric (data), graphical (Desmos), to symbolic (equations) (Lesh et al., 2020). This design also supports the development of visual thinking, which is important in understanding quadratic functions (Oktaviyanthi & Agus, 2023).

In the pilot experiment stage, the activity sheets given were still in English so that many prospective teacher students did not understand the purpose of the activity. So that in the teaching experiment stage, all activity sheets were made into Indonesian, both on the activity sheets and in the Desmos application. In addition, in the pilot experiment stage, prospective teacher students worked on the activities individually, while in the teaching activity 3 stage, they worked in groups with the hope that a discussion process would occur to train students' flexibility skills, as shown in Figure 5 and Figure 6.

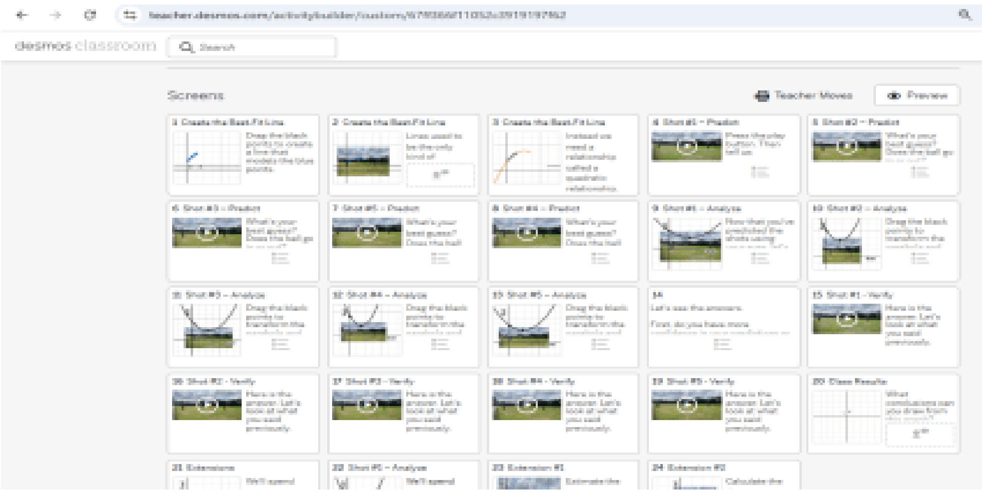


Figure 5. Initial view of Desmos in the Experimental Pilot stage

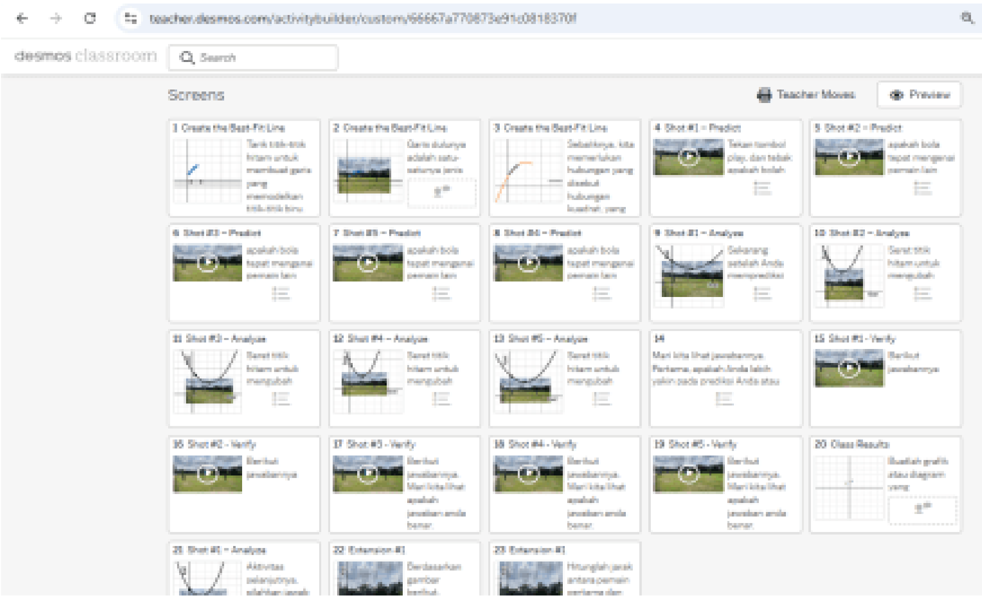


Figure 6. Initial view of Desmos at the Teaching Experiment stage

At the pilot experiment stage, there were several prospective teacher students who did not understand the intent or purpose of the questions so that students could not answer the questions given and when the teaching experiment stage was carried out, the language change was overcome so that students could complete activity 4 well. This can be seen in the Figure 5 and Figure 6 for example, for question no. 1 at the pilot experiment stage, more students did not answer compared to the teaching experiment stage, as shown in Figure 7 and Figure 8.

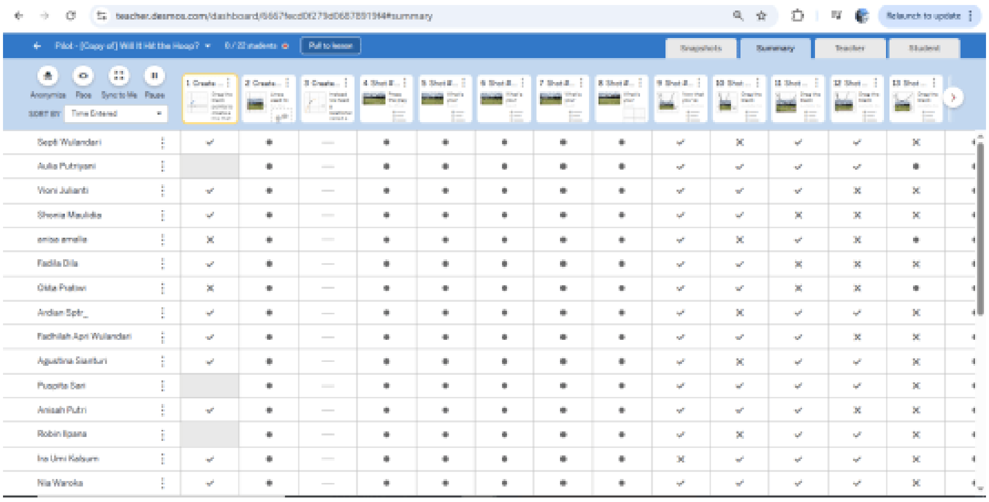


Figure 7. Student Answers at the Pilot Experiment Stage

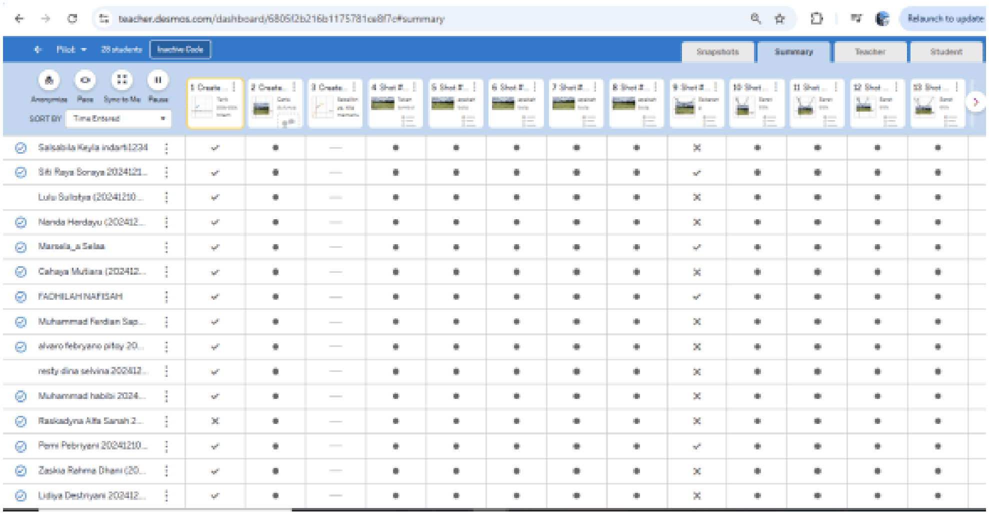


Figure 8. Student Answers at the Teaching Experiment Stage

d. Determining the Parabola Equation

In this activity, students construct a parabola equation based on three points of the observed ball trajectory, then verify the graph using Desmos. This activity strengthens procedural flexibility, where students are able to choose the right procedure, and integrate conceptual understanding to explain and test the results. The use of interactive visual applications in solution verification has also been shown to support improved reasoning and conceptual understanding (Lee et al., 2021).

In this activity 4, the aim is for students to be able to predict the parabola equation from a known parabola graph. Students will discuss with their groups to determine the parabola equation from the given ball throw image after which it will be proven using the Desmos application. In the pilot experiment stage in question 2, no image was given again so that when the proof was not seen whether the parabola equation formed was correct. The following are students' answers at the pilot experiment stage, as shown in Figure 9.

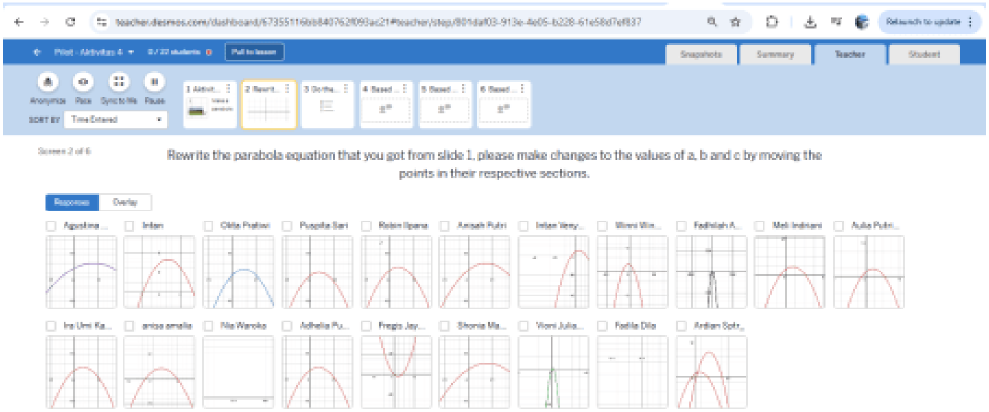


Figure 9. Student responses at the pilot experiment stage

Based on the student's answer above, it is not apparent that the parabola equation obtained from the calculation is a parabola equation that matches the given ball trajectory. So that at the teaching experiment stage, the image in the second activity was added with a throw-in photo so that when the student teacher proves it, they will see a parabola graph formed from the parabola equation covering the existing ball trajectory, as shown in Figure 10.

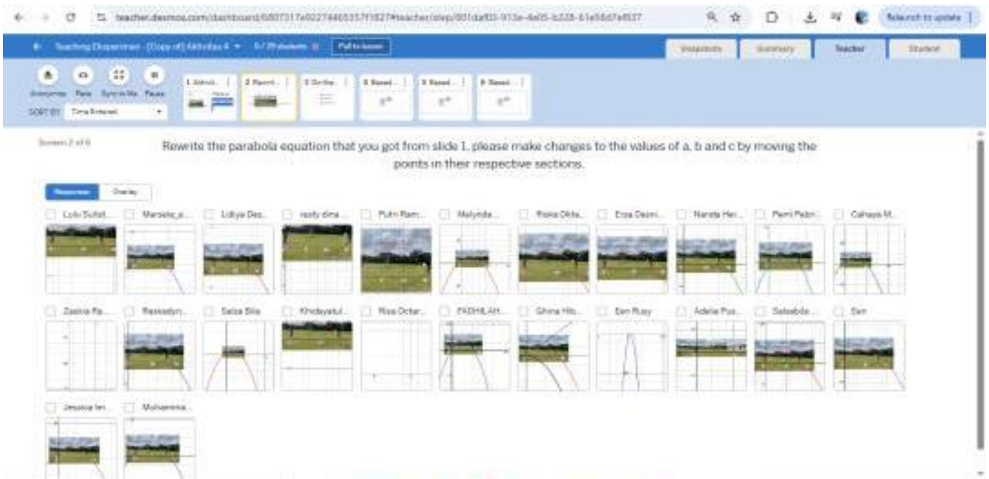


Figure 10. Student Answers at the Teaching Experiment Stage

The improvements designed and implemented at the teaching experiment stage aim to improve the understanding of prospective teacher students regarding the parabolic equation material and students' flexibility abilities. The first improvement made was in the language, in the pilot experiment in Desmos activities using English, but it was an obstacle for prospective teacher students to understand activities 4 and 5. After improvements were made at the teaching experiment stage, prospective mathematics teacher students were able to complete it well. The second improvement was made to the throwing practice carried out by prospective teacher students to find factors that affect the distance of a throw. Changes in the measuring instruments used in the field by students at the teaching experiment stage so that the measurements obtained were more precise.

The third improvement was made in the process of working on activities 3 and 4, which at the pilot teaching stage, students worked individually so that there was no discussion among prospective teacher students, resulting in difficulties in finding solutions to the problems given in activities 3 and 4. At the experimental teaching stage, activities up to activity four were carried out in groups so that students' thinking skills, especially flexibility, became more trained and developed. The improvement that occurred at the teaching experiment stage can be seen from the students' flexibility abilities which have become more developed. This can be seen from the many ways that students use to obtain solutions to the problems given.

3. Discussion

The learning trajectory design developed in this study demonstrates effective integration between real-world contexts (Arhan's throw-in) and STEM-based learning principles to build students' mathematical thinking flexibility. Based on the results of the pilot and teaching experiments, the four activities in HLT from video analysis, elevation angle experiments, parabola graphic representation with Desmos, to compiling parabola equations successfully facilitated a shift in students' understanding from concrete experiences to formal representations.

Each activity contains essential elements of the STEM component: students apply the principles of parabolic motion (Science), use technology applications such as Desmos (Technology), analyze the effectiveness of throwing techniques (Engineering), and build quadratic equation models from contextual situations (Mathematics). This approach supports Bybee (2013) and English (2016)'s view that integrated STEM learning strengthens critical thinking, collaborative, and problem-solving skills. Specifically, the design of this activity is able to develop representational, conceptual, and procedural flexibility of student teachers. In activities 1 and 2, students begin to move from visual observation to conceptual understanding and experimental data collection, in line with the principle of guided reinvention (Gravemeijer, 2004). In activities 3 and 4, the use of Desmos facilitates students to move between representations (video, graphics, symbolic), build mathematical models, and evaluate their accuracy. This indicates the development of representational and conceptual flexibility (Lesh et al., 2020; Star & Rittle-Johnson, 2008).

Flexibility ability in the context of mathematics learning reflects students' ability to move between representations, apply different strategies, and adapt approaches according to the context of the problem at hand (Rittle-Johnson et al., 2017). In this design context, the integration of real-world contexts allows students to experience multiple forms of representation visual, verbal, numeric, and symbolic which supports the development of representational flexibility. Then, through exploration of elevation angle and throwing distance in experimental activities, students begin to build an understanding of the relationships between concepts, which strengthens conceptual flexibility.

Support for procedural flexibility emerged in the final activity when students determined the equation of a parabola from three points and tested it using interactive visual technology. This process demonstrated that students not only followed algorithmic procedures, but also made adjustments based on conceptual understanding and the visual context observed. Adaptive expertise theory Hatano & Inagaki (1984) explained that flexible learners are not only able to solve problems efficiently, but also innovate and adapt strategies in new situations which is seen in student learning outcomes. In addition, Realistic Mathematics Education (RME) as proposed by Freudenthal (1991) also supports the idea that learning that starts from real contexts and moves towards mathematical formality can strengthen the connectedness of concepts and strategies used by students. In this context, video-based activities and physical experiments form a bridge between concrete experiences and more abstract mathematical understandings, creating space for flexible thinking to develop naturally.

The results at the teaching experiment stage showed significant improvements compared to the pilot experiment stage, both in terms of student participation, quality of discussion, and variations in strategies used. Improvements in language formats, measuring instruments, and forms of group work also increase the effectiveness of learning. This increase is in line with the findings of Rittle-Johnson et al. (2017) who emphasized that engagement in meaningful contextual and representational activities directly supports thinking flexibility. Thus, the designed learning trajectory not only succeeds in contextualizing the parabola material, but also broadens the scope of students' abilities in understanding and modelling mathematical concepts flexibly and applicatively. This confirms that authentic context-based STEM approaches such as football throw-ins contribute significantly to strengthening pre-service teacher student mathematical *flexibility* (Queiruga-Dios et al., 2025).

D. CONCLUSION AND SUGGESTIONS

This study successfully developed a STEM-based learning trajectory using a soccer throw-in context to enhance mathematical flexibility among prospective mathematics teachers. The learning design proved effective in promoting three core aspects of flexibility: (1) conceptual understanding of parabolas, (2) ability to shift between representations, and (3) strategic adaptability in selecting appropriate problem-solving procedures. Key findings demonstrate that structured integration of real-world contexts, physical exploration, and digital tools such as Desmos can meaningfully support the transition from informal reasoning to formal mathematical modeling. The trajectory's phased structure fostered deep engagement and encouraged students to view mathematical problems through an interdisciplinary and applied lens.

Practically, this research highlights the potential of sports-based contexts as an engaging and pedagogically rich medium for mathematics instruction. It also affirms the value of technology-enhanced and context-oriented HLT design in developing flexible mathematical thinking, especially for teacher preparation programs. Future studies are encouraged to adapt and scale this trajectory across other mathematical topics, explore its impact on different student populations, and integrate additional technologies (e.g., augmented reality) to further enrich representational learning and collaborative reasoning in STEM-based environments.

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