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Synchonized DAMRI Public Tourist Transportation Route Design using Max-Plus Algebra

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ABSTRACT

The transportation system is a crucial infrastructure for supporting connectivity between each National Strategic Tourist Area (NSTA) tourist destination of Yogyakarta. Management of transportation network and scheduling departure of transportation services are not yet optimal. This study aims to design a synchronized DPTT route and schedule that covers the entire service area using max-plus algebra. This type of study is applied research. The scheduling problem will focus on determining the number of fleets serving all routes with departure periods below 60 minutes. This research was conducted through literature review, field observations and online maps using Google Maps, and computation using the *Scilab* program. The results show that time travel between 8 tourist destinations are organized into a strongly connected directed graph with 20 routes. Departures are modeled as a linear discrete-event system over max-plus algebra. Computation in Scilab produce a baseline departure period of 90 minutes. We evaluate by adding 2, 8, 10, or 12 of buses by reinforcing the longest route. The simulation shows that the addition of 10 and 12 buses in certain section can reduce the departure period to 56 and 48 minutes respectively. The results demonstrate that targeted fleet additions and network reconnection, guided by max-plus synchronization, can substantially improve service regularity and passenger connectivity.

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A. INTRODUCTION

National Strategic Tourism Area (NSTA) of Indonesia is an area that has the main function of tourism or has the potential for national tourism development that has an important influence in one or more aspects, such as economic, social and cultural growth, empowerment of natural resources, environmental carrying capacity, and defense and security. NSTA is regulated in the national tourism development master plan of Indonesia. NSTA development strategies are carried out by the government to improve the quality of national tourism, such as supporting infrastructure (Widiarsih et al., 2018), building tourism ecosystem (Muhamad et al., 2024), and also participating of local communities (Wahyudi et al., 2023). Furthermore, the development of transportation infrastructure continues to be optimized to enhance accessibility to tourism site (Sinaga & Kusumawardahni, 2024).

Transportation services are one of the supporting factors in the development of NSTA (Putra et al., 2025). The public transportation system must be managed to reach all tourist attraction in NSTA. The government has implemented various strategies in managing the public transportation system to provide tourist satisfaction and economic benefits to the community (Fiardi et al., 2025). They include vehicle types, routes, scheduling, and affordability. The transportation service provider for NSTA is DAMRI Public Tourism Transportation (DPTT), a state-owned enterprise of Indonesia.

Yogyakarta is one of the Indonesian provinces with numerous NSTAs. The tourist area in MSTA Yogyakarta is developed through the provincial infrastructure development plan 2025-2034 (Ministry of Public Works and Public Housing, 2024). The DPTT connects tourist destinations in Yogyakarta, such as Yogyakarta International Airport (YIA), Imogiri Cemetery, Parangtritis Beach, Baron Beach, the Yogyakarta Zero Kilometer Point, the Breksi Cliffs Nature Tourism Area, the Prambanan Temple, and the Borobudur Temple. These tourist destinations are connected by five DPTT routes. Based on information from *DamriAps* and *Google Maps*, the network of DPTT routes does not optimal. This network represented as a disconnected graph. The graph consists of two connected components, YIA and Imogiri in one component and the rest in another components. This network causes some problem in affordability of tourist area. For instance, the DPTT cannot serve direct trips from YIA to Borobudur.

The DPTT scheduling synchronization is still a problem in the NSTA Yogyakarta transportation service system. Bus departure schedules on each route are irregular. Based on table schedule, some trips have a different number of departures from the origin point to a particular destination (*DamriApps*). This can lead to inefficiencies in service management. Passengers may also experience confusion if the schedule is not organized regularly.

Connectivity between tourist destinations and synchronized departures of DPTT are two key considerations in managing transportation services. This research will design a new DPTT route and then mathematically model it for scheduling synchronization. Max-Plus algebra is one of the tools that is widely used for scheduling problems (Dirza et al., 2019; Sagawa, 2020; Žužek, 2019). In addition, synchronization problems also often use max-plus algebra (Martinez, 2022; Riess, 2023). Max-plus Algebra is an idempotent semiring (Gonçalves, 2019; Heidergott et al., 2006; Watanabe, 2020). Max-plus algebra is also widely used in other fields such as economics (Mrugalski, 2023), supply chain (Gonçalves, 2021), internet network (Eramo, 2023) and also traffic regulation (Joelianto, 2020).

The problem of scheduling and synchronizing DPTT departure times in this study is categorized as a discrete event problem. Several studies have been conducted on modeling discrete events using max-plus algebra (Heidergott, 2000; Schutter, 2020). The synchronization process in this study is carried out by analyzing the values and eigenvectors of the max-plus system (Markkassery, 2024; Umer et al., 2020). Computation of values and eigenvectors of max-plus systems using the *Scilab* program (Kudryashova et al., 2020; Nagar, 2017; Subiono, 2015a).

The DPTT network currently exhibits disconnected graph with two components and uncoordinated departures, limiting direct trips and reliable transfer. Timetable synchronization in a classic challenge in public transportation, and max-plus algebra offers a compact framework to model event timings and propagate transfer constraints. This paper contributes a route design that connects key terminals into a strongly connected graph, max-plus synchronization model to compute schedule of departures, and making scenario analyses showing how targeted fleet additions improve network headway and connectivity.

B. METHODS

This research is applied research with the aim of modeling and simulating the results of the DPTT network modeling using Max-Plus Algebra. The stages in this research are as follows.

1. Literature Study

A literature study was conducted to identify problems in transportation networks. The literature on Max-Plus Algebra reviewed includes articles on network problems (Al Bermanei et al., 2024; Hoekstra, 2020; Krivulin, 1995), basic theories of max-plus algebra (Baccelli, 1992; Rudhito, 2016; Subiono, 2015a), and the application of algorithms related to eigenvalues and vectors to network problems.

2. Collecting Information

Various data were collected and processed into relevant information for the research. The collected data included the DPTT route network, DPTT departure schedules, DPTT travel times to each route, and the number of terminals. This data was collected through the *DAMRI Apps*, *MitraDarat Apps*, and *Google Maps*. The number of terminals is 8, equal to the number of tourist site in NSTA. The total route is 20 where differentiate in 5 lane The collected data was then processed to identify problems and provide supporting information for modeling the DPTT transportation network.

3. Design Route of Transportation Network

Connectivity and synchronization are the primary foundations for designing new routes. The designed routes ensure that all tourist destinations are connected by the DPTT route. Furthermore, synchronization of transportation arrivals and departures was also considered in the development of the Max-plus Algebra model.

4. Modelling with Max-plus Algebra

The connected DPTT network is modeled using Max-Plus Algebra. The system of equations in Max-Plus Algebra is the model to be built. The model building process begins with an analysis of the DPTT network structure, a graph representation of the DPTT network, and ends with the compilation of a system of equations in Max-Plus Algebra.

5. Analyzing and Simulating the Model

The established DPTT network model is analyzed for scheduling synchronization. Scheduling of departure is determined by computation assisted by the *Scilab* program version 5.5.2 with Toolboxes *MAXPLUSV04032016*. The results of the eigenvalue and vector computations are analyzed to determine the DPTT departure period. Several interventions will be implemented if the DPTT departure period is deemed too long (>60 minutes). Additional buses on certain routes will be implemented to reduce the period value. Interventions are considered optimal if the departure period is <60 minutes.

C. RESULT AND DISCUSSION

1. Modelling the Network of DPTT using Max-Plus Algebra

The graph representation of the DPTT origin network is a disconnected graph. Furthermore, the DPTT departure schedules for each route are not synchronized. This is the basis for this research to design a new route for the DPTT network. The new route was designed by adding several possible routes so that the representation graph becomes a strongly connected graph. The addition of routes takes into account the distance between the two terminals and the accessibility of direct access. The results of the new route design can be seen in Figure 1 below.

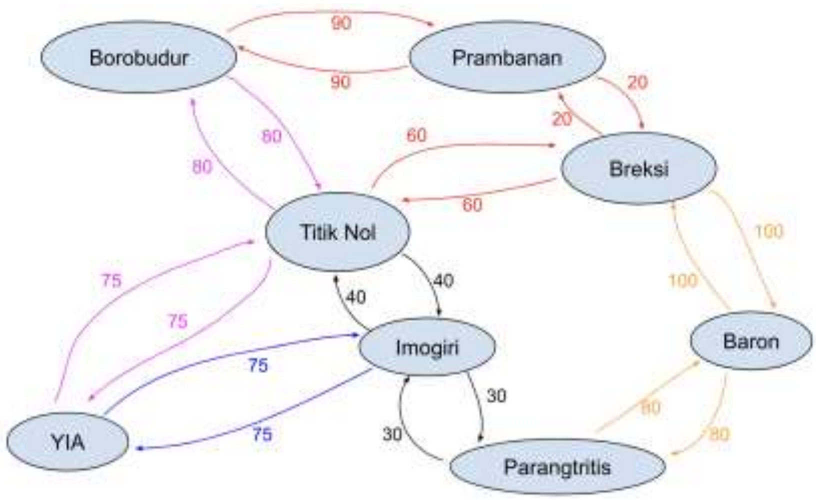

Figure 1. New Design of DPTT Transportation Network Graph

The transportation network representation graph in Figure 1 above is a strongly connected graph. Next, a synchronization process was performed for bus departures on each route, taking into account the travel duration for each route. Based on the network graph above, 20 variables were obtained that indicate synchronization between routes. The definitions of these variables can be seen in the following Table 1.

Table 1. Defining Route Variables for Synchronization

Lane	Variable	Route		Duration (minutes)
		Origin	Destination	
1	x_1	Titik Nol	Imogiri	40
1	x_2	Imogiri	Titik Nol	40
1	x_3	Imogiri	Parangtritis	30
1	x_4	Parangtritis	Imogiri	30
2	x_5	Imogiri	YIA	75
2	x_6	YIA	Imogiri	75
3	x_7	Titik Nol	YIA	75
3	x_8	YIA	Titik Nol	75
3	x_9	Titik Nol	Borobudur	80
3	x_{10}	Borobudur	Titik Nol	80
4	x_{11}	Baron	Parangtritis	80
4	x_{12}	Parangtritis	Baron	80
4	x_{13}	Baron	Breksi	100
4	x_{14}	Breksi	Baron	100

Lane	Variable	Route		Duration (minutes)
		Origin	Destination	
5	x_{15}	Titik Nol	Breksi	60
5	x_{16}	Breksi	Titik Nol	60
5	x_{17}	Breksi	Prambanan	20
5	x_{18}	Prambanan	Breksi	20
5	x_{19}	Prambanan	Borobudur	90
5	x_{20}	Borobudur	Prambanan	90

The information in Figure 1 about the network graph and Table 2 about the variables for synchronization are used to construct a DPTT network model with Max-Plus Algebra. Suppose $x_i(k)$ is departure time of the kth bus with the route in Table 4 above with $i \in \{1,2,\dots,20\}$. The resulting model is a system of linear equations over max-plus algebra. This model can be seen in equation (1).

$$\begin{cases} x_1(k+1) = x_2(k) \otimes 40 \oplus x_{10}(k) \otimes 80 \oplus x_{16}(k) \otimes 60 \\ x_2(k+1) = x_4(k) \otimes 30 \\ x_3(k+1) = x_1(k) \otimes 40 \oplus x_6(k) \otimes 75 \\ x_4(k+1) = x_3(k) \otimes 30 \oplus x_{11}(k) \otimes 80 \\ x_5(k+1) = x_4(k) \otimes 30 \oplus x_6(k) \otimes 75 \\ x_6(k+1) = x_5(k) \otimes 75 \\ x_7(k+1) = x_{10}(k) \otimes 80 \oplus x_{16}(k) \otimes 60 \\ x_8(k+1) = x_7(k) \otimes 75 \\ x_9(k+1) = x_2(k) \otimes 40 \oplus x_8(k) \otimes 75 \\ x_{10}(k+1) = x_9(k) \otimes 80 \\ x_{11}(k+1) = x_{14}(k) \otimes 100 \\ x_{12}(k+1) = x_3(k) \otimes 30 \oplus x_{11}(k) \otimes 80 \\ x_{13}(k+1) = x_{12}(k) \otimes 80 \\ x_{14}(k+1) = x_{13}(k) \otimes 100 \oplus x_{15}(k) \otimes 60 \oplus x_{18}(k) \otimes 20 \\ x_{15}(k+1) = x_2(k) \otimes 40 \oplus x_8(k) \otimes 75 \oplus x_{16}(k) \otimes 60 \\ x_{16}(k+1) = x_{13}(k) \otimes 100 \oplus x_{18}(k) \otimes 20 \\ x_{17}(k+1) = x_{13}(k) \otimes 100 \oplus x_{15}(k) \otimes 60 \\ x_{18}(k+1) = x_{20}(k) \otimes 90 \\ x_{19}(k+1) = x_{17}(k) \otimes 20 \\ x_{20}(k+1) = x_{19}(k) \otimes 90 \end{cases} \tag{1}$$

Equation (1) is transformed into a matrix equation with the form

$$[x_i(k+1)]_{i=1}^{20} = A_{20\times 20}[x_i(k+1)]_{i=1}^{20}$$

with matrix A as follows.

$$\begin{bmatrix}
\varepsilon & 40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 30 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 30 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 30 & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 100 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 30 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 80 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 100 & \varepsilon & 60 & \varepsilon & \varepsilon & 20 & \varepsilon & \varepsilon \\
\varepsilon & 40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 75 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 100 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 20 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 100 & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 90 \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 20 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 90 & \varepsilon
\end{bmatrix}$$

Computation assisted by *Scilab* program to determine the eigenvalues of matrix A and also the solution of the system of equations (1). Computation to determine the eigenvalues uses the *maxplusalgol* command. The computation results show that the eigenvalue of matrix A above is 90. This indicates that the departure period of each bus at each terminal is 90 minutes. One of the eigenvectors of matrix A is,

$$v = [50, 20, 0, 80, 20, 5, 50, 35, 20, 10, 90, 80, 70, 80, 50, 80, 80, 10, 10, 10]^T$$

Vector v is used as the initial vector to determine other solutions of system (1). The computation determines the solution using the *maxplussys* command in *Scilab*. These results are then used to compile the DPTT departure schedule at each terminal. The first departure starts at 04.30 and the last departure at the latest at 20.00. The DPTT departure schedule can be seen in Table 3 below.

Table 3. Synchronized DPTT Departure Schedule

Origin	Destination	1	2	3	4	5	6	7	8	9	10
Titik Nol	Imogiri	05:20	06:50	08:20	09:50	11:20	12:50	14:20	15:50	17:20	18:50
Imogiri	Titik Nol	04:50	06:20	07:50	09:20	10:50	12:20	13:50	15:20	16:50	18:20
Imogiri	Parangtritis	04:30	06:00	07:30	09:00	10:30	12:00	13:30	15:00	16:30	18:00
Parangtritis	Imogiri	05:50	07:20	08:50	10:20	11:50	13:20	14:50	16:20	17:50	19:20
Imogiri	YIA	04:50	06:20	07:50	09:20	10:50	12:20	13:50	15:20	16:50	18:20
YIA	Imogiri	04:35	06:05	07:35	09:05	10:35	12:05	13:35	15:05	16:35	18:05
Titik Nol	YIA	05:20	06:50	08:20	09:50	11:20	12:50	14:20	15:50	17:20	18:50
YIA	Titik Nol	05:05	06:35	08:05	09:35	11:05	12:35	14:05	15:35	17:05	18:35
Titik Nol	Borobudur	04:50	06:20	07:50	09:20	10:50	12:20	13:50	15:20	16:50	18:20
Borobudur	Titik Nol	04:40	06:10	07:40	09:10	10:40	12:10	13:40	15:10	16:40	18:10
Baron	Parangtritis	06:00	07:30	09:00	10:30	12:00	13:30	15:00	16:30	18:00	19:30
Parangtritis	Baron	05:50	07:20	08:50	10:20	11:50	13:20	14:50	16:20	17:50	19:20
Baron	Breksi	05:40	07:10	08:40	10:10	11:40	13:10	14:40	16:10	17:40	19:10
Breksi	Baron	05:50	07:20	08:50	10:20	11:50	13:20	14:50	16:20	17:50	19:20
Titik Nol	Breksi	05:20	06:50	08:20	09:50	11:20	12:50	14:20	15:50	17:20	18:50

Origin	Destination	1	2	3	4	5	6	7	8	9	10
Breksi	Titik Nol	05:50	07:20	08:50	10:20	11:50	13:20	14:50	16:20	17:50	19:20
Breksi	Prambanan	05:50	07:20	08:50	10:20	11:50	13:20	14:50	16:20	17:50	19:20
Prambanan	Breksi	04:40	06:10	07:40	09:10	10:40	12:10	13:40	15:10	16:40	18:10
Prambanan	Borobudur	04:40	06:10	07:40	09:10	10:40	12:10	13:40	15:10	16:40	18:10
Borobudur	Prambanan	04:40	06:10	07:40	09:10	10:40	12:10	13:40	15:10	16:40	18:10

Table 5 above shows that there are 10 departures per day on each DPTT route. The first departure is at 4:30 a.m. on the Imogiri-Parangtritis route. The last departure is at 7:30 p.m. on the Baron-Parangtritis route.

2. Optimizing Departure Periods by Adding Buses on Certain Routes

The DPTT departure schedule in Table 3 is synchronized. The departure period is 90 minutes. Some routes take less time, such as Imogiri-Parangtritis, which only takes 30 minutes. Therefore, shorter departure periods are needed. This improves DPTT service on each route. The addition of buses to certain routes is done to optimize the departure period. In this study, four simulations of adding buses to specific routes were conducted. The addition of buses to specific routes will impact the addition of variables to the model. These variables are known as auxiliary state variables.

a. Additional 2 Buses

Buses were added to routes with a travel time of 100 minutes, the Baron-Breksi and Breksi-Baron routes. The auxiliary state variables for this condition are x_{21} and x_{22}. The max-plus model of this condition, namely the system of equations (1) is added by two with 2 equations as follows.

$$x_{21}(k+1) = x_{13}(k-1) \otimes 0$$
$$x_{22}(k+1) = x_{14}(k-1) \otimes 0$$

The computational results show that the eigenvalue of this max-plus system is 77.5. It can be said that adding one bus to the Baron-Breksi and Breksi-Baron routes each results in a departure period of 77.5 minutes for each bus at each terminal. One of the eigenvectors of this system is

$$v = [150, 27.5, 112.5, 75, 27.5, 25, 150, 147.5, 145, 147.5, 72.5, 75, 77.5, 127.5, 145, 37.5, 127.5, 95, 70, 82.5, 0, 50]^T$$

The results of the scheduling computation using the initial value of vector v show that there are 11 departures per day. The last departure is at 7:55 PM on the Titik Nol-Imogiri route.

b. Additional 8 Buses

Routes with a minimum travel duration of 80 minutes are added with 1 bus each. Based on Table 1, there are 8 routes with this condition. System (1) is added with equations involving 8 auxiliary state variables $x_{21}, x_{22}, \ldots, x_{28}$. System (1) is added the following equations

$$x_{21}(k+1) = x_9(k-1) \otimes 0$$
$$x_{22}(k+1) = x_{10}(k-1) \otimes 0$$
$$x_{23}(k+1) = x_{11}(k-1) \otimes 0$$
$$x_{24}(k+1) = x_{12}(k-1) \otimes 0$$
$$x_{25}(k+1) = x_{13}(k-1) \otimes 0$$
$$x_{26}(k+1) = x_{14}(k-1) \otimes 0$$
$$x_{27}(k+1) = x_{19}(k-1) \otimes 0$$
$$x_{28}(k+1) = x_{20}(k-1) \otimes 0$$

Through computation, the eigenvalue of this system is 75. The addition of six buses from condition (a) does not significantly reduce the periodicity. In fact, this condition causes a decrease in the number of bus departures per day. Under these conditions, buses on each route can only depart nine times per day.

c. Additional 10 Buses

Routes with a minimum travel duration of 75 minutes are added with one bus each. There are 12 routes that meet this requirement. Routes with a travel duration of 90 minutes still only have one bus serving them. In this condition, 10 buses are added with the provisions for the routes in Table 1. Analogous to conditions (a) and (b), 10 auxiliary state variables are required to obtain the following additional equations.

$$x_{22}(k+1) = x_6(k-1) \otimes 0$$
$$x_{23}(k+1) = x_7(k-1) \otimes 0$$
$$x_{24}(k+1) = x_8(k-1) \otimes 0$$
$$x_{25}(k+1) = x_9(k-1) \otimes 0$$
$$x_{26}(k+1) = x_{10}(k-1) \otimes 0$$
$$x_{27}(k+1) = x_{11}(k-1) \otimes 0$$
$$x_{28}(k+1) = x_{12}(k-1) \otimes 0$$
$$x_{29}(k+1) = x_{13}(k-1) \otimes 0$$
$$x_{30}(k+1) = x_{14}(k-1) \otimes 0$$

The eigenvalue computation is 56. This indicates a significant decrease in periodicity from conditions (a) and (b). The scheduling results show that each bus on each route can depart up to 13 times a day.

d. Additional 12 Buses

All routes with a duration of more than 60 minutes involve 2 buses in service. Based on Table 1, there are 12 routes that meet these conditions. Twelve auxiliary state variables are added to the model in system (1). The added equations are as follows.

$$x_{21}(k+1) = x_5(k-1) \otimes 0$$
$$x_{22}(k+1) = x_6(k-1) \otimes 0$$
$$x_{23}(k+1) = x_7(k-1) \otimes 0$$
$$x_{24}(k+1) = x_8(k-1) \otimes 0$$
$$x_{25}(k+1) = x_9(k-1) \otimes 0$$

$$x_{26}(k+1) = x_{10}(k-1) \otimes 0$$
$$x_{27}(k+1) = x_{11}(k-1) \otimes 0$$
$$x_{28}(k+1) = x_{12}(k-1) \otimes 0$$
$$x_{29}(k+1) = x_{13}(k-1) \otimes 0$$
$$x_{30}(k+1) = x_{14}(k-1) \otimes 0$$
$$x_{31}(k+1) = x_{19}(k-1) \otimes 0$$
$$x_{32}(k+1) = x_{20}(k-1) \otimes 0$$

The eigenvalue of this system is 48. This means that the DPTT departure periodicity on each route is 48 minutes. This is quite ideal when viewed based on the transportation network in Figure 3. The computational results also show that each route can serve up to 18 departures per day.

The max-plus algebraic model of the DPTT transportation network yields synchronized bus departure schedules at each terminal. Synchronized network management makes transportation management more efficient. This aligns with Dirza et al.(2019) and Sagawa (2020) research, which demonstrated that network synchronization and control through a max-plus algebraic model yields more optimal results in departure scheduling. Furthermore, simulation results show that adding buses to certain routes can reduce system periodicity. Reduced system periodicity results in shorter passenger wait times. Simulation results indicate that adding buses to routes with the longest travel times significantly reduces periodicity. This also aligns with Subiono (2015).

D. CONCLUSION AND SUGGESTIONS

This study has designed a new DPTT route connected to each tourist destination. Using a max-plus algebraic model, a synchronized bus departure schedule was obtained. Analyzing on the eigen value of matrices over max-plus algebra give information about the departure periodicity. The model solution also shows that the DPTT can serve departures on each route up to 10 times a day. Adding buses to routes with long travel durations is an alternative to optimize departure periodicity. Several simulations have been done by adding various number of buses in certain route. Adding 12 buses gives optimal solution to reduce periodicity. The number of DPTT departure services per day can reach 18 times. Further research can develop routes that need additional buses to optimize both vehicle efficiency and departure periodicity. Furthermore, an analysis of the impact of departure delays on the DPTT network system can also be further investigated.

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