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Process of Proportional Reasoning Students' Errors in Solving Mathematical Problems

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ABSTRACT

Proportional reasoning plays a crucial role in mathematical reasoning, yet many students struggle to coordinate multiplicative relationships when solving mathematical problems. This study aimed to examine the processes behind students' errors in proportional reasoning and to describe the types of incorrect strategies they used when working through a contextual joint-work problem. Using a qualitative exploratory descriptive design, data were collected from students' written solutions, think-aloud explanations, and interview responses to capture their reasoning processes in depth. The participants were 15 first-semester students from the Mathematics Education Department, Universitas Islam Negeri (UIN) Mataram. Results showed four major categories of incorrect reasoning: intuitive reasoning based on misleading but salient information, additive reasoning that relied on differences rather than multiplicative structures, proportion attempts that identified proportional cues but applied them incorrectly, and other incomplete or unsupported strategies. Additive reasoning emerged as the most dominant pattern across students of varying proficiency, indicating a strong tendency to default to non-proportional interpretations even when the situation required multiplicative thinking. Although some students recognized structural features such as periodic assistance, they struggled to coordinate unit work or rates, leading to systematically flawed conclusions. These findings suggest that students' proportional reasoning errors stem from entrenched intuitive and additive tendencies. The study highlights the importance of instructional approaches that explicitly develop unit-rate reasoning, strengthen multiplicative understanding, and support accurate representation of proportional situations.

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A. INTRODUCTION

Reasoning is a foundational competency in mathematics that supports systematic, logical, and analytical thinking and thereby undergirds problem solving and decision-making across academic and everyday contexts. Within mathematics education, reasoning links cognition and intellect to enable learners to formulate claims, evaluate information, and draw warranted conclusions, making it central to higher-order thinking and to students' long-term disciplinary development (Rogers & Steele, 2016). Classroom studies show that when instruction foregrounds reasoning and, relatedly, justification and proving students' opportunities to construct and refine mathematical ideas expand, and teachers' enactments can either amplify or constrain these opportunities (Mata-Pereira & da Ponte, 2017). Experimental and design-based research further indicates that engaging students in creative mathematical reasoning can deepen conceptual understanding beyond what is achieved through imitative procedures alone, elucidating the distinctive contribution of reasoning to durable learning (Jonsson et al., 2020). At the same time, reasoning is intimately tied to mathematical communication; difficulties in articulating ideas verbally or in writing often signal gaps in the underlying reasoning processes that support problem solving (Maulyda, Annizar, et al., 2020). Collectively, this evidence situates reasoning not merely as an instructional goal but as a core epistemic practice of school mathematics (Maulyda, Annizar, et al., 2020).

A particularly consequential form of mathematical reasoning is proportional reasoning the capacity to understand and operate with multiplicative relationships in ratio and proportion situations. Proportional reasoning is indispensable for interpreting rational number structures and for developing coherence across topics such as rates, scaling, similarity, and linear relationships (Brahier, 2016). It supports flexible comparison, prediction, and decision-making by enabling learners to switch units, coordinate covarying quantities, and recognize invariant ratios (Brahier, 2016). Its relevance extends beyond mathematics to science, geography, and technology-rich fields, where interpreting data, rates of change, and multiplicative relations is routine (e.g., the analysis of climate anomalies and extreme events relies on quantitative reasoning with ratios and departures from baseline conditions) (Dole et al., 2011), and where modular, scalable engineering systems demand robust quantitative modeling and inference (Bonte et al., 2017). In teacher education, practice-based interventions have documented gains in prospective teachers' proportional reasoning, underscoring the teachability and malleability of this competence when instruction is explicitly oriented to multiplicative structures (Pişkin Tunç & Çakıroğlu, 2022). Moreover, systematic reviews of interventions for students with learning disabilities and persistent mathematics difficulties show that well-designed proportional reasoning supports can improve student outcomes, while also highlighting the complexity and heterogeneity of learner needs in this domain (Nelson et al., 2022).

Despite its centrality, proportional reasoning is demonstrably challenging for many learners. Classic and contemporary studies document persistent misconceptions, including overreliance on additive strategies in multiplicative contexts, difficulties coordinating units and unit rates, and challenges with the order and equivalence of rational numbers (Arican, 2019). Preservice teachers themselves often struggle to differentiate proportional from nonproportional relationships, a distinction that is critical for appropriate model selection and problem representation (Arican, 2019), and they may show inconsistent understanding of inverse proportionality, which requires recognizing that the product of quantities remains invariant as one increases and the other decreases (Cabero et al., 2020). Teachers' knowledge of unitizing the ability to conceive and reconceive composite units is especially important for interpreting students' reasoning and for making productive instructional decisions, yet it is frequently underdeveloped among novices (Buforn et al., 2022). Furthermore, research on teachers' epistemic analysis of proportionality tasks suggests that even mathematically strong candidates may not fully appreciate the range of solution paths, inferential warrants, and justifications that proportional situations invite (Burgos & Godino, 2022). These reasoning difficulties are often compounded by communication barriers in problem contexts rich in language or multiple representations, reinforcing the need to integrate attention to reasoning and representation with attention to mathematical discourse practices (Burgos & Godino, 2022).

The literature also highlights instructional and professional learning designs that can foster proportional reasoning by deliberately cultivating reasoning-and-proving practices. Studies point to specific teacher moves such as pressing for generalization, eliciting and comparing justifications, and orchestrating discussion around structure that create opportunities for students to construct multiplicative relationships and to warrant claims about invariance (Weiland et al., 2021). In teacher education, practice-based approaches that situate proportional reasoning within authentic instructional routines (e.g., rehearsals, approximations of practice) have been shown to strengthen both content knowledge and pedagogical decision-making related to ratio and rate (Cabero et al., 2020; Pişkin Tunç & Çakıroğlu, 2022) Concurrently, methodological advances such as topic modeling have opened promising avenues for investigating teachers' knowledge at scale, enabling researchers to trace patterns in how teachers conceptualize and communicate about key topics including proportionality across large corpora of responses (Copur-Gencturk et al., 2023). This body of work converges on the view that proportional reasoning development is not simply a matter of exposure to tasks but depends on the quality of teacher facilitation, the explicit surfacing of multiplicative structures, and the cultivation of classroom norms for explanation and justification (Copur-Gencturk et al., 2023).

A persistent challenge concerns how best to measure and track students' proportional reasoning in ways that are sensitive to both conceptual understanding and common misconceptions. Assessment innovations, such as two-tier instruments that combine selected responses with justification prompts, offer a means to capture not only whether students can produce correct answers but also how they reason thereby providing more diagnostic information for instruction (AÇIKGÜL, 2021). Complementary assessments of teachers' epistemic analysis competence on proportionality tasks reveal the extent to which future teachers can anticipate solution strategies, evaluate arguments, and align tasks with learning goals capabilities that mediate the translation of assessment evidence into pedagogical action (Burgos & Godino, 2022). Evidence that preservice teachers have difficulty distinguishing proportional from nonproportional situations underscores the importance of assessments that foreground structural features and invite explanation rather than mere calculation (Arican, 2019). Such instruments, when coupled with analyses of classroom discourse and task enactment, can illuminate the interplay between students' reasoning processes and teachers' instructional moves (AÇIKGÜL, 2021).

Prior research has often focused on identifying error types rather than unpacking the reasoning processes that generate them, leaving insufficient understanding of why intuitive and additive tendencies persist even among higher-proficiency students. This study contributes by providing fine-grained, process-level evidence drawn from written work, think-alouds, and interviews showing not only the presence but the mechanisms of intuitive, additive, and misapplied-proportional reasoning. It further demonstrates that students' difficulty lies not merely in computational skill but in representing and coordinating units, rates, and multiplicative structures within a realistic joint-work context. By clarifying these mechanisms, the study offers an urgently needed foundation for designing instructional interventions that directly target entrenched non-proportional reasoning habits and strengthen students' abilities to construct accurate proportional representation.

B. METHODS

This study employed an exploratory descriptive design with a qualitative approach, as the primary data consisted of verbal expressions and students' written work in solving mathematical problems. A qualitative approach was selected to explore students' proportional reasoning processes in depth, allowing for a detailed examination of their strategies and misconceptions. Such an approach is appropriate when the goal is to uncover patterns of thought and reasoning rather than to measure outcomes quantitatively (Alma et al., 2025; Nisa et al., 2022).

The participants were 15 first-semester students from the Mathematics Education Department, Universitas Islam Negeri (UIN) Mataram. They were chosen to represent varying levels of mathematical ability categorized as sufficient, medium, and high based on prior academic performance. Students were asked to solve a contextual mathematical problem involving proportional reasoning: determining the time required for a worker to complete a task with intermittent assistance from others. This problem was designed to elicit reasoning strategies commonly associated with proportional thinking, which previous studies have shown to be prone to errors such as intuitive and additive reasoning (Maharani & Murtiyasa, 2023; Maulyda, Sukoriyanto, et al., 2020).

Data collection involved administering problem-solving test sheets, followed by think-aloud protocols and semi-structured interviews to capture students' cognitive processes. The researcher served as the primary instrument, consistent with qualitative research principles. Data analysis was conducted through an iterative process comprising data reduction, categorization based on four indicators of incorrect reasoning (Intuitive, Additive, Proposition Attempt, and other errors), synthesis, and formulation of interpretative insights. Analysis continued until data saturation was achieved, indicated by the absence of new information. This systematic approach ensured the reliability and depth of the findings, aligning with established qualitative research practices (Sabat et al., 2021).

C. RESULTS AND DISCUSSION

Students' incorrect proportional reasoning was coded into four strategy categories adapted from the study's a priori framework: (a) intuitive using salient but inappropriate information to decide relationships between quantities and failing to determine equivalence correctly; (b) additive focusing on differences (or other additive cues) instead of multiplicative relations; (c) proportion attempt articulating an intended proportional relation yet failing to map quantities or operations correctly; and (d) other errors miscellaneous or metacognitive lapses not captured by the previous codes (e.g., incomplete attempts, unsubstantiated guesses, or unproductive strategies). These categories align with well-documented difficulties in distinguishing proportional from non-proportional structures and with the predominance of additive schemas where multiplicative thinking is required (Callingham & Siemon, 2021).

1. Subject 1 (S1; Low Mathematical Proficiency): Dominant Intuitive Reasoning with Additive Intrusions

S1 treated the comparison problem as a "significant fraction" task and proceeded by subtracting a maximum number of days from a self-determined "own workdays," then dividing by "shared workdays." The transcribed explanation "I subtracted the maximum number of days from my own workdays, then divided the result by my shared workdays" shows that S1 did not anchor operations in the provided constraints and misinterpreted the relationships among the three workers' contributions. The written work also used an addition step (e.g., "29 + 2 = 31 days") disconnected from a multiplicative model of shared productivity.

Coding: S1's performance was primarily intuitive (misuse of salient but irrelevant information, failure to test equivalence) with additive intrusions (using addition/subtraction as if they preserved the proportional structure). This profile is consistent with research showing that learners often privilege additive cues in multiplicative contexts and conflate comparison-of-quantities tasks with arbitrary arithmetic operations (Callingham & Siemon, 2021; Sari et al., 2024). It also resonates with findings that fraction or part–whole intuitions can bias reasoning about discrete proportional situations (Abreu-Mendoza et al., 2023).

2. Subject 2 (S2; Average Proficiency): Pattern Construction with Additive Focus and Partial Proportional Schema

S2 attempted to construct a pattern ("help every three days") to compute how often Adi and Beta assist, deriving "6" and "9" assistance days, respectively. However, S2 then added these assistance counts to obtain "9 days working together" and subtracted that from 60 to claim "51 days," incorrectly concluding that more help still yields a near-maximal total duration. The interview showed that S2 recognized the periodic structure but failed to coordinate it with the multiplicative impact of joint work on overall time; logically, periodic help should decrease the completion time.

Coding: S2's work was coded as additive (summing assistance counts and subtracting from a fixed 60-day baseline) with a proportion attempt (identifying a periodic proportional relation but failing to map it to the aggregate rate/total work correctly). This mixed profile mirrors prior reports that students can name proportional features while persisting with additive computations especially in discretized, pattern-based contexts (Abreu-Mendoza et al., 2023; Nugraha et al., 2023). The result illustrates a common "across-the-table" or tallying heuristic rather than coordinating rates or unit work (Pelen, 2025).

3. Subject 3 (S3; High Proficiency): Confident but Misplaced Additive Coordination

S3 described "adding A + B + C working together" and then subtracting large day counts (e.g., "60 – 30 = 30 days"), concluding "10 days faster" from the "every three days" condition and finally asserting "12 days" as shown in Figure 1 and Figure 2. The procedure aggregates and subtracts day totals without representing rate or unit work, indicating a misplaced focus on addends rather than multiplicative composition of productivity. The interview corroborates that S3 partitioned the work into "own" versus "assisted" days but coordinated them arithmetically rather than proportionally.

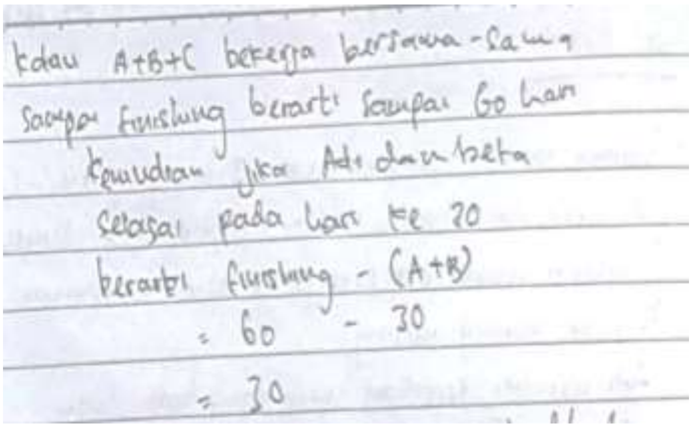

Figure 1. Results of the work of subject S3 on incorrect reasoning

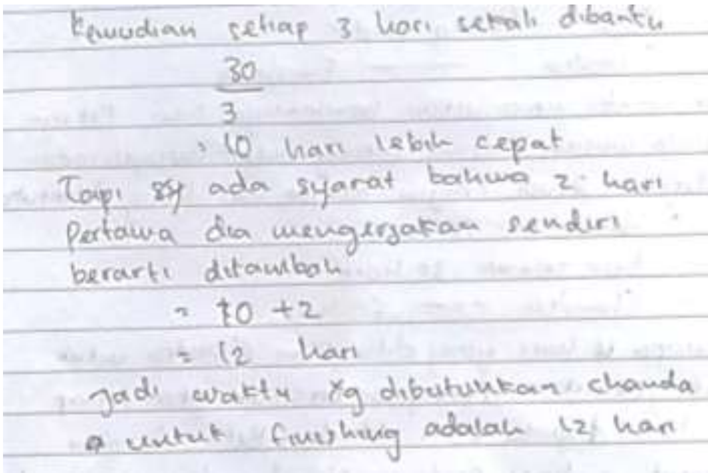

Figure 2. Results of subject S3's work on incorrect reasoning at the additive stage

Coding: S3's reasoning was categorized as additive. The confident articulation of steps, while structurally misaligned, is typical of learners who have rich procedural resources but apply them to the wrong quantitative structure (Callingham & Siemon, 2021). This pattern underscores how more advanced students can stably persist in non-proportional schemas even when they fluently explain their approach (Izzatin et al., 2021; McMillan, 2025). Across the three cases, additive reasoning exerted a strong pull: S1 mixed arbitrary addition/subtraction with misread information; S2 constructed a periodic pattern yet collapsed back to summation; S3 systematically coordinated totals additively. Only S2 exhibited a clear proportion attempt identifying structure (help every three days) but mis-mapping it to completion time. Intuitive reasoning dominated S1's entry and decision stages (misidentifying relevant givens). No unambiguous instance of other errors (as purely metacognitive or non-engagement) emerged; the observed errors were strategic rather than non-committal.

These results replicate and extend evidence that distinguishing proportional from non-proportional situations is a central hurdle, that additive schemas are overgeneralized, and that discretized contexts (e.g., "every third day help") heighten bias toward counting rather than reasoning with rates or unit fractions of work (Abreu-Mendoza et al., 2023; Nugraha et al., 2023). They reinforce the documented linkage between multiplicative thinking and successful proportional reasoning (Callingham & Siemon, 2021), and they echo reports that students' productive language of proportion (e.g., “every three days”) does not guarantee correct mapping to quantitative relations (Pelen, 2025).

The intuitive profile in S1 parallels findings that learners often lean on salient but inappropriate cues in discretized settings, leading to fraction-like or part whole biases that distort proportional judgments (Hurst et al., 2022). The additive dominance in S2 and S3 is emblematic of the persistent “additive trap,” where students treat completion time as a fixed baseline adjusted by adding/subtracting counts rather than integrating joint work rates (Callingham & Siemon, 2021). S2’s proportion attempt suggests partial schema activation recognizing regular assistance without coordinating the unit rate or unit work underpinning the global solution, a difficulty also noted in learners who describe proportional patterns yet compute with tallies (Pelen, 2025).

Beyond immediate task performance, the cases align with broader accounts of how dispositions and prior experiences shape non-routine proportional problem solving (Izzatin et al., 2021), and how even “high-ability” students may default to efficient but structurally inapt procedures when multiplicative connections are not made explicit (McMillan, 2025). In terms of reasoning processes, the observed strategies reveal limited metarepresentational monitoring (e.g., “more help ⇒ fewer days”) and difficulty in building or selecting representations that encode rates rather than counts, consistent with research linking spatial/numerical representations and exact/approximate calculation to early proportional competencies (Gunderson & Hildebrand, 2021).

The task’s “every third day” structure can be framed as a mixture of individual and joint work rates, conceptually akin to distinguishing between direct and inverse proportional relationships depending on how “days,” “work amount,” and “help events” are modeled. Literature on inverse proportional problems shows students often rely on informal strategies that prioritize addends or tallies over multiplicative invariants (Cabero et al., 2020). The present cases exhibit similar tendencies, underscoring the need to scaffold the choice of representation (e.g., unit work per day versus counts of help days) so that the multiplicative structure becomes salient (Callingham & Siemon, 2021).

Make the multiplicative structure explicit. Instruction should foreground unit-rate or unit-work formulations (e.g., “fraction of job completed per day”), then aggregate via multiplication, not addition. Authentic contexts (e.g., nutrition labels for added sugar interpreted through proportional comparisons) can help students connect rates to outcomes (Basu & Nguyen, 2021; Foley et al., 2023). Leverage schema-based instruction (SBI). SBI that teaches students to recognize problem schemas (ratio, rate, proportion) and map quantities to representations has shown benefits, including for students with mathematics difficulties, and can generalize across settings (Jitendra et al., 2021, 2022). The S2 case partial schema activation with additive computation illustrates where SBI can target representational mapping. Use iterative partitioning and structured representations. Iterative partitioning connects symbolic fractions to underlying proportional structure and can reduce fraction biases in discretized tasks (Abreu-Mendoza et al., 2023; Hurst et al., 2022). Visual/spatial scaffolds (number lines, bar models, double number lines) may strengthen alignment between representation and multiplicative reasoning (Gunderson & Hildebrand, 2021).

Support diagnostic classification. To move beyond global correctness, error codes like "intuitive," "additive," and "proportion attempt" can be linked to a Q-matrix of cognitive attributes for diagnostic assessment, enabling principled inferences about which subskills require support (de la Torre et al., 2022). Design for sustained reasoning and reflection. Structured collaborative or technology-mediated tasks (e.g., robotics or online group study) can cultivate logical and scientific reasoning that emphasizes coordination of representations and justifications skills directly relevant to coordinating rates and totals (Cheng et al., 2021; Shofiyah et al., 2024). Incorporating culturally meaningful problem contexts may further strengthen sense-making and transfer (Foley et al., 2023; Rodríguez-Nieto et al., 2025). Anticipate informal strategies in inverse or mixed-rate problems. Lessons should surface, test, and refine students' informal strategies to prevent stable but incorrect additive procedures from masquerading as "efficient" solutions (Pelen, 2025). Teacher moves that probe the consequence of "more help ⇒ fewer days" can trigger productive disequilibrium and realignment.

D. CONCLUSION AND SUGGESTIONS

The findings of this study reveal that students' errors in proportional reasoning are dominated by intuitive and additive strategies, with only limited evidence of emerging proportional thinking. Across subjects with low, medium, and high mathematical proficiency, the tendency to rely on additive procedures rather than reasoning with multiplicative relationships was persistent and robust. Intuitive reasoning led students to focus on salient but irrelevant information, while additive reasoning caused them to treat the problem as a matter of tallying or adjusting fixed quantities rather than coordinating rates or units of work. Even when students recognized proportional cues, as seen in the proportion attempt strategy, they struggled to appropriately map these cues onto the underlying multiplicative structure of the situation. These patterns reinforce earlier findings that distinguishing proportional from non-proportional contexts is a central challenge for learners and that additive schemas are often overgeneralized inappropriately. The main contribution of this study lies in its detailed, process-oriented analysis of students' reasoning patterns when confronted with a non-routine proportional problem involving mixed individual and joint work rates. By classifying students' reasoning into four error categories intuitive, additive, proportion attempt, and other errors the study provides a nuanced framework that may assist educators in diagnosing specific misconceptions and cognitive tendencies. This fine-grained characterization adds to the existing literature by showing how proportional reasoning difficulties manifest not only in incorrect final answers but in the structure and sequencing of students' problem-solving steps. The results underscore the importance of supporting students in constructing representations that highlight unit rates and multiplicative relationships rather than counts or differences.

The implications of this study point toward instructional approaches that make the multiplicative structure of proportional situations explicit, foreground unit-rate reasoning, and scaffold students' ability to select representations aligned with the quantitative relationships in the problem. Educators may benefit from incorporating schema-based instruction and diagnostic assessments that target specific misconceptions, as well as designing tasks that encourage sustained reflection on the consequences of proportional relationships (e.g., "more help leads to fewer days"). Future research could extend this work by examining a broader range of task types, exploring how instructional interventions influence the development of proportional reasoning, and investigating how metacognitive awareness supports students' ability to monitor and evaluate their reasoning. Additionally, studies that trace changes in students' reasoning over longer periods or across multiple domains may provide further insights into how proportional thinking develops and how persistent errors can be effectively addressed.

REFERENCES

Abreu-Mendoza, R. A., Powell, A. B., Renninger, K. A., Rivera, L. M., Vulic, J., Weimar, S., & Rosenberg-Lee, M. (2023). Middle-schoolers' misconceptions in discretized nonsymbolic proportional reasoning explain fraction biases better than their continuous reasoning: Evidence from correlation and cluster analyses. *Cognitive Psychology*, *143*. https://doi.org/10.1016/j.cogpsych.2023.101575

AÇIKGÜL, K. (2021). Developing a Two-Tier Proportional Reasoning Skill Test: Validity and Reliability Studies. *International Journal of Assessment Tools in Education*, *8*(2), 357–375. https://doi.org/10.21449/ijate.909316

Alma, A. S., Rochaminah, S., Ismaimuza, D., & Fajriani, F. (2025). Mathematical Literacy and Learning Styles: A Study on Student Geometry Learning. *Desimal Jurnal Matematika*, *8*(2), 247–256. https://doi.org/10.24042/qnh6ps42

Arican, M. (2019). Preservice Mathematics Teachers' Understanding of and Abilities to Differentiate Proportional Relationships from Nonproportional Relationships. *International Journal of Science and Mathematics Education*, *17*(7), 1423–1443. https://doi.org/10.1007/s10763-018-9931-x

Basu, D., & Nguyen, H. B. (2021). Eating Healthy: Understanding Added Sugar Through Proportional Reasoning. *International Journal of Environmental Research and Public Health*, *18*(23), 12821. https://doi.org/10.3390/ijerph182312821

Bonte, P., Ongenae, F., De Backere, F., Schaballie, J., Arndt, D., Verstichel, S., Mannens, E., Van de Walle, R., & De Turck, F. (2017). The MASSIF platform: a modular and semantic platform for the development of flexible IoT services. *Knowledge and Information Systems*, *51*(1), 89–126. https://doi.org/10.1007/s10115-016-0969-1

Brahier, D. (2016). *Teaching Secondary and Middle School Mathematics*. Routledge. https://doi.org/10.4324/9781315685502

Buforn, À., Llinares, S., Fernández, C., Coles, A., & Brown, L. (2022). Pre-service teachers' knowledge of the unitizing process in recognizing students' reasoning to propose teaching decisions. *International Journal of Mathematical Education in Science and Technology*, *53*(2), 425–443. https://doi.org/10.1080/0020739X.2020.1777333

Burgos, M., & Godino, J. D. (2022). Assessing the Epistemic Analysis Competence of Prospective Primary School Teachers on Proportionality Tasks. *International Journal of Science and Mathematics Education*, *20*(2), 367–389. https://doi.org/10.1007/s10763-020-10143-0

Cabero, I., Santágueda, M., Villalobos Antúnez, J. V, & Roig-Albiol, A. I. (2020). Understanding of inverse proportional reasoning in pre-service teachers. *Education Sciences*, *10*(11), 1–19. https://doi.org/10.3390/educsci10110308

Callingham, R., & Siemon, D. (2021). Connecting multiplicative thinking and mathematical reasoning in the middle years. *The Journal of Mathematical Behavior*, *61*, 100837. https://doi.org/10.1016/j.jmathb.2020.100837

Cheng, P.-J., Liao, Y.-H., & Yu, P.-T. (2021). Micro:bit Robotics Course: Infusing Logical Reasoning and Problem-Solving Ability in Fifth Grade Students Through an Online Group Study System. *International Review of Research in Open and Distributed Learning*, *22*(1), 21–40. https://doi.org/10.19173/irrodl.v22i1.4844

Copur-Gencturk, Y., Choi, H.-J., & Cohen, A. (2023). Investigating teachers' understanding through topic modeling: a promising approach to studying teachers' knowledge. *Journal of Mathematics Teacher Education*, *26*(3), 281–302. https://doi.org/10.1007/s10857-021-09529-w

de la Torre, J., Qiu, X.-L., & Santos, K. C. (2022). An Empirical Q-Matrix Validation Method for the

Polytomous G-DINA Model. *Psychometrika*, *87*(2), 693–724. https://doi.org/10.1007/s11336-021-09821-x

Dole, R., Hoerling, M., Perlwitz, J., Eischeid, J., Pegion, P., Zhang, T., Quan, X.-W., Xu, T., & Murray, D. (2011). Was there a basis for anticipating the 2010 Russian heat wave? *Geophysical Research Letters*, *38*(6), n/a-n/a. https://doi.org/10.1029/2010GL046582

Foley, G. D., Budhathoki, D., Thapa, A. B., & Aryal, H. P. (2023). Instructor perspectives on quantitative reasoning for critical citizenship. *ZDM - International Journal on Mathematics Education*, *55*(5), 1009–1020. https://doi.org/10.1007/s11858-023-01520-4

Gunderson, E. A., & Hildebrand, L. (2021). Relations among spatial skills, number line estimation, and exact and approximate calculation in young children. *Journal of Experimental Child Psychology*, *212*. https://doi.org/10.1016/j.jecp.2021.105251

Hurst, M. A., Butts, J. R., & Levine, S. C. (2022). Connecting Symbolic Fractions to Their Underlying Proportions Using Iterative Partitioning. *Developmental Psychology*, *58*(9), 1702–1715. https://doi.org/10.1037/dev0001384

Izzatin, M., Waluya, S. B., Kartono, n., Dwidayati, N., & Dewi, N. R. (2021). Students' proportional reasoning in solving non-routine problems based on mathematical disposition. *Journal of Physics: Conference Series*, *1918*(4). https://doi.org/10.1088/1742-6596/1918/4/042114

Jitendra, A. K., Harwell, M. R., & Im, S.-H. (2022). Sustainability of a Teacher Professional Development Program on Proportional Reasoning Skills of Students With Mathematics Difficulties. *Exceptional Children*, *89*(1), 79–100. https://doi.org/10.1177/00144029221094053

Jitendra, A. K., Harwell, M. R., Karl, S. R., Im, S.-H., & Slater, S. C. (2021). Investigating the Generalizability of Schema-Based Instruction Focused on Proportional Reasoning: A Multi-State Study. *Journal of Experimental Education*, *89*(4), 587–604. https://doi.org/10.1080/00220973.2020.1751580

Jonsson, B., Granberg, C., & Lithner, J. (2020). Gaining Mathematical Understanding: The Effects of Creative Mathematical Reasoning and Cognitive Proficiency. *Frontiers in Psychology*, *11*. https://doi.org/10.3389/fpsyg.2020.574366

Maharani, R. I., & Murtiyasa, B. (2023). Analysis of Students' Error in Solving Trigonometry Comparison Problems With the Polya Criteria Guiden. *Prima Jurnal Pendidikan Matematika*, *7*(2), 157. https://doi.org/10.31000/prima.v7i2.8482

Mata-Pereira, J., & da Ponte, J.-P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, *96*(2), 169–186. https://doi.org/10.1007/s10649-017-9773-4

Maulyda, M. A., Annizar, A. M., Hidayati, V. R., & Mukhlis, M. (2020). Analysis of students' verbal and written mathematical communication error in solving word problem. *Journal of Physics: Conference Series*, *1538*(1), 012083. https://doi.org/10.1088/1742-6596/1538/1/012083

Maulyda, M. A., Sukoriyanto, S., Hidayati, V. R., Erfan, M., & Umar, U. (2020). Student Representation in Solving Story Problems Using Polya Steps. *Formatif Jurnal Ilmiah Pendidikan Mipa*, *10*(1). https://doi.org/10.30998/formatif.v10i1.4629

McMillan, B. (2025). Connections within the Multiplicative Field: A Case Study of Adán's Mathematical Thinking. *Investigations in Mathematics Learning*, *17*(1), 1–21. https://doi.org/10.1080/19477503.2024.2372976

Nelson, G., Hunt, J. H., Martin, K., Patterson, B., & Khounmeuang, A. (2022). Current Knowledge and Future Directions: Proportional Reasoning Interventions for Students with Learning Disabilities and Mathematics Difficulties. *Learning Disability Quarterly*, *45*(3), 159–171. https://doi.org/10.1177/0731948720932850

Nisa, T. F., Budayasa, I. K., Lukito, A., & Abadi, A. (2022, January). The Dynamics of Students' Mathematical Cognition Process in Solving Multitasking-based Problems: A Typical PISA Problem Used to Examine Students' Numerical Literacy. In *International Conference on Madrasah Reform 2021 (ICMR 2021)* (pp. 37-44). Atlantis Press. https://doi.org/10.2991/assehr.k.220104.007

Nugraha, Y., Sa'dijah, C., Susiswo, S., & Chandra, T. D. (2023). Proportional and Non-Proportional Situation: How to Make Sense of Them. *International Journal of Educational Methodology*, *9*(2), 355–365. https://doi.org/10.12973/ijem.9.2.355

Pelen, M. S. (2025). Beyond the Across Algorithm: Informal Strategies for Inverse Proportional

Problems. *International Journal of Science and Mathematics Education*, *23*(5), 1341–1368. https://doi.org/10.1007/s10763-024-10524-9

Pişkin Tunç, M., & Çakıroğlu, E. (2022). Fostering prospective mathematics teachers' proportional reasoning through a practice-based instruction. *International Journal of Mathematical Education in Science and Technology*, *53*(2), 269–288. https://doi.org/10.1080/0020739X.2020.1844909

Rodríguez-Nieto, C. A., Pabón-Navarro, M. L., Cantillo-Rudas, B. M., Sudirman, S., & Font, V. F. (2025). The potential of ethnomathematical and mathematical connections in the pre-service mathematics teachers' meaningful learning when problems-solving about brick-making. *Infinity Journal*, *14*(2), 419–444. https://doi.org/10.22460/infinity.v14i2.p419-444

Rogers, K. C., & Steele, M. D. (2016). Graduate Teaching Assistants' Enactment of Reasoning-and-Proving Tasks in a Content Course for Elementary Teachers. *Journal for Research in Mathematics Education*, *47*(4), 372–419. https://doi.org/10.5951/jresematheduc.47.4.0372

Sabat, A. O., & Pramudya, I. (2021, November). Gender Difference: Students' Mathematical Literacy in Problem Solving. In *International Conference of Mathematics and Mathematics Education (I-CMME 2021)* (pp. 25-30). Atlantis Press. https://doi.org/10.2991/assehr.k.211122.004

Sari, R. N., Rosjanuardi, R., Isharyadi, R., & Nurhayati, A. (2024). Level of students' proportional reasoning in solving mathematical problems. *Journal on Mathematics Education*, *15*(4), 1095–1114. https://doi.org/10.22342/jme.v15i4.pp1095-1114

Shofiyah, N., Suprapto, N., Prahani, B. K., Jatmiko, B., Anggraeni, D. M., & Nisa', K. (2024). Exploring undergraduate students' scientific reasoning in the force and motion concept. *Cogent Education*, *11*(1), 2365579. https://doi.org/10.1080/2331186X.2024.2365579

Weiland, T., Orrill, C. H., Nagar, G. G., Brown, R. E., & Burke, J. (2021). Framing a robust understanding of proportional reasoning for teachers. *Journal of Mathematics Teacher Education*, *24*(2), 179–202. https://doi.org/10.1007/s10857-019-09453-0