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Application of Kernel Nonparametric Biresponse Regression with the Nadaraya-Watson Estimator in Poverty Analysis in South Sulawesi

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ABSTRACT

Poverty is a complex social issue that requires in-depth analysis to identify its contributing factors. South Sulawesi, as one of the provinces in Indonesia, continues to face various challenges in poverty alleviation. This study is a quantitative research that aims to model the poverty rate and poverty severity index using a biresponse nonparametric kernel regression with the Nadaraya-Watson estimator and Gaussian kernel function. The analysis is based on 2024 data form the Central Bureau of Statistics (BPS), which includes poverty indicators as response variables and socio-economic factors, processed using R Studio 2025. The nonparametric biresponse kernel regression analysis yielded optimal bandwidths of $h_1 = 0{,}188$; $h_2 = 0{,}083$; $h_3 = 0{,}159$; and $h_4 = 0{,}028$. Model accuracy is demonstrated by a Generalized Cross-Validation (GCV) value of 5.515 and a Mean Squared Error (MSE) of 0.585, indicating high stability and low prediction error. The model demonstrates adaptive accuracy in simultaneously modeling the two response variables and highlights the strength of kernel-based biresponse regression as an evidence-based tool for policymakers to design targeted, region-specific poverty alleviation strategies.

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A. INTRODUCTION

Indonesia faces poverty as a significant challenge in its national development efforts. According to the World Bank, poverty is not solely measured by financial deficits but is also characterized as a condition in which individuals or community groups fail to gain adequate access to essential resources needed to fulfil their basic needs (Purwanti, 2024; Hidayat et al., 2023; Aliem & Jamil, 2023). This includes limitations in access to education, healthcare services, and infrastructure that support the improvement of quality of life. According to data from the Central Bureau of Statistics (BPS) (2024), the number of poor people in March 2024 was recorded at 25.22 million individuals. Although this represents a decrease of 0.68 million compared to the previous year, the poverty rate remains high at 9.03 percent. This condition indicates that economic disparity and limited access to basic services continue to be serious issues that must be addressed through inclusive and sustainable policies (Kadir et al., 2019; Irfan et al., 2024).

One of provinces that is challenged by this complexity is South Sulawesi province. South Sulawesi is one of the provinces characterized by complex poverty, marked by disparities between urban and rural areas as well as limitations in infrastructure and public services (Kadir et al., 2019). Its diverse geographical conditions further complicate community access to education, healthcare, and employment opportunities, thereby affecting welfare levels (Pasarella et al., 2022). Data indicate that poverty levels in South Sulawesi have fluctuated during the period 2019-2023. In 2019, the poverty rate reached 16.12%, equivalent to approximately 759,580 individuals; this figure surged in 2020 due to the pandemic, then declined in 2021, but rose again the following year (Sabar et al., 2024). Such fluctuations suggest that poverty is influenced by various factors, both external such as global crises, and internal such as development inequality (Said et al., 2022). Therefore, a more comprehensive data-driven approach is necessary to more accurately understand poverty patterns.

In terms of investigating the pattern of poverty, we employ the nonparametric regression. It is a relevant statistical approach for analysing the pattern of poverty since it does not require presumptions about the form of the relationship between predictor variables and response variables (Rosalina et al., 2023). This approach provides high flexibility to capture complex, nonlinear patterns between variables (Husain et al., 2025). One commonly used method in nonparametric regression is the Nadaraya-Watson kernel estimation. Its advantage lies in the ability to smoothly capture local data structure without specifying the functional form of the model in advance (Husain et al., 2021; Pembargi et al., 2023). By using kernel functions and bandwidth parameters, this method provides local average estimates of response values that reflect the true conditions around observation points. This allows for more adaptive and precise analysis, particularly in the context of poverty which is affected by multiple factors with non-simple relationships.

Previous studies have demonstrated that kernel nonparametric regression is an effective approach for data modelling. For example, research by Pembargi et al. (2023) successfully applied kernel nonparametric regression to model Regional Original Revenue (PAD) in Central Lombok Regency. Using a Gaussian kernel function and optimal bandwidth, they achieved accurate results with a coefficient of determination (R^2) of 87.55% and a Mean Absolute Percentage Error (MAPE) of 5.4%. This finding indicates that kernel nonparametric methods effectively capture nonlinear economic data dynamics without requiring predefined functional forms. Similarly, Utami & Lahdji (2022) used a polynomial kernel approach to model Covid-19 patient numbers in Semarang City and obtained excellent results with an R^2 of 97%. In the context of poverty analysis, Adrianingsih et al. (2025) examined poverty in Nusa Tenggara Timur Province using kernel regression with various kernel functions (Gaussian, Epanechnikov, Triangle, and Quartic). Their findings showed that the Gaussian kernel function provided the most accurate predictions, with a strong balance between model complexity and error. In a broader methodological context, Vinod (2022) highlights the potential of nonlinear, nonparametric kernel regressions to overcome the problem of p-hacking by focusing not only on statistical significance but also on practical significance through scale-free generalized partial correlation coefficients (GPCCs). These studies corroborate that kernel approaches can model complex nonlinear data patterns, especially when relationships between predictor and response variables cannot be adequately explained by conventional parametric models.

Nevertheless, the previous research have been mostly limited to modelling with a single response variable, which fails to depict the simultaneous relationships among two or more interrelated important aspects. In the context of poverty phenomena, analyses often require simultaneous consideration of multiple indicators influenced by the same predictor variables. Hence, a univariate response approach is insufficient to grasp the complexities linking these indicators.

This study employs biresponse kernel nonparametric regression as a methodological approach capable of simultaneously analyzing two poverty indicators. From an academic perspective, the research contributes to the existing body of knowledge by expanding the application of nonparametric methods to complex socio-economic phenomena that are often difficult to capture with conventional models. From a practical standpoint, the findings are expected to generate data-driven insights that can assist policymakers in formulating poverty alleviation strategies that are more effective, inclusive, and tailored to local conditions. The urgency of this research is reinforced by the persistent poverty challenges in South Sulawesi, which require more adaptive and sustainable analytical solutions.

B. THEORETICAL REVIEW

Nonparametric regression is an approach employed when the form of the relationship between the response variable and predictor variables is unknown (Ramli et al., 2023). This method is flexible because it does not assume a specific functional form, in contrast to parametric regression which typically requires linear or polynomial models. Suppose there are n independent observations, paired data $(x_i, y_i), i = 12, \dots, n$ following the nonparametric regression model as formulated in equation (1) (Ni'matuzzahroh & Dani, 2024):

$$y_i = g(x_i) + \varepsilon_i \qquad ; \ i = 1,2, \cdots, n \tag{1}$$

where y_i represents the response variable, x_i the predictor variable, dan ε_i is a random error term that is independent, identically distributed with a normal distribution having mean zero and variance σ^2. The function $g(x_i)$ represents the regression curve whose form is unknown but assumed to be smooth meaning continuous and differentiable. Various estimators can be used to approximate this unknown regression function, including Fourier series, kernel methods, wavelets, truncated splines, among other . The principal advantage of nonparametric regression lies in its capability to capture complex and nonlinear data structures without fixing the functional form from the outset (Eubank R L, 1999).

The kernel estimator is an advancement of the histogram estimator. In the context of density or regression function estimation, the kernel function assigns local weights to observations surrounding the estimation point (Nakarmi et al., 2021). Mathematically, the kernel density function $\hat{g}_h(x)$ is defined in equation (2):

$$\hat{g}_h = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}K\left(\frac{x - x_i}{h}\right) \tag{2}$$

Similarly, the multivariate kernel density estimator $\hat{g}_h(x)$ is expressed in equation (3):

$$\hat{g}_{h_1,h_2,\cdots,h_p}(x_{1i}, x_{2i}, \cdots, x_{pi}) = \frac{1}{n}\sum_{i=1}^{n}\prod_{j=1}^{p}\frac{1}{h_j}K\left(\frac{x_j - x_{ji}}{h_j}\right) \tag{3}$$

where $i = 1,2,\cdots,n$ indexes observation, $j = 1,2,\cdots,p$ indexes predictor variable, K is the kernel function l, and h represents the bandwidth parameter that controls the smoothness by defining the bias-variance tradeoff of the estimator $\hat{g}_h(x)$. Let $x_1, x_2, \cdots, x_n$ be a random sample from distribution g and K be a positive and bounded kernel function. The kernel must satisfy five conditions (Tenri Ampa et al., 2024):

1. For all x, $K(x) \geq 0$;
2. $\int_{-\infty}^{\infty} x_i K(x)dx = \begin{cases} 1 & ,i = 0 \\ 0 & ,1 \leq i \leq r \\ \neq 0 & ,i = r \end{cases}$;
   where r is the kernel order
3. $\int_{-\infty}^{\infty} x^2 K(x)dx = \sigma^2 > 0$;
4. $\int_{-\infty}^{\infty} K(x)dx = 1$;
5. $K(x)$ is symmetric about zero, i.e.,$K(x) = K(-x)$

Common kernel functions include uniform, Epanechnikov, quartic, triangular, triweight, and Gaussian kernels as follows (Saidi et al., 2021):

1. Kernel uniform $\left(K(x) = \frac{1}{2}, |x| \leq 1\right)$
2. Kernel Epanechnikov $\left(K(x) = \frac{3}{4}(1 - x^2), |x| \leq 1\right)$
3. Kernel Quartic $\left(K(x) = \frac{15}{16}(1 - x^2)^2, |x| \leq 1\right)$
4. Kernel triangular $(K(x) = 1 - |x|, |x| \leq 1)$
5. Kernel Triweight $\left(K(x) = \frac{35}{32}(1 - u^2)^3, |x| \leq 1\right)$
6. Kernel Gaussian $\left(K(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}x^2\right), -\infty < x < \infty\right)$

To obtain suitable regression estimates, proper weighting of observations is essential. In kernel regression, the unknown regression function is approximated via kernel-based nonparametric estimation. A widely used estimator in this approach is the Nadaraya-Watson estimator, which computes a locally weighted average of the response variable, with weights derived from the kernel function (Aliu et al., 2022). The Nadaraya-Watson estimator assigns higher weights to data points x_i closer to the point of estimation x Mathematically, it is expressed as equation (4):

$$\hat{m}(x) = \frac{\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) y_i}{\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)} \tag{4}$$

where $\hat{m}(x)$ denotes the nonparametric regression function estimate at predictor.

The strength of the Nadaraya-Watson estimator lies in its flexibility to model nonlinear relationships without requiring assumptions about the functional form (Sadek & Mohammed, 2024). However, its accuracy critically depends on selecting an appropriate kernel function and bandwidth. An excessively small bandwidth may cause overfitting, closely following noise in the data, whereas too large a bandwidth may lead to underfitting, over smoothing key features (Mardianto et al., 2020). Therefore, optimizing the bandwidth is vital and is often determined by minimizing the Generalized Cross Validation (GCV) criterion, which balances bias and variance (Afifah et al., 2017). The GCV is defined as in equation (5):

$$GCV(h) = \frac{MSE(h)}{\left(n^{-1}tr\left(\boldsymbol{I} - \boldsymbol{A}(h)\right)\right)^2} \tag{5}$$

with

$MSE(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{m}(x_i)\,)^2$, dan

$\boldsymbol{A}(h) = \boldsymbol{X}[\boldsymbol{X^T}\boldsymbol{X}]^{-\boldsymbol{1}}\boldsymbol{X^T}$

where h is *bandwidth*, $\boldsymbol{I}$ is the $n \times n$ identity matrix, $\boldsymbol{X}$ is the nonparametric kernel component matrix of size nxn and n is the total number of observations.

C. METHODS

This study employs a quantitative approach utilizing secondary data sourced from the official publication of the Central Bureau of Statistics (BPS) 2024, obtained through the BPS official website and the publication *Sulawesi Selatan dalam Angka 2025*. The data type applied is cross-sectional, covering 24 regencies/cities in the South Sulawesi Province. The data used in this research includes the Poverty Percentage and Poverty Severity Index in South Sulawesi Province for the year 2024, alongside several influencing factors. The study involves two response variables (Y) and four predictor variables (X), as detailed in Table 1 below:

Table 1. Variables

Variables	Description
Y_1	Poverty Percentage
Y_2	Poverty Severity Index
X_1	Labor Force Participation Rate (TPAK)
X_2	Open Unemployment Rate (TPT)
X_3	Human Development Index (IPM)
X_4	Gross Regional Domestic Product (GRDP)

The analytical steps for constructing a biresponse nonparametric kernel regression model using the Nadaraya-Watson estimator and Gaussian kernel function on poverty data from South Sulawesi, implemented in R Studio 2025 with R version 4.3.1, are as follows:

1. Conduct a dependency test between the first response variable and the second response variable to ensure that the two responses are statistically related and justify the use of a biresponse model.

2. Perform exploratory data analysis (EDA) on both response and predictor variables, which includes by generating descriptive statistics, including minimum, maximum, mean, and standard deviation.
3. Create scatter plots between each response and predictor variable to visually examine the overall relationship patterns. The scatter plots are particularly useful for detecting nonlinearity in the data, which justifies the use of nonparametric regression.
4. Determine the optimal bandwidth value by minimizing the Generalized Cross-Validation (GCV) criterion. This step is crucial since the selection of bandwith affects the smoothness and accuarcy of the regression curve.
5. Model the poverty data using the biresponse nonparametric kernel regression with the Nadaraya-Watson estimator, applying the Gaussian kernel function and the selected optimal bandwith.
6. Evaluate the accuracy of the fitted model, focusing on the performance of the biresponse kernel regression in capturing the relationship between poverty indicators and predictor variables.
7. Calculate the coefficient of determination (R^2) and Mean Square Error (MSE) to assess the goodness of fit, where R^2 explain the proportion of variance in the response variables accounted for by the predictor variables, while MSE measures the average squared difference between observed and predicted values to evaluate model accuracy.

D. RESULT AND DISCUSSION

1. Dependency Test Between Response Variables

Before constructing the biresponse regression model, a test was conducted to examine the relationship between the two response variables: Poverty Percentage (Y_1) and Poverty Severity Index (Y_2). Based on Pearson correlation analysis, a coefficient of 0.68 was obtained, indicating a strong positive relationship between the two variables. This means that areas with more people living under the poverty line tend to also experience deeper levels of poverty. Therefore, employing a biresponse regression model is a relevant approach because it simultaneously considers the interdependence between the two response variables.

2. Descriptive Statistics

Descriptive statistics for all analyzed variables provide a general overview of the socio-economic conditions in the province of South Sulawesi, as shown in Table 2.

Table 2. Descriptive Statistics

Variables	Minimum	Maximum	Mean	Std. Deviation
Poverty Index	4.97	12.41	8.60	2.31
Poverty Severity Index	0.08	0.58	0.26	0.13
TPAK	59.92	78.58	68.46	5.01
TPT	1.51	9.71	3.71	1.96
IPM	69.45	85.23	74.49	3.55
PDRB	3.27	6.03	4.67	0.72

Table 2 shows significant disparities in socio-economic indicators. Poverty percentage ranges from 4.97% to 12.41%, while poverty severity index ranges from 0.08 to 0.58. These variations reveal that some districts are affected not only by high poverty incidence but also by severe economic deprivation. Furthermore, the large variations observed in Labor Force Participation Rate (TPAK), Open Unemployment Rate (TPT), Human Development Index (IPM), and Gross Regional Domestic Product (PDRB) reflect economic and social development disparities across South Sulawesi Province. In policy terms, this highlights the urgency of designing targeted interventions that address both poverty incidence and depth, while also improving access to employment and education.

Figure 1 illustrates the geographic distribution of the poverty rate and poverty severity index in South Sulawesi Province. Luwu, Enrekang, and North Luwu Regencies are the areas with the heaviest poverty burden, characterized by both high poverty rates and poverty severity. This reflects that in these regions, not only are many people living below the poverty line but also in dire economic conditions. Meanwhile, Pangkep and Jeneponto Regencies show high poverty rates, but their poverty severity indexes are still in the moderate category. This indicates that despite the large number of poor people, the level of economic deprivation is not as severe as in regions that simultaneously fall into the high poverty category. On the other hand, Sidrap Regency stands out as the only region with a combination of both low poverty rates and poverty severity indexes, reflecting the effectiveness of economic development and a relatively more equitable distribution of prosperity.

This spatial pattern demonstrates a clear concentration of poverty in certain regencies, indicating development disparities between regions in South Sulawesi. This demonstrates that although the aggregate provincial poverty rate shows a downward trend, some regions remain significantly behind. The interpretation is that the burden of poverty is not evenly distributed, but rather concentrated in certain areas with weak structural conditions, such as limited access to education, productive employment, and basic infrastructure.

Therefore, targeted and contextual intervention strategies are needed. Areas with a double burden (poverty percentage and poverty severity index) should be prioritized in poverty alleviation programs, for example through strengthening the local economy, improving human resource quality, and improving access to basic infrastructure. Conversely, areas with a high poverty rate but moderate poverty severity, such as Pangkep and Jeneponto, require strategies to expand employment opportunities and economic empowerment to reduce the number of poor people. Meanwhile, development practices in Sidrap can serve as a successful model for replication in other regions, demonstrating that inclusive development and equitable prosperity can be achieved with the right strategies.

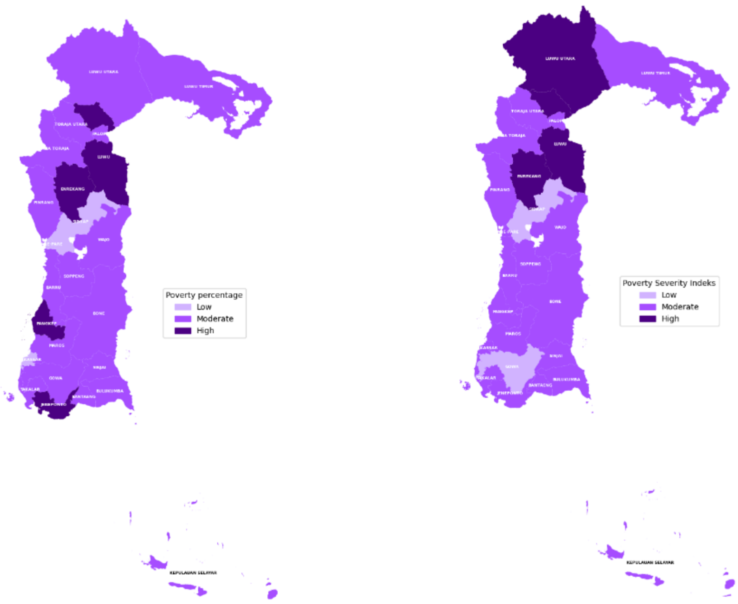

Figure 1. Spatial Visualization of Poverty Percentage and Poverty Severity Index

2. Kernel Nonparametric Biresponse Regression Modeling

Before constructing the kernel nonparametric biresponse regression model, an exploratory analysis was conducted to examine the relationship between the two response variables: poverty percentage and poverty severity index.

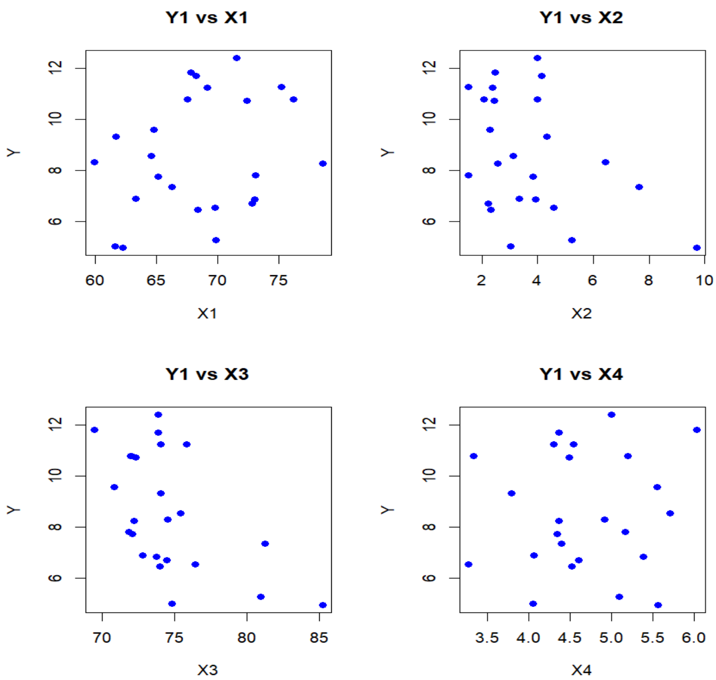

Figure 2. Scatterplot of Poverty Percentage Against Influencing Factors

Figures 2 and Figure 3 depict the relationship patterns between the response variables, namely poverty percentage and poverty severity index, and each predictor variable. The scatterplots reveal that the relationships do not conform to identifiable patterns such as linear, quadratic, or exponential forms. The data points appear dispersed randomly without a clear directional trend, indicating that parametric regression is unsuitable for capturing the dynamics among these variables. This finding validates the decision to employ a kernel nonparametric regression approach, which does not assume any specific functional form and is capable of capturing complex and context-specific nonlinear interactions. Substantively, this suggests that poverty in South Sulawesi is influenced by diverse socio-economic factors whose effects vary across regions, thereby requiring flexible methods to uncover and model these dynamics more accurately.

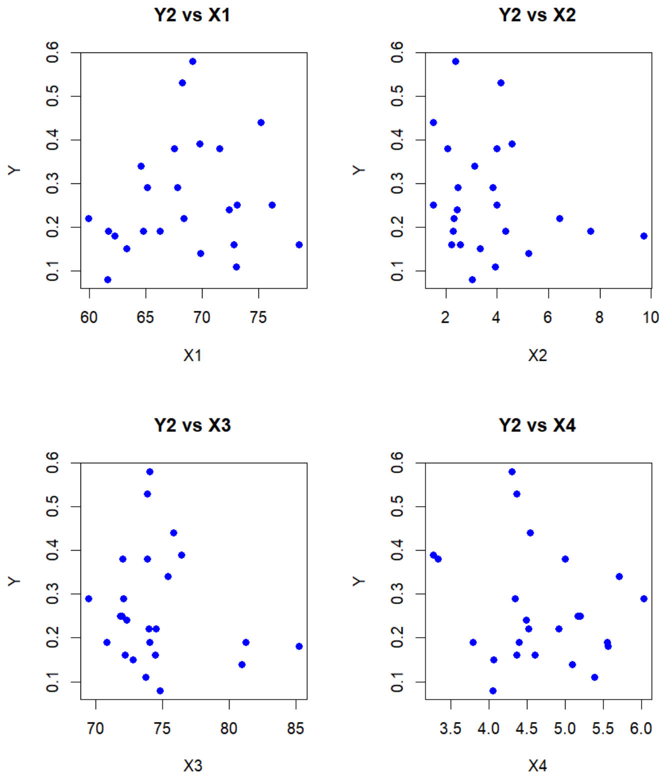

Figure 3. Scatterplot of Poverty Severity Index Against Influencing Factors

The nonparametric regression modelling proceeds using the Gaussian kernel approach with the Nadaraya-Watson estimator to estimate the values of each response variable based on their predictors. This approach is chosen because the Gaussian kernel provides smoothness and effectively handles varied data distributions. Additionally, the model includes the selection of optimal bandwidth for each predictor variable based on the criterion of minimizing Generalized Cross Validation (GCV) values, as shown in Table 3.

Table 3. Optimal Bandwidth Value

h_1	h_2	h_3	h_4	GCV
0,188	0,083	0,159	0,028	**5,515***
0,377	0,166	0,319	0,056	5,547
0,754	0,331	0,638	0,111	5,548
0,566	0,248	0,478	0,084	5,557
0,943	0,414	0,797	0,139	5,568

h_1	h_2	h_3	h_4	GCV
1,131	0,497	0,956	0,167	5,560
1,319	0,580	1,116	0,195	5,629
1,508	0,663	1,275	0,223	5,651
1,697	0,745	1,434	0,251	5,666
1,885	0,828	1,594	0,279	5,679

Table 3 presents the results of optimal bandwidth selection obtained through the minimization of the Generalized Cross Validation (GCV) criterion, which serves as an indicator of model performance in the kernel nonparametric biresponse regression. Among the tested bandwidth combinations, the smallest GCV value of 5.515 was achieved with the bandwidth parameters $h_1 = 0{,}188$; $h_2 = 0{,}083$; $h_3 = 0{,}159$; and $h_4 = 0{,}028$. This lowest GCV value indicates that the chosen combination provides the best balance between bias and variance, yielding optimal estimates without overfitting. Therefore, this bandwidth combination was selected as the smoothing parameter in the kernel regression modeling process to ensure accurate and efficient capture of the underlying relationship structure among variables. Substantively, the small GCV value shows that the model is able to represent poverty dynamics in South Sulawesi with stability, avoiding results that are either too simplified or too extreme, and thus producing findings that are more reliable for poverty analysis.

Following the determination of the optimal bandwidth values, the kernel nonparametric biresponse regression equation using the Gaussian kernel Nadaraya-Watson estimator is formulated as shown in equation (6):

$$\hat{y}_{ki} = \frac{\sum_{i=1}^{n} \prod_{j=1}^{4} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x_j - x_{ji}}{h_j}\right)^2\right) y_{ki}}{\sum_{i=1}^{n} \prod_{j=1}^{4} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x_j - x_{ji}}{h_j}\right)^2\right)} \tag{6}$$

where $k = 1{,}2$, represents the number of response variables $j = 1{,}2{,}3{,}4$ denotes the number of predictor variables, $i = 1{,}2, \cdots, n$ represents the number of observations in the dataset, dan h_j stands for the optimal bandwidth value corresponding to each predictor variable $h_1 = 0{,}188$; $h_2 = 0{,}083$; $h_3 = 0{,}159$; and $h_4 = 0{,}028$.

The resulting model from this approach has a coefficient of determination $R^2 = 93.82\%$, indicating that the model explains the majority of the variability in the data. This means that nearly all variations in poverty levels and severity across districts can be explained by the selected socio-economic factors, providing strong evidence for identifying the main drivers of poverty. The Mean Square Error (MSE) value of 0.585 reflects a relatively low prediction error, suggesting that the model performs well in representing the relationships among the variables in the dataset. In substantive terms, this low MSE indicates that the predictions produced by the model are close to the actual conditions, making the results more trustworthy as a reference for policymakers in designing targeted poverty alleviation strategies.

Figure 4 presents a comparison between actual and predicted values, demonstrating that for both response variables the poverty percentage and the poverty severity index the predicted trajectories align closely with the observed data. This outcome confirms the robustness of the kernel biresponse regression model in capturing the overarching poverty dynamics and dominant trends across districts. Nonetheless, the model exhibits a tendency to smooth local fluctuations, which may lead to an understatement of extreme conditions in certain regions.

From a substantive perspective, this suggests that the model is highly reliable for macro-level policy formulation and strategic planning, while further refinement is required to enhance its sensitivity to micro-level variations and localized poverty pockets. Such refinement could improve the model's capacity to inform more precise, community-based interventions, as shown in Figure 4.

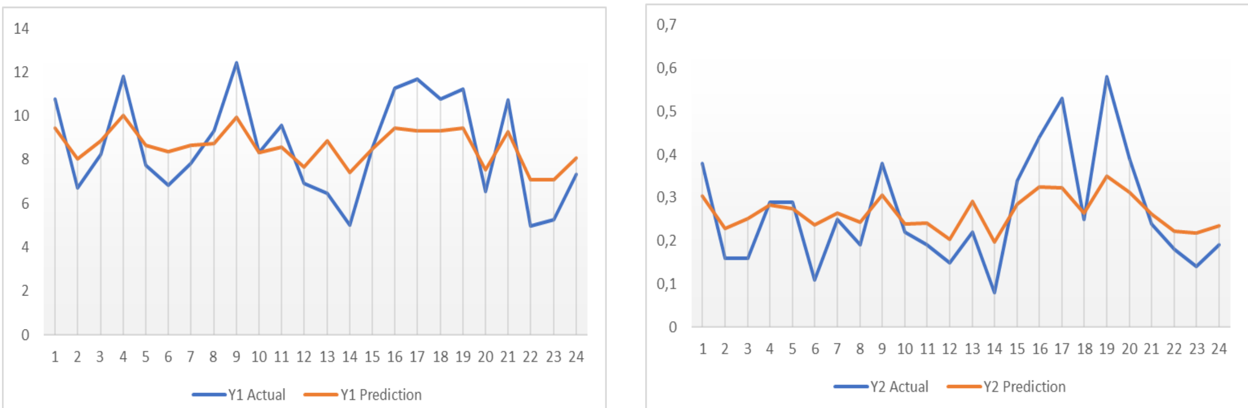

Figure 4. The comparison between actual value dan predicted value

The application of kernel nonparametric biresponse regression with the Nadaraya-Watson estimator in this study demonstrates its superiority in modeling two interrelated response variables simultaneously—poverty percentage and poverty severity index—that exhibit different distributional characteristics. In the context of poverty analysis in South Sulawesi, this approach enables smoother, more stable, and more realistic predictions, providing a more accurate depiction of the socioeconomic conditions of the region. Consequently, the predictive results can serve as a solid basis for designing more targeted policy interventions aimed at both reducing the number of poor people and addressing the severity of poverty in districts requiring special attention. Nevertheless, this study has a limitation in that the predictor variables were assigned the same bandwidth values for both response variables. This assumption may restrict the flexibility of the model in fully capturing the potentially different relationships between each predictor and the two distinct responses. Future research is therefore encouraged to explore approaches that allow for varying bandwidths across responses, which would enhance the adaptiveness and accuracy of the estimations in representing poverty dynamics.

Overall, the findings of this study are in line with previous research that emphasizes the effectiveness of kernel-based nonparametric regression in capturing complex and nonlinear socio-economic relationships. This indicates that the present study not only supports earlier evidence but also extends it by applying a biresponse framework, offering a more comprehensive perspective on poverty analysis.

E. CONCLUSION AND SUGGESTIONS

The results of the kernel nonparametric biresponse regression analysis with the Gaussian Nadaraya-Watson estimator are expressed in the following equation:

$$\hat{y}_{ki} = \frac{\sum_{i=1}^{n} \prod_{j=1}^{4} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x_j - x_{ji}}{h_j}\right)^2\right) y_{ki}}{\sum_{i=1}^{n} \prod_{j=1}^{4} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x_j - x_{ji}}{h_j}\right)^2\right)}$$

where the model has two response variables ($k = 1,2$), four predictor variables ($j = 1,2,3,4$) and n observations ($i = 1,2, \cdots, n$), The optimal bandwidth values for each predictor variable are $h_1 = 0{,}188$; $h_2 = 0{,}083$; $h_3 = 0{,}159$; and $h_4 = 0{,}028$. Model accuracy is indicated by a Generalized Cross Validation (GCV) score of 5.515 and a Mean Square Error (MSE) of 0.585. These results imply that the method can effectively capture poverty dynamics in South Sulawesi and provide reliable evidence for policy interventions.

This study contributes by demonstrating the usefulness of kernel-based biresponse regression in capturing complex socio-economic relationships and providing policymakers with a tool to identify poverty incidence and severity. A limitation is the use of identical bandwidth values for both responses, which may reduce model flexibility. Future research should consider varying bandwidths across responses and explore spatial or longitudinal data to improve accuracy and policy relevance.

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