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Learning Trajectory of Whole Numbers using the Context of *Muara Enim Melemang* Based on Realistic Mathematics Education (PMRI)

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ABSTRACT

Learning whole numbers in the early grades of elementary school often poses challenges because students must understand abstract concepts and represent them symbolically. This study focuses on developing a Local Instructional Theory (LIT) for learning whole numbers using the Muara *Enim* me*lemang* cultural context within the *Pendidikan Matematika Realistik Indonesia* (PMRI) framework. The research contributes both to the development of learning theory (LIT) and to improving the effectiveness of culturally based mathematics learning in Indonesian primary schools. The study employed design research (validation study) consisting of three stages: (1) the preliminary design formulated a Hypothetical Learning Trajectory (HLT) through curriculum analysis and contextual task design; (2) the teaching experiment tested the HLT in classroom practice to explore students' strategies and interactions; and (3) the retrospective analysis compared the HLT and the Actual Learning Trajectory (ALT) to refine the emerging instructional theory. Findings reveal that the predicted HLT aligned with the ALT. Students successfully developed number sense counting, comparing, ordering, and performing simple operations through concrete representations of *lemang*. Difficulties mainly appeared in symbolic notation, such as using inequality signs and subtraction symbols. The study's novelty lies in establishing a local culture-based LIT that bridges PMRI principles with ethnomathematics. Practically, it provides a contextual model teachers can adopt to enhance meaningful mathematics learning in early grades.

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A. INTRODUCTION

Learning whole numbers in elementary school plays a crucial role because it provides the foundation for understanding more advanced mathematical concepts such as place value, operations, and algebraic reasoning (Kilpatrick et al., 2001; NCTM, 2000). However, various studies have revealed that many young learners face persistent difficulties in grasping whole number concepts, including representing quantities, understanding the relationship between numbers, and applying basic operations meaningfully (Fuson, 2019; Askew, 2019). These challenges often occur because instruction tends to emphasize procedural fluency such as memorizing number sequences and calculation rules rather than building deep conceptual understanding (Clarke, 2015; McIntosh, 2020). Consequently, students develop limited number sense and struggle to connect symbols with quantities in real-life contexts.

Number sense, which refers to the intuitive understanding of numbers and their relationships, is a key aspect of mathematical literacy and an important predictor of later mathematical achievement (Jordan et al., 2013; Heuvel-Panhuizen, 2020). To cultivate number sense, mathematics instruction should be designed to help students construct knowledge through exploration, reasoning, and reflection on meaningful experiences (Sowder, 2021). In this regard, researchers have proposed the use of learning trajectories to guide teachers in understanding how students' mathematical thinking develops progressively from informal to formal levels (Simon, 1995; Clements & Sarama, 2014). A learning trajectory typically consists of three components: learning goals, developmental progressions, and instructional activities designed to achieve those goals. When applied effectively, this approach helps teachers anticipate students' strategies and misconceptions, enabling them to provide appropriate scaffolding during learning (Doorman et al., 2012; Bakker, 2018).

In Indonesia, the *Pendidikan Matematika Realistik Indonesia* (PMRI) approach adapted from the Dutch Realistic Mathematics Education (RME) has been shown to effectively support learning trajectories by connecting mathematical concepts with students' real-world experiences (Sembiring et al., 2008; Zulkardi & Putri, 2019). PMRI emphasizes guided reinvention through contexts familiar to students, the use of models as bridges between the real and abstract worlds, student contributions, interactivity, and intertwinement between topics (Gravemeijer, 1994; Wijaya, 2017). By situating learning within meaningful situations, PMRI allows students to develop abstract mathematical knowledge through realistic and context-based reasoning.

Despite its potential, research implementing PMRI and learning trajectories in Indonesia has been predominantly concentrated on topics such as fractions, geometry, and measurement (Clements & Sarama, 2014; Wijaya, 2017; Putri & Zulkardi, 2020). Only a few studies have explored the development of learning trajectories for whole numbers, particularly in the early grades where conceptual understanding of numbers forms the basis for all subsequent mathematical learning. Moreover, existing studies often use general classroom contexts rather than local cultural settings, even though ethnomathematical perspectives highlight the pedagogical value of culture as a source of mathematical meaning (Rosa & Orey, 2011; Alangui, 2019; Seah, 2020). This indicates a methodological and contextual research gap; there is a lack of empirically validated Local Instructional Theories (LIT) that integrate the principles of PMRI with the richness of Indonesian local culture to enhance number learning.

From a methodological perspective, previous PMRI design research has not systematically analyzed how local culture can be embedded within the stages of a learning trajectory, especially for developing number sense and symbolic understanding among early graders. From a contextual perspective, local traditions such as *melemang, a communal* activity in Muara *Enim* involving measuring, counting, and dividing *lemang* (sticky rice cooked in bamboo) have rarely been used as learning contexts, despite their strong mathematical relevance. Meanwhile, from a substantive perspective, the concept of whole numbers, including counting, ordering, comparison, and simple operations, remains insufficiently linked to meaningful real-life experiences that could help students transition from concrete to abstract understanding.

Therefore, this study seeks to fill these gaps by developing a Local Instructional Theory (LIT) for whole number learning using the *Muara Enim melemang* context based on the PMRI approach. The LIT developed in this research aims to describe the learning trajectory that connects students' informal strategies such as counting and comparing physical objects to formal representations using mathematical symbols. Methodologically, it extends design research by incorporating cultural elements as part of the HLT (Hypothetical Learning Trajectory) design and validation process. Theoretically, it contributes to the development of PMRI by elaborating how ethnomathematical contexts can function not merely as illustrations but as epistemological foundations for mathematical reinvention.

In practical terms, the *melemang* context is expected to help students understand the meaning of numbers more concretely through activities such as counting pieces of *lemang*, comparing quantities between plates, ordering the number of *lemang*, and performing addition or subtraction through the idea of "adding" or "eating" *lemang*. These culturally familiar activities align with the PMRI principle of guided reinvention, where students rediscover mathematical concepts embedded in daily life. Consequently, this research not only aims to construct a theoretically grounded LIT for whole number learning but also provides a practical, culturally responsive model that teachers can adopt to design contextual mathematics instruction in Indonesian primary schools.

In summary, this study is significant because it bridges theoretical, methodological, and cultural gaps in existing PMRI-based research. It contributes to the development of learning theory (LIT) by validating a culturally grounded learning trajectory and to the improvement of instructional practice by offering a concrete example of integrating local culture into mathematics learning. Through this integration, the study aspires to enhance the effectiveness, meaningfulness, and cultural relevance of whole number learning in early elementary education.

B. METHODS

This study employed a design research approach of the validation study type as proposed by Gravemeijer & Eerde (2009). Design research was chosen because it is suitable for developing and refining learning theories in authentic classroom contexts through iterative design and analysis cycles. The main objective of this approach is to produce a Local Instructional Theory (LIT) that connects the theoretical understanding of learning with practical instructional design. The research consisted of three main stages: (1) Preliminary Design, (2) Teaching Experiment, and (3) Retrospective Analysis. Each stage was interrelated in a cyclic manner where the findings from one stage served as the basis for refining the next stage ensuring coherence between design, implementation, and theoretical interpretation.

1. Preliminary Design

At this stage, the researchers conducted (a) curriculum analysis to identify learning goals and competencies related to whole numbers in the early grades; (b) a literature review on learning trajectories, PMRI principles, and ethnomathematics integration; and (c) classroom observations and teacher interviews to identify students' common misconceptions and contextual familiarity. Based on these data, a Hypothetical Learning Trajectory (HLT) was formulated, which included learning objectives, predicted student strategies, expected learning difficulties, and proposed instructional activities using the *melemang* context. The HLT functioned as a theoretical blueprint that would later be tested and refined in the teaching experiment stage.

2. Teaching Experiment

The teaching experiment stage consisted of two phases: pilot experiment and main teaching experiment (Cobb et al., 2003).

a. The pilot experiment involved a small group of students to test the feasibility of the HLT and the clarity of learning materials such as worksheets, *lemang* representations, and visual aids. Data from this phase were used to revise and refine the HLT design, including adjustments to the task sequence and wording of instructions.
b. The main teaching experiment was then conducted in a first-grade classroom with 28 students. During this phase, the revised HLT was implemented collaboratively by the researcher (as facilitator) and the classroom teacher (as co-teacher). Classroom activities were recorded through video observation, field notes, and student worksheets, focusing on students' strategies, interactions, and conceptual progress.

The purpose of this stage was not only to test the effectiveness of the designed learning trajectory but also to collect empirical evidence of students' actual learning paths, which later became the basis for comparison with the hypothetical trajectory.

3. Retrospective Analysis

The retrospective analysis aimed to systematically compare the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT) observed during classroom implementation. Following the analytical framework of Gravemeijer & Cobb (2006), this stage involved qualitative data analysis consisting of three main steps:

a. Data reduction – organizing field notes, transcripts, and student work by categorizing learning episodes according to each activity stage in the HLT.
b. Data display – developing narrative descriptions and diagrammatic maps showing the evolution of students' strategies and misconceptions across lessons.
c. Conclusion drawing and verification – interpreting patterns of students' reasoning and validating emerging findings through triangulation between data sources (observation, interviews, and student artifacts).

To enhance trustworthiness, the analysis employed methodological triangulation (comparing multiple data sources), peer debriefing with two PMRI experts, and iterative coding to ensure consistency in interpreting student learning processes. The findings of the retrospective analysis were then used to refine the initial HLT into a validated Local Instructional Theory (LIT**)**, a context-based theoretical model for teaching whole numbers using the *melemang* cultural context, as shown in Table 1.

Table 1. Methodological Flow

Research Stage	Main Activity	Output	Contribution to Next Stage
Preliminary Design	Curriculum & context analysis, formulation of HLT	Draft of Hypothetical Learning Trajectory	Serves as design prototype for classroom implementation
Teaching Experiment	Pilot and main implementation, classroom observation	Actual student responses & strategies (ALT)	Provides empirical data for comparison and refinement
Retrospective Analysis	Qualitative data coding, triangulation, and theory refinement	Validated Local Instructional Theory (LIT)	Produces theoretical and practical contribution for PMRI

With this structured procedure, the methodology ensured a clear connection between design, implementation, and analysis. Each stage progressively contributed to the development and validation of the Local Instructional Theory, demonstrating how the integration of PMRI principles and the *melemang* cultural context supports students' conceptual understanding of whole numbers in a meaningful and culturally relevant way.

C. RESULT AND DISCUSSION

1. Preliminary Design

Curriculum analysis and teacher interviews revealed that most early-grade students experienced difficulties in linking numbers to quantities. They tended to memorize number sequences without understanding their meaning and often made errors when comparing numbers or writing inequality symbols. Based on these findings, the researchers designed a Hypothetical Learning Trajectory (HLT) that integrated the *melemang* cultural context as a meaningful representation to support conceptual understanding.

The HLT consisted of six learning stages: (1) recognizing numbers through *lemang* pieces, (2) understanding place value, (3) comparing and ordering numbers, (4) composing and decomposing numbers, (5) performing addition and subtraction using *lemang*, and (6) an introduction to simple fractions. The design followed PMRI principles guided reinvention and progressive mathematization allowing students to discover mathematical concepts through familiar, context-based activities. The HLT also predicted possible learning difficulties, such as confusion with the direction of inequality signs and the transition from concrete representations to abstract symbols. This hypothetical model became the theoretical foundation tested in the teaching experiment phase, as shown in Table 2.

Table 2. Hypothetical Learning Trajectory

Stage	Learning Goals	Learning Activities (Contextual Tasks)	Predicted Student Strategies/Learning Hypothesis	Teacher's Role	PMRI Principles Involved
1. Recognizing Whole Numbers	Students can recognize and count the number of *lemang* pieces (1–20).	Students count *lemang* pieces on a plate or in pictures and say the numbers aloud.	• Count with repetition or skipping numbers. • Gradually use one-to-one correspondence.	Provide *lemang* models, guide counting one by one, and encourage	Real context, student contribution

Stage	Learning Goals	Learning Activities (Contextual Tasks)	Predicted Student Strategies/Learning Hypothesis	Teacher's Role	PMRI Principles Involved
			• Understand that the last number represents total quantity.	verification among peers.	
2. Understanding Place Value (Tens and Ones)	Students understand the composition of numbers using groups of tens and ones.	Students group *lemang* into bundles of ten and count remaining singles (e.g., 25 = 2 tens + 5 ones).	• Initially count all objects. • Begin to group by tens. • Recognize patterns and connect to positional notation.	Guide grouping by tens and ask probing questions ("How many tens? How many ones?").	Modeling, progressive mathematization
3. Comparing and Ordering Numbers	Students can compare two numbers and arrange them in ascending or descending order.	Students compare the number of *lemang* on two plates and arrange number cards from smallest to largest.	• Use visual or one-to-one comparison. • State "more" or "less" correctly but may misuse symbols (> <). • Gradually connect relational meaning to symbols.	Facilitate discussion, highlight comparison symbols, and use visual aids (e.g., crocodile mouth model).	Interactivity, reflection
4. Composing and Decomposing Numbers	Students can compose and decompose numbers into parts (e.g., 10 + 5 = 15).	Students use *lemang* pieces to form different combinations (e.g., 12 = 10 + 2 or 6 + 6).	• Initially find one combination. • Later explore multiple decompositions. • Understand flexible number partitioning.	Encourage exploration of multiple solutions and peer comparison.	Student contribution, intertwinement
5. Addition and Subtraction through *Melemang* Context	Students can perform simple addition and subtraction (≤20) meaningfully.	• Addition: Combine *lemang* from two plates. • Subtraction: Model "eating" or "removing" *lemang*.Example: "If there are 15 *lemang* and 5 are eaten, how many remain?"	• Start with counting all, then counting on. • Physically remove *lemang* for subtraction. • Possible confusion with subtraction symbols.	Demonstrate processes, provide guiding questions, and support transition to symbolic notation.	Guided reinvention, realistic modeling
6. Introduction to Fractions through *Lemang* Sharing	Students understand fractions as parts of a whole.	Students cut *lemang* into halves or quarters and discuss fair sharing.	• Think smaller pieces mean fewer. • Gradually understand equal parts as fair division. • Recognize simple fractions (½, ¼).	Use real or visual *lemang* models, emphasize equal size, and connect to fractional notation.	Intertwinement, contextualization

2. Teaching Experiment

This stage consisted of two phases: a pilot experiment and a main teaching experiment.

a. Pilot Experiments

The pilot experiments stage is an initial trial conducted to evaluate the feasibility of learning activities, instruction in the worksheet, and the use of the *melemang* context as a medium for learning whole numbers. Cobb et al. (2003) emphasize the importance of pilot experiments in design research to adapt the initial design to classroom reality, because the learning trajectory is essentially still hypothetical.

In this stage, the researcher involved 8 first-grade students of SDN 1 Lawang Kidul with varying abilities (low, medium, high). The activities tested included: counting the number of *lemang*, comparing the number of *lemang* on two plates, and simple addition by adding pieces of *lemang*. During the implementation, the researcher observed the strategies used by students, such as counting one by one, matching one to one, or directly stating the results based on memorized numbers. The data obtained were in the form of observation notes, video recordings, student work results, and short interviews regarding the clarity of instructions.

The results of the pilot experiments showed that the *lemang* context was able to attract students' attention because it was close to their culture. However, several obstacles were found, for example, some students were still confused about writing numbers greater than 20, and had difficulty using the symbols <, >, and =. Based on these findings, the LKPD was then revised by adding clearer visual examples, using small numbers first (≤20), and emphasizing concrete concepts before transitioning to symbols. This is in accordance with the view of Gravemeijer & van Eerde (2009) that design research is cyclical and requires revision based on the results of the initial implementation.

b. Teaching Experiments

Following revisions to the pilot experiments, the teaching experiments phase was implemented on a full-class scale, involving 28 first-grade students at SDN 1 Lawang Kidul. According to Gravemeijer & Cobb (2006), teaching experiments aim to test and validate the designed learning trajectory and understand how students' learning develops in a real-world context. The teaching experiments, covering Whole Numbers up to 100, using *lemang* as a context, began with counting the number of *lemang* in each picture, as shown in Figure 1.

Figure 1. Activity 1 number sense in whole numbers up to 100

In activity 1, students were able to identify that Picture A contains 10 *lemang*, Picture B 15 *lemang*, Picture C 20 *lemang*, Picture D 30 *lemang*, and Picture E 25 *lemang*. Most students counted the objects individually, although some tried to group them according to a pattern. This is in line with the findings of Fuson & Beckmann (2021) that counting concrete objects can strengthen number sense and early understanding of numbers, as shown in Figure 2.

Figure 2. Activity 2 determine place value

In the place value activity, students are guided to break down the number of *lemang* into tens and units. For example, 10 *lemang* is written as 1 tens and 0 units, 15 as 1 tens and 5 units, and 25 as 2 tens and 5 units. Some students incorrectly wrote 15 = 15 tens, but after guidance, they were able to correct it. Ross (2020) emphasized that understanding place value must be built through concrete representations, so that students do not simply memorize symbols. In the dialogue, the teacher asked: "If 25 *lemang* can be written as how many tens and how many units?" and the students answered "Two tens and five units." The teacher then emphasized the importance of separating tens and units to make numbers easier to understand. The next activity is to compare and sort the number of *lemang*. Students are asked to sort the pictures from the largest to the smallest number, so that the order obtained is: Picture D (30), Picture E (25), Picture C (20), Picture B (15), and Picture A (10), as shown in Figure 3.

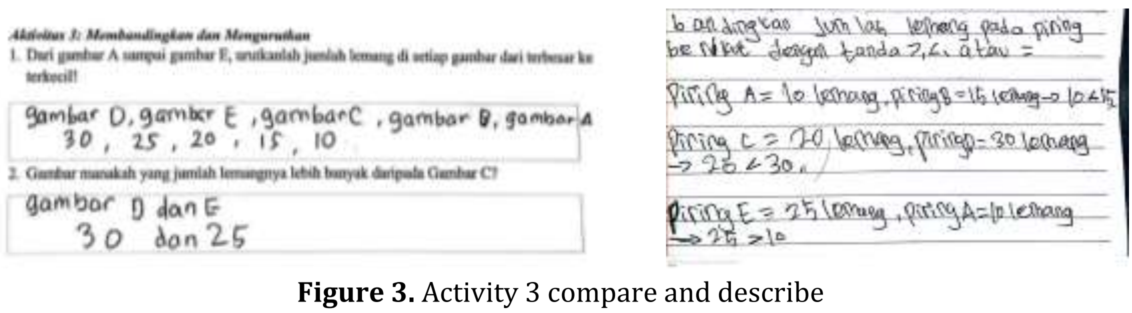

Figure 3. Activity 3 compare and describe

Some students were initially confused about the difference between 25 and 30, but eventually solved the problem correctly. According to Van de Walle et al. (2019), the ability to compare numbers is crucial for developing number sense, especially when supported by visual representation. The teacher reinforced understanding by asking, "*Why is 30 the largest number?*" and the students answered, "*Because it has the most lemang*." as shown in Figure 4.

Figure 4. Addition and subtraction

Finally, addition and subtraction activities were carried out using the context of *lemang*. Students were able to correctly solve problems, for example, 10 + 9 = 19, 15 – 5 = 10, and 20 + 30 = 50. Some students still counted manually, but the concrete context helped them understand simple arithmetic operations. NCTM (2020) emphasizes the importance of using real-world contexts in addition and subtraction to make the concepts more meaningful. In the dialogue, the teacher asked: "If there are 10 *lemang* and then 9 are added, how much is left?" and the student answered, "19, Ma'am." The teacher then asked another problem: "If 15 is subtracted from 5, what is left?" and the student answered, "10, Ma'am." Thus, the teaching experiments demonstrated that using the local cultural context of *lemang* helps students understand the concept of whole numbers more concretely, both in counting, determining place value, comparing, arranging and decomposing, and performing simple arithmetic operations.

3. Retrospective Analysis

The retrospective analysis stage was conducted by comparing the learning trajectories predicted in the HLT with the actual trajectories (ALT) encountered during classroom implementation. In line with Gravemeijer & Cobb (2006), the retrospective analysis aims to assess the extent to which the learning hypothesis aligns with actual practice and to identify aspects that need improvement. The analysis results indicate that, in general, the learning of whole numbers using the Muara *Enim melemang* context proceeded according to the predictions in the HLT. In the activity of counting *lemang*, students were able to count forwards and backwards effectively up to 20. In the activity of comparing the number of *lemang*, students were able to use both one-to-one correspondence and direct counting strategies, in line with initial predictions. Similarly, in the activity of arranging numbers in order, most students successfully arranged the numbers in ascending and descending order without experiencing significant difficulties.

In additions and subtraction activities using *lemang*, students were able to grasp basic operational concepts through concrete representations. The context of "eating *lemang*" proved to facilitate understanding of the concept of subtraction, and several students began to demonstrate a transition from the "counting all" strategy to the "counting on" strategy. Thus, the development of students' number sense in recognizing, comparing, and operating with simple numbers followed the predicted path in HLT.

However, slight differences were found in the use of mathematical symbols, particularly in writing the greater-than (>) and less-than (<) signs. Some students still experienced confusion in determining the direction of the sign even though they were conceptually able to compare two numbers correctly. For example, when learning that 30 is greater than 25, some students still wrote "30 < 25." This suggests that the difficulty arises not in understanding quantity, but rather in symbolic representation skills.

Thus, the retrospective analysis confirmed that the designed learning trajectory proved effective and in line with the predictions of the HLT, with the exception of the aspect of writing comparison symbols that required additional reinforcement. The resulting LIT revision emphasized the need to provide more scaffolding in the introduction of mathematical signs (> and <), for example by using visual aids such as crocodile mouths or directional arrows, so that the transition from concrete understanding to symbolic representation can be smoother.

Table 3. Comparison of HLT and ALT

Stage of Activity	HLT (Hypothetical Learning Trajectory)	ALT (Actual Learning Trajectory)
1. Counting *Lemang*	Students are able to count forward and backward from 1–20 using concrete *lemang*.	Students fluently counted up to 20, with only a few performing skip counting.
2. Place Value (Tens & Ones)	Students understand the composition of tens and ones, for example 14 = 1 ten + 4 ones.	Most students understood the concept, but some still counted all from the beginning.
3. Comparing Numbers	Students compare two numbers and write the symbols >, <, = correctly.	Students could compare quantities correctly, but some still reversed the direction of the > and < symbols.
4. Ordering Numbers	Students are able to arrange numbers in ascending and descending order up to 20.	Most students successfully arranged numbers in both ascending and descending order, with only a few still needing help in backward order.
5. Adding *Lemang*	Students add by combining two groups of *lemang*, then write the result with the + symbol.	Students were able to add, but most still used the counting all strategy, while some had started using counting on.
6. Subtracting *Lemang*	Students understand subtraction as taking away/eating *lemang*, then write the result with the – symbol.	Students were able to subtract correctly through the “*lemang* eaten” context, but some still reversed the symbol (e.g., writing 8–3 as 3–8).

Stage of Activity	HLT (Hypothetical Learning Trajectory)	ALT (Actual Learning Trajectory)
7. Introduction to Fractions	Students understand fractions as parts of a whole (½, ¼).	Students recognized simple fractions through pieces of *lemang*, although some thought smaller pieces meant fewer even though equal in size.

The results of the study indicate that the learning trajectory of whole numbers using the Muara *Enim melemang* cultural context generally aligns with predictions in the Hypothetical Learning Trajectory (HLT). However, in the actual implementation (ALT), obstacles were still encountered in the use of mathematical symbols, especially in writing the greater than (>) and less than (<) signs, as well as in subtraction which was sometimes written backwards. Overall, the *melemang* context proved effective in providing meaningful concrete experiences, in line with the principles of Indonesian Realistic Mathematics Education (PMRI) which emphasize the importance of real contexts and local culture (Zulkardi & Putri, 2019).

a. Counting Numbers with *Lemang*

   In the counting activity, most students were able to count forwards and backward to 20. However, some students still skipped counting, for example, forgetting to say "12." This aligns with Fuson's (2019) findings that young children tend to rely on memorizing number sequences, making it easy to make mistakes with numbers greater than 10. The teacher then provided scaffolding by directing students to point to each piece of *lemang* when counting, in accordance with the one-to-one correspondence theory (Clarke, 2015). This strategy effectively reinforces students' understanding that each object is counted only once.

b. Comparing Quantities

   When comparing the number of *lemang*, students can conceptually understand which plate is larger. However, difficulties arise when writing comparison symbols. Some students still miswrite the symbols, for example, writing "30 < 25" even though they have correctly stated "six is more than four" verbally. This phenomenon is common among early grade students, as explained by McIntosh (2020), who notes that the error lies not in understanding quantity, but rather in the direction of the symbol. The use of visual scaffolding in the form of a "crocodile mouth" metaphor has been shown to help students internalize the meaning of the greater-than and less-than signs, in line with research by Yang (2022).

c. Arranging Numbers

   In the number ordering activity, the majority of students were able to arrange the numbers in ascending order correctly, but some still had difficulty arranging the numbers backward. This confirms Van den Heuvel-Panhuizen's (2020) observation that backward ordering requires greater flexibility in thinking because students must understand the inverse relationships between numbers. Classroom interactions, where students collaboratively corrected each other's mistakes, demonstrated how mathematical discussions contribute to collective learning (Cobb et al., 2003).

d. Addition Operations with *Lemang*

In addition, students were able to correctly solve problems using the context of combining *lemang* pieces. However, the strategies used varied: most used counting all, while some students began to use counting on. This development is consistent with the developmental models of addition strategies according to Carpenter & Moser (1984) and Clements & Sarama (2014). The teacher encouraged the use of the counting on strategy because it is more efficient, for example, starting with the first number and then adding the second.

e. Subtraction with *Lemang*

Subtraction activities using the context of "eating *lemang*" make it easier for students to understand that subtraction means taking a part from a whole.

f. Integration of Local Culture in Mathematics Learning

Using *lemang* as a local cultural context has been shown to increase student engagement and facilitate understanding of number concepts. This finding supports the ethnomathematics approach (Rosa & Orey, 2016), where cultural context provides additional meaning and bridges abstract understanding with students' real-life experiences. Furthermore, this approach aligns with the PMRI principle, which emphasizes the importance of guided reinvention through contexts relevant to everyday life (Zulkardi & Putri, 2019). The following is a synthesis of HLT, ALT, discussion, and implications, as shown in Table 4.

Table 4. Synthesis of HLT, ALT, Discussion, and Implications

Activity	HLT (Prediction)	ALT (Classroom Findings)	Discussion (Theoretical Analysis)	Implications/ Improvements
Counting *Lemang*	Students are able to count forward and backward from 1–20 using *lemang*.	Most students counted fluently, only a few skipped *counting*.	Errors occurred because students still relied on memorization (Fuson, 2019). *One-to-one correspondence* scaffolding helps (Clarke, 2015).	Practice counting while pointing to each *lemang*, and use backward counting games.
Comparing Numbers	Students compare two quantities and write the >, <, = symbols correctly.	Students could compare quantities but some wrote the symbols in the wrong direction.	The issue lies in symbolic representation, not concept (McIntosh, 2020). The "crocodile mouth" visual is effective (Yang, 2022).	Use visual scaffolding (crocodile) and exercises writing symbols in story contexts.
Ordering Numbers	Students are able to arrange numbers in ascending and descending order up to 20.	Students successfully arranged numbers in ascending order; some still made errors in descending order.	Descending order requires higher numerical flexibility (Van den Heuvel-Panhuizen, 2020). Classroom discussion strengthens understanding (Cobb et al., 2003).	Provide more practice with backward counting through games or songs.

Activity	HLT (Prediction)	ALT (Classroom Findings)	Discussion (Theoretical Analysis)	Implications/ Improvements
Adding *Lemang*	Students combine two groups of *lemang* and write the result using the + symbol.	Students added correctly; most still used *counting all*, while a few began to use *counting on*.	Strategy development aligns with Carpenter & Moser's (1984) and Clements & Sarama's (2014) models.	Train *counting on* by giving the starting number, then letting students continue the count.
Subtracting *Lemang*	Students understand subtraction as taking away/eating *lemang* and then write the – symbol.	Students understood the concept, but some reversed the symbol (e.g., wrote 3–8 instead of 8–3).	Children often grasp the concept but miswrite the symbol (Askew, 2019). The transition from concrete to symbolic must be gradual (Clements, 2020).	Focus remedial lessons on writing operation symbols by linking story problems → concrete models → symbols.
Integrating *Lemang* Culture	The *lemang* context supports whole number learning.	Students were actively engaged and more easily understood the concepts.	Cultural contexts enhance relevance and motivation for learning (Rosa & Orey, 2016; Zulkardi & Putri, 2019).	Integration of local culture should be maintained and expanded in PMRI for other topics.

D. CONCLUSION AND SUGGESTIONS

This study shows that the learning trajectory for whole numbers using the Muara *Enim melemang* cultural context based on the Indonesian Realistic Mathematics Education (PMRI) generally aligns with the predictions of the Hypothetical Learning Trajectory (HLT). First-grade elementary school students were able to count, compare, sequence, and perform addition and subtraction effectively using the concrete representation of *lemang*. Only minor discrepancies were observed in the Actual Learning Trajectory (ALT), specifically in students' difficulties writing the greater-than (>) and less-than (<) signs, as well as the subtraction symbol. These findings indicate that the gap lies not in conceptual understanding but in symbolic representation. The *melemang* context has proven effective in enhancing student engagement, motivation, and meaningful mathematical understanding. Therefore, integrating local culture within the PMRI framework can effectively bridge students' transition from concrete experiences to abstract reasoning. Future research is recommended to explore how other ethnomathematical contexts across diverse Indonesian regions can be systematically incorporated into PMRI-based design research to develop culturally grounded learning trajectories for different mathematical domains.

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REFERENCES

Alangui, W. V. (2020). Beyond songs and dances: Ethnomathematics and the challenge of culture. *Revista Latinoamericana de Etnomatemática, 13*(3), 88–107. Retrieved from https://www.redalyc.org/journal/2740/274065861006/274065861006.pdf

Askew, M. (2019). *Transforming primary mathematics: Understanding classroom tasks, tools and talk.* Routledge. https://doi.org/10.4324/9780203806746

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers.* Routledge. Retrieved from api.taylorfrancis.com

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 7–44). Academic Press. https://doi.org/10.5951/jresematheduc.15.3.0179

Clarke, D. (2015). Early number sense: The key to later mathematics learning. *Australian Primary Mathematics Classroom, 20*(4), 4–9. https://primarystandards.aamt.edu.au/

Clements, D. H. (2020). Linking research and practice: The case of learning trajectories in early mathematics education. *ZDM Mathematics Education, 52*(1), 33–45. https://doi.org/10.1007/s11858-019-01122-z

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). Routledge. https://www.taylorfrancis.com/books/mono/10.4324/9780203520574/

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. https://doi.org/10.3102/0013189X032001009

Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM Mathematics Education, 46*(5), 719–733. https://doi.org/10.1007/s11858-014-0632-0

Doorman, M., Drijvers, P., Gravemeijer, K., Boon, P., & Reed, H. (2012). Design research: A powerful tool to improve education. *ZDM Mathematics Education, 44*(3), 501–512. https://doi.org/10.1007/s11858-012-0418-4

Fuson, K. C. (2019). Early whole number learning and development. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 423–456). NCTM. https://doi.org/10.1007/s11858-012-0418-4

Gravemeijer, K. (1994). *Developing realistic mathematics education.* Freudenthal Institute. https://research.tue.nl/en/publications/developing-realistic-mathematics-education?utm_source=chatgpt.com

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105–128. https://doi.org/10.1207/s15327833mtl0602_2

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17–51). Routledge. https://www.fi.uu.nl/publicaties/literatuur/EducationalDesignResearch.pdf?utm_source=chatgpt.com

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The Elementary School Journal, 109*(5), 510–524. https://doi.org/10.1086/597397

Jordan, N. C., Glutting, J., & Ramineni, C. (2013). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 23*, 123–128. https://doi.org/10.1016/j.lindif.2012.11.003

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* National Academy Press. https://www.nap.edu/catalog/9822/adding-it-up-helping-children-learn-mathematics

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* NCTM. https://www.nctm.org/standards/

Rosa, M., & Orey, D. C. (2011). Ethnomathematics: The cultural aspects of mathematics education. *Revista Latinoamericana de Etnomatemática, 4*(2), 32–54. https://revista.etnomatematica.org/index.php/RevLatEm/article/view/32

Rosa, M., & Orey, D. C. (2016). Ethnomathematics and its pedagogical action in mathematics education. *Journal of Mathematics and Culture, 10*(3), 75–102. DOI: https://doi.org/10.1007/978-3-319-30120-4_3

Seah, W. T. (2020). Values in mathematics education: Its place and potential. *International Journal of Science and Mathematics Education, 18*(1), 1–18. https://doi.org/10.1007/s10763-019-09983-0

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM Mathematics Education, 40*(6), 927–939. https://doi.org/10.1007/s11858-008-0125-9

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145. https://doi.org/10.2307/749877

Sowder, J. T. (2021). Developing number sense in young children: A research-based approach. *Early Childhood Research Quarterly, 54*, 25–38. https://doi.org/10.1016/j.ecresq.2020.101021

Van den Heuvel-Panhuizen, M. (2020). Didactical phenomenology applied to mathematics education. In M. van den Heuvel-Panhuizen (Ed.), *National reflections on the Netherlands didactics of mathematics* (pp. 75–92). Springer. DOI: https://doi.org/10.1007/978-3-030-33824-4

Yang, D. C. (2022). Using visual scaffolds to support young children's understanding of mathematical symbols. *Early Childhood Education Journal, 50*(5), 865–878. https://doi.org/10.1007/s10643-021-01274-2