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Arithmetic Adventure: A Role-Playing Game to Support Students' Mathematical Thinking on Arithmetic Sequences

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ABSTRACT

Students often experience difficulties in mathematical thinking, particularly in specializing, generalizing, conjecturing, and convincing, when learning arithmetic sequences. At the same time, research on narrative based Role Playing Games in developing these skills within Indonesian classrooms remains limited. This study develops an educational Role Playing Game titled Arithmetic Adventure: The Mystery of Polaria Village that embeds mathematical thinking processes into contextual storyline tasks. A Development Research approach using the ADDIE model was selected because its systematic and iterative structure supports continuous refinement of digital learning media. Participants were 34 tenth grade students from SMA Negeri 10 Palembang. Data were collected through expert validation, student practicality questionnaires, and pretest and posttest assessments of mathematical thinking. The game achieved a validity score of 4.45 which is categorized as very valid across content, construct, language, and ICT aspects. Practicality testing showed a score of 86.25% which is categorized as very practical. Students' mathematical thinking improved significantly from the pretest mean of 54.06 to the posttest mean of 73.65. A paired sample t test confirmed the improvement as statistically significant with a value of t equal to minus 10.461 and a significance value less than 0.001. The effect size was large with a Cohen value of 1.79. These findings indicate that the game effectively supported students' progression from pattern recognition to formal justification. The study contributes theoretically by illustrating how narrative Role Playing Games can strengthen mathematical thinking and practically by providing an innovative digital medium aligned with curriculum needs.

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A. INTRODUCTION

Arithmetic sequences are one of the fundamental topics in mathematics education that form the basis for mastering more advanced concepts such as infinite series, limits, and exponential functions. This topic also plays an important role in connecting arithmetic reasoning with real-life contexts, for example, in calculating loan interest, installments, and population growth (Shah, 2024). However, despite its importance, students' understanding of arithmetic sequences remains relatively low in many educational contexts. Various studies have reported that students experience persistent misconceptions and difficulties when learning this topic (Aminah et al., 2023; Ferdiani et al., 2022; Hidayat et al., 2022). These difficulties include connecting concepts to real situations, determining the nth-term formula, applying the formula correctly in problem-solving, and formulating or solving verbal arithmetic problems that require higher levels of mathematical thinking (Mateus-Nieves & Díaz, 2021).

In addition, many studies highlight that students' skills in dealing with abstract mathematical concepts, including arithmetic sequences and series, have not developed optimally such as those by (Ishartono et al., 2022; Setiana et al., 2021). This condition is partly caused by the dominance of teacher-centered instruction and the lack of learning activities that promote exploration and contextual reasoning (Cevikbas & Kaiser, 2022; Khasawneh et al., 2023; Woods & Copur-Gencturk, 2024). Consequently, students are often unable to apply arithmetic sequence concepts to unfamiliar or real-world problems (Kohen & Orenstein, 2021; Uyen et al., 2021). This issue is further reflected in the 2022 PISA results, where Indonesia remains at a low performance level with an average mathematics score of 366, which is 106 points below the OECD average, indicating that most students only reach level 2 (Lestari et al., 2021). In line with these findings, Indonesian students still struggle to grasp problem contexts, connect symbolic representations with contextual patterns, gather relevant information, and apply or evaluate problem-solving strategies effectively (Kania et al., 2024; Utomo & Syarifah, 2021). These findings underscore the importance of rethinking mathematics instruction to better support conceptual understanding, critical thinking, and higher-order thinking (Angraini et al., 2024; Kania et al., 2025).

This issue highlights the need for a learning approach that not only emphasizes mastery of procedures, but also develops students' mathematical thinking skills in greater depth. Mathematical thinking skills include specializing (using examples in a problem), generalizing (identifying patterns), conjecturing (formulating assumptions or predictions), and convincing (presenting logical and evidence-based arguments) (Susanti et al., 2025). However, current instructional practices remain dominated by procedural exercises and one-way explanation, which limit students' opportunities to construct and test their own ideas (Boaler et al., 2022). Hence, strengthening students' mathematical thinking through exploratory and contextual learning experiences has become a key priority in 21st-century mathematics education (Ningrum et al., 2025).

Building on this need, the challenges of mathematics learning in Indonesia are becoming increasingly complex as students are now expected to master not only mathematical thinking but also broader 21st-century competencies in line with the national vision of the Merdeka Curriculum (Suryaningsih et al., 2023). The 6C competencies (Critical thinking, Creativity, Communication, Collaboration, Citizenship, and Character) can only be developed through dynamic, interactive, and learner-centered learning (Fitriani, 2024). The use of learning technology is one way to achieve these goals, as it can provide interactive media that supports student engagement, facilitates independent learning, and encourages the development of higher-order thinking skills (Abu-Rasheed et al., 2023). This perspective aligns with the previous discussion on mathematical thinking, as both emphasize active exploration and reasoning rather than rote procedures.

In line with the direction of the Merdeka Curriculum, integrating technology in mathematics learning has become increasingly relevant as it enhances student motivation, creativity, and active participation (Kusumawati & Umam, 2025). The main challenge, however, is ensuring that this integration aligns with the Pancasila Student Profile, which values curiosity, collaboration, and reflective thinking (Nurdiana et al., 2024). Innovative digital learning designs such as Role-Playing Games (RPGs) offer a promising solution, allowing students to engage in

contextual exploration and narrative-based problem solving that mirror real-life values (Cullinan & Genova, 2022; Westborg, 2022).

This connection also bridges global literature on game-based learning with Indonesia's educational priorities, demonstrating how technology can serve both global and national educational agendas (Wardoyo et al., 2021). Despite the growing interest in game-based learning, few studies have specifically explored how RPG designs can foster mathematical thinking processes, such as specializing, generalizing, conjecturing, and convincing, within the context of the Merdeka Curriculum. This study addresses that gap by developing an RPG-based learning design that integrates mathematical reasoning with 21st-century competencies. The novelty of this research lies in its dual focus: promoting deep mathematical thinking while concretely supporting the competency-based and exploratory learning principles of the Merdeka Curriculum.

Building upon this foundation, one of the most promising technological innovations is the use of educational games, particularly in the form of Role-Playing Games (RPGs). RPGs as a learning medium provide an immersive and contextual learning experience through storylines, exploration, and decision-based challenges. Previous studies have shown that RPGs can increase student motivation, engagement, and conceptual understanding (Huynh et al., 2020; Suryani et al., 2025; Zhao et al., 2022), while also supporting strategic thinking and logical decision-making skills (Gui et al., 2023). With the availability of development platforms such as RPG Maker, teachers and researchers can design flexible educational RPGs that are easily accessible via Android devices and support mobile learning (Arianti et al., 2024). Compared to other digital learning media such as GeoGebra, Kahoot, or interactive simulations, RPGs provide a deeper learning experience by combining stories, freedom of choice, and emotional engagement that help students learn through meaningful exploration (Korkeila & Harviainen, 2023).

However, research on the use of RPGs in mathematics learning, especially on the topic of arithmetic sequences, is still relatively limited. Most previous studies have emphasized student motivation or interest in learning, while the explicit development of mathematical thinking skills through RPGs has not been widely studied. In fact, the integration of RPGs with mathematical thinking indicators such as specializing, generalizing, conjecturing, and convincing has the potential to provide a learning experience that is not only enjoyable but also conceptually meaningful. Previous studies have mostly focused on motivational outcomes or general engagement without explicitly addressing the development of higher-order thinking or mathematical reasoning through game-based learning (Nurdiana et al., 2024; Rahim et al., 2024; Rifayanti et al., 2024)**.** Therefore, this research differs by positioning RPGs not merely as motivational tools but as structured learning environments designed to cultivate students' mathematical thinking in arithmetic sequences (Anagnostopoulou, 2024). Thus, there is a research gap that can be filled through the development of Arithmetic Adventure.

This study aims to develop a valid and practical RPG learning medium titled Arithmetic Adventure that can support students' mathematical thinking skills in arithmetic sequences. Through the integration of narrative, exploration, and interactivity elements, Arithmetic Adventure is expected to address the challenges of mathematics learning today, while providing a fun, contextual, and meaningful learning experience. The validity and practicality

of the game will be evaluated through expert reviews and student trials, while its effectiveness will be examined using pretest–posttest designs (Amir et al., 2023) In line with the findings of So & Gaydos (2024) on the effectiveness of digital games in education, this study is expected to provide a theoretical contribution in the form of strengthening the literature on the role of RPGs in developing mathematical thinking skills, as well as a practical contribution in the form of providing innovative alternative learning media that can be used by teachers in the classroom and by students independently.

B. METHODS

1. Research Type and Design

This study employed a Development Research approach using the ADDIE model, which consists of five systematic stages: Analyze, Design, Development, Implementation, and Evaluation (Branch, 2010). The ADDIE model was chosen instead of other instructional design models such as 4D, Kemp, or Hannafin and Peck because it provides a comprehensive and iterative framework that combines product development and evaluation within a continuous cycle. Its flexibility enables formative evaluation at every stage, ensuring that the developed educational product, an RPG-based learning game, meets the criteria of validity, practicality, and potential effectiveness in improving students' mathematical thinking skills

The development process followed the five stages sequentially. In the Analyze stage, the researchers conducted a needs analysis, curriculum analysis, and an analysis of students' mathematical difficulties related to arithmetic sequences, as well as an examination of school facilities and infrastructure that support game-based learning. The Design stage involved creating the material structure, storyline, and game storyboard aligned with mathematical thinking indicators, along with preparing the research instruments. The Development stage resulted in the first prototype (Prototype I), which was validated by subject-matter experts, media experts, and language experts, and then revised based on their feedback. The Implementation stage included field testing of the revised prototype to measure its practicality and to assess the potential impact of the educational RPG on students' mathematical thinking skills through classroom application. Finally, the Evaluation stage was conducted formatively throughout each phase and summatively at the end to assess the overall quality of the product, as shown in Figure 1.

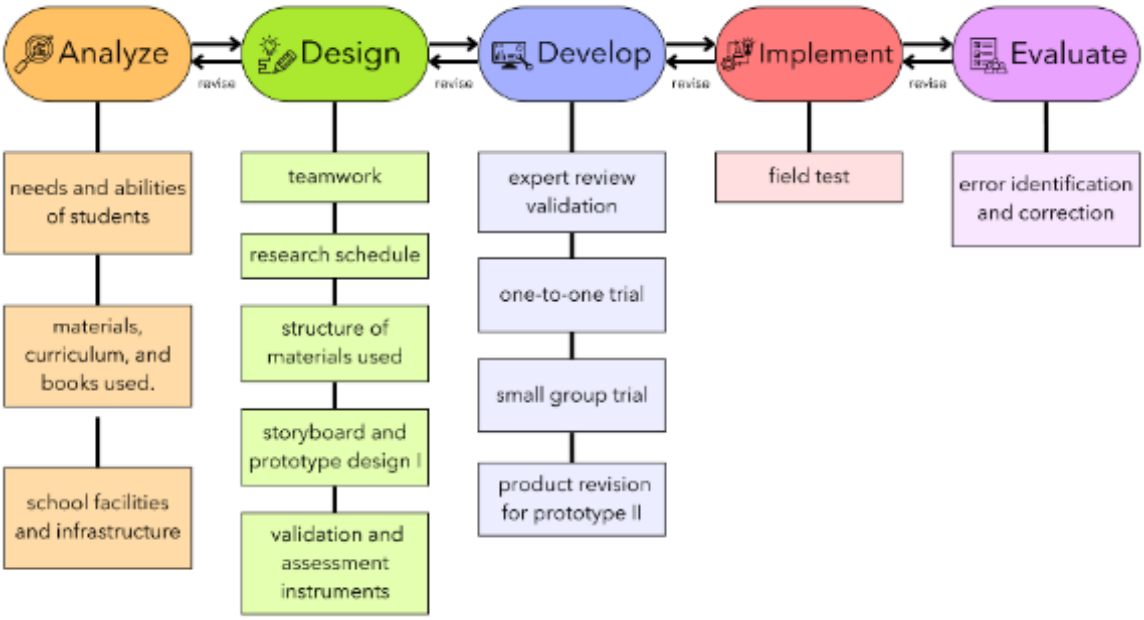


Figure 1. Flowchart of development stages using the ADDIE model

2. Research Subjects and Context

The research was conducted at SMA Negeri 10 Palembang during the first semester of the 2025/2026 academic year (September 2025). The participants were 34 tenth-grade students selected through purposive sampling based on the following criteria: (1) students had studied arithmetic sequences, (2) represented heterogeneous academic abilities, and (3) had implemented the Merdeka Curriculum. This curriculum emphasizes student-centered and competency-based learning, making it compatible with interactive and exploratory digital learning media such as RPGs. The expert validators consisted of three groups: content experts (mathematics education specialists), media experts (educational technology professionals), and language experts (Indonesian language and educational linguistics specialists).

3. Research Instruments

The research instruments used in this study consisted of three main types: expert validation sheets, student practicality questionnaires, and mathematical thinking tests. In addition to these primary instruments, supporting tool such as the student worksheet (LKPD) was incorporated into the learning process to strengthen procedural guidance and ensure consistent assessment. The descriptions of each instrument are provided below. The expert validation form was designed to evaluate the content, construct, technological, and linguistic aspects of the developed RPG using a five-point Likert scale, ranging from Strongly Disagree (1) to Strongly Agree (5) (Ličen et al., 2023). The validation data were analyzed quantitatively using the mean score formula, and qualitatively through written feedback and suggestions from validators as a basis for product improvement. The results were interpreted based on the validity criteria adopted from Cruchinho et al. (2024), as shown in Table 1.

Table 1. Validity Criteria

Average Score (R)	Category
$4.21 < R \leq 5$	Very Valid
$3.41 < R \leq 4.21$	Valid
$2.61 < R \leq 3.40$	Quite Valid
$1.81 < R \leq 2.60$	Invalid
$1.00 < R \leq 1.81$	Very Invalid

The student practicality questionnaire contained ten statements (seven positive and three negative), measured using a five-point Likert scale from Strongly Disagree (1) to Strongly Agree (5). The questionnaire assessed effectiveness, interactivity, efficiency, and creativity of the educational RPG during learning. The practicality scores were analyzed in percentage form and interpreted using the criteria shown in Table 2. Descriptive analysis of student comments was also conducted to identify improvement suggestions.

Table 2. Practicality Criteria

Average Score (N)	Category
$85 < N \leq 100$	Very Practical
$70 < N \leq 85$	Practical
$55 < N \leq 70$	Quite Practical
$70 < N \leq 55$	Less Practical

The mathematical thinking test consisted of essay-type questions designed to measure students' thinking skills based on four indicators proposed by Stacey (2006): specializing, generalizing, conjecturing, and convincing. Each item was scored on a 0–4 scale and converted to a 0–100 scale using the formula**:**

$$Score = \frac{obtained\ score}{maximal\ score} \times 100 \quad (1)$$

Students' mathematical thinking ability was classified using the value categories shown in Table 3.

Table 3. Value Category Range

Value	Category
$81 - 100$	Very Good
$61 - 80$	Good
$41 - 60$	Fair
$21 - 40$	Poor
$0 - 20$	Very Poor

A product is considered to have a potential effect if at least 70% of students achieve a Good or Fair category on the posttest results, in accordance with educational effectiveness standards that state instructional materials should enable most students to reach acceptable performance levels. As additional learning support, a student worksheet (LKPD) was also developed to accompany the RPG. The LKPD guided students in recording observations, writing intermediate reasoning, and documenting mathematical patterns discovered during gameplay. While the

LKPD was not evaluated as a standalone product, it functioned as a structured learning scaffold to assist students in engaging with the RPG tasks and demonstrating their reasoning processes. Together with the RPG and test instruments, the LKPD provided a cohesive set of materials designed to strengthen the learning experience and ensure alignment with the mathematical thinking indicators.

4. Validation, Testing Design, and Data Collection Procedure

Instrument validation involved three expert validators, each providing assessments using the validation form and rubric. Inter-rater agreement and consistency were checked qualitatively to ensure instrument reliability, while the quantitative scores were averaged for validity interpretation. Product validation was followed by limited trials (one-to-one and small-group testing) to obtain initial feedback, which informed the revision into Prototype II before full implementation. To assess the potential effect of the RPG on students' mathematical thinking, a quantitative pre-experimental method was used, specifically the One-Group Pretest–Posttest Design (Sugiyono, 2013). This design measures the same group of students before and after the intervention to observe learning improvement. The design is described in Table 4.

Table 4. One-Group Pretest–Posttest Design

Pretest	Treatment	Posttest
O_1	X	O_2

Notes:
O_1 = pretest results (initial mathematical thinking ability)
X = treatment using the educational Role-Playing Game
O_2 = posttest results (final mathematical thinking ability)

The research procedure included three main phases: (1) preparation, including validation of the test instruments and expert review; (2) implementation, beginning with pretest administration, followed by learning using the RPG-based media, and ending with a posttest of equivalent difficulty; and (3) data analysis, where quantitative results were statistically processed. To assess the potential effect of the RPG on students' mathematical thinking, a quantitative pre-experimental method was used, specifically the One-Group Pretest–Posttest Design (Sugiyono, 2013). This design measures the same group of students before and after the intervention to observe learning improvement. The design is described in Table 4.

5. Data Analysis Techniques

Data obtained from the validation sheets, questionnaires, and tests were analyzed both quantitatively and qualitatively. Validation and practicality scores were processed using mean and percentage formulas to determine product validity and usability levels. Meanwhile, students' test data were analyzed using SPSS version 26 for Windows. Normality of pretest and posttest data was tested using the Shapiro–Wilk test, appropriate for samples under 50 participants. The normality criterion was: if $Sig. > 0.05$, the data are normally distributed; if $Sig. \leq 0.05$, they are not. Once normality was confirmed, a paired sample t-test was applied to

determine whether there was a significant difference between pretest and posttest mean scores. The t-test formula used is as follows:

$$t_{test} = \frac{\bar{D}}{SD/\sqrt{n}} \tag{2}$$

The hypotheses tested were:

- H_0: There is no significant effect of the RPG on students' mathematical thinking skills.
- H_1: There is a significant effect of the RPG on students' mathematical thinking skills.

The decision criterion was: if $Sig. < 0.05$, H_0 is rejected. To complement the significance test, an effect size analysis was also conducted to measure the magnitude of learning improvement. The effect size was calculated using Cohen's d (1998), which is recommended for pretest–posttest designs:

$$d = \frac{(M_{post} - M_{pre})}{SD_{pooled}} \tag{3}$$

where:

$$SD_{pooled} = \frac{\sqrt{SD_{pre}{}^2 + SD_{post}{}^2}}{2} \tag{4}$$

The interpretation used the commonly accepted criteria:

- $d \geq 0.80$ = large effect
- $0.50 \leq d < 0.80$ = medium effect
- $0.20 \leq d < 0.50$ = small effect

In this study, the RPG was considered to have a meaningful potential effect if the paired t-test showed a significant difference (Sig. < 0.05) and the effect size reached at least the medium category.

C. RESULT AND DISCUSSION

This study focuses on the development of interactive learning media based on Role-Playing Games (RPG) designed to enhance students' mathematical thinking skills in the topic of arithmetic sequences. The results are presented following the five stages of the ADDIE model: Analyze, Design, Development, Implementation, and Evaluation, along with an overall discussion and limitations. This research specifically highlights the indicators of mathematical thinking, such as specializing, generalizing, conjecturing, and convincing, as the theoretical foundation for both game design and learning activities. The novelty of this study lies in its integration of RPG mechanics with mathematical-thinking–driven tasks in the local Indonesian curriculum context.

1. Analyze

In the Analyze stage, researchers conducted a series of analyses to establish a basis for the development of Role-Playing Game (RPG)-based learning media. This analysis included identifying the learning context, student characteristics, the curriculum used, and the availability of supporting facilities and infrastructure. First, the location and subject of the research were determined. Preliminary communication with the school showed support for the research, including openness in providing data and facilitating learning activities involving the development product. Observations and informal interviews with mathematics teachers revealed that students often struggled with arithmetic sequences, particularly in connecting concepts to real-life contexts and solving problems. Learning was still passive and teacher-centered, highlighting the need for a more interactive approach that fosters mathematical thinking. This aligns with previous findings reporting students' difficulties in generalizing patterns and constructing formulas in arithmetic sequences.

Curriculum analysis showed that arithmetic sequences are part of Phase E of the Merdeka Curriculum, requiring both conceptual understanding and problem-solving in everyday contexts. However, textbooks emphasized procedural approaches and lacked support for higher-order thinking. An analysis of the facilities and infrastructure shows that the school has adequate supporting facilities for the implementation of technology-based media, including the availability of projectors and internet connections. This allows the developed RPG to be optimally integrated into learning, both in classical and group-based learning.

Based on the results of this analysis, researchers obtained a strong foundation for designing learning media that suits the needs and characteristics of students in order for them to help reach one's math learning goals in the arithmetic sequences topic. The urgency of using games in learning comes from the fact that students are generally more interested in activities that are interactive, challenging, and fun. Arithmetic sequences are often considered abstract and difficult to understand when delivered through conventional methods. By utilizing games, particularly role-playing games, symbolic concepts can be presented in the form of narrative contexts, visuals, and meaningful play experiences. This analysis also informed the alignment between game tasks and indicators of mathematical thinking, ensuring each stage of the game explicitly targets a cognitive process.

2. Design

At this stage, researchers designed a storyboard for Arithmetic Adventure: The Mystery of Polaria Village. This storyboard serves as a narrative and visual guide that contains the storyline, character conversations, challenges, and mathematical problems integrated into the game. Through the storyboard, researchers can arrange the sequence of challenges in stages, starting from the introduction of simple number patterns to understanding the nth term formula in arithmetic sequences. Each challenge is intentionally mapped to a specific mathematical thinking indicator to ensure that every gameplay decision has a learning purpose. The game flow is designed so that students can explore the concept of arithmetic sequences naturally through play, allowing their mathematical thinking abilities to develop systematically and continuously. The storyboard design in question can be seen in Figure 2.

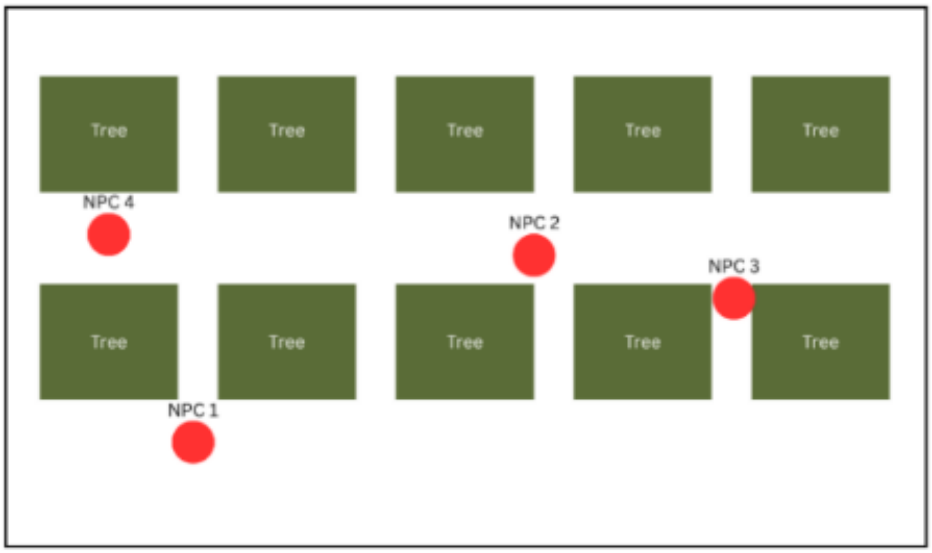


Figure 2. Visual Storyboard of Arithmetic Adventure

OUTLINE STORYBOARD ARITHMETIC ADVENTURE – ARITHMETIC SEQUENCE

MAP001 – Opening Scene (Introduction & Character Selection)
- **Setting**: black screen, only text dialogue.
- **Characters**: Arka introduces the story, the village of Polaria in chaos due to a missing pattern.
- **Action**:
  - The player selects the hero character to use.
  - The player inputs the character's name.
  - Arka gives directions to meet Lyra in the garden → continue to MAP002.

MAP002 – Mission 1 Stage 1 (Specializing) - Counting Trees in Specific Grids
- **Setting**: Village garden with tree grids (3, 6, 9, 12, grid 5 is empty).
- **Characters**: Lyra (guiding observation), Master Sequentia (giving questions).
- **Question 1**: How many trees are in the 5th grid?
- **Correct answer:** 15 → Master explains the concept of arithmetic sequences (a, b, Un).
- **If incorrect:** players can go around again, then try answering again.
- **Next action:** Meet Arka → MAP003.

MAP003 – Mission 1 Stage 2 (Generalizing) - Determining Trees in Distant Grids
- **Setting:** Garden with pattern (2, 4, 6, then empty until grid 10).
- **Characters:** Arka → Lyra → Master Sequentia.
- **Question 2a (Lyra):** The difference between each grid? (+2 → correct).
- **Question 2b (Master):** How many trees are in the 9th grid? (Answer: 18).
- **If correct:** Master introduces the general formula for the sequence: $U_n = a + (n-1)b$ and gives examples
- **up to U_{300} .**
- **If incorrect:** the player can re-observe, then answer again.
- **Next action:** MAP004.

MAP004 – Mission 1 Phase 3 (Conjecturing) - Predicting Hidden Planting Patterns
- **Background** : Grid 4 (11 trees), grid 8 (19 trees), the rest are empty.
- **Characters** : Arka → Lyra → Master Sequentia → Lord Chaos.
- **Questions 3a–3c (Lyra)** :
  - Grid distance (4 steps → correct).
  - Difference in the number of trees (8 → correct).
  - Increment per step (b = 2 → true).
- **3d Question (Master)** : Determine the first term (a=5 → correct).
- **Question 3e (Lord Chaos)** : Predict the tree in the 12th grid (27 → correct).
- **If wrong** : the player can go around and repeat.
- **Further action** : MAP005.

MAP005 – Mission 1 Phase 4 (Convincing) - Convincing the Lord of Pattern Consistency
- **Setting** : Lord Chaos challenges players to prove the truth of the pattern.
- **Characters** : Lord Chaos → Lyra → Arka → Master Sequentia.
- **Question 4a (Lyra)** : Determine a and b (a=5, b=2).
- **Question 4b (Arka)** : Determine the formula for the th term−n ($U_n = 5 + (n-1)22$).
- **Question 4c (Master)** : Select the statement supporting the formula (Grid 11=25, Grid 12=27).
- **If all is true** :
  - Master congratulates → mission completed.
  - Players are considered to be able to think mathematically.
  - The game returns to the Title Screen.
- **If wrong** : you can try the question again.

Figure 3. Storyboard for the Arithmetic Adventure Storyline

The storyboards shown in Figure 2 and Figure 3 illustrate the design of Arithmetic Adventure, which focuses on arithmetic sequences. The game flow is designed through player interaction with non-player characters (NPCs) that represent stages of mathematical thinking, namely specializing, generalizing, conjecturing, and convincing. Each mission guides students to understand the regularity of sequences in stages, starting from counting the number of trees in a certain grid, determining the difference between terms, predicting the nth term, to proving the consistency of the pattern. This explicit linkage between game stages and cognitive indicators addresses the need for conceptual clarity in the research design. Additionally, the platform used, RPG Maker MV, and its capacity for event-based interaction is fully utilized to support structured problem-solving and branching dialogue. With this design, Arithmetic Adventure provides an interactive learning experience to foster conceptual understanding of arithmetic sequences.

3. Development

After a comprehensive overview of the storyline, challenges, and math problems through the storyboard, the design was then turned into a Role-Playing Game (RPG). At this stage, researchers began developing the initial design for the Arithmetic Adventure game, taking into account the narrative flow, game mechanics, and problem integration. This initial design serves as a prototype that will form the basis for further development on the RPG Maker MV platform, so that the game can be played interactively in accordance with the learning objectives. Importantly, each task embedded in the prototype was mapped to indicators of mathematical thinking (specializing, generalizing, conjecturing, convincing) to ensure alignment between pedagogical goals and game mechanics, as shown in Table 5.

Table 5. Results of Prototype I Arithmetic Adventure

Design Results	Description
	Opening and map display
	Display of dialog, questions, and feedback on answers provided.

After a comprehensive overview of the storyline, challenges, and math problems through the storyboard, the design was then turned into a Role-Playing Game (RPG). At this stage, researchers began developing the initial design for the Arithmetic Adventure game, taking into account the narrative flow, game mechanics, and problem integration. This initial design serves as a prototype that will form the basis for further development on the RPG Maker MV platform, so that the game can be played interactively in accordance with the learning objectives.

a. Expert Review

After the development of prototype I, the Role-Playing Game (RPG) product that was designed went through a validation process by three validators consisting of two experts in the field of mathematics education and one education practitioner. The validation assessment was conducted comprehensively on four main aspects, namely content validity (suitability and accuracy of material), construct validity (structural design and pedagogical framework), ICT integration (technology and user interface functions), and

language validity (clarity and accuracy of linguistic elements). The validation instrument used contained 28 items covering all of these aspects.

This stage also ensured that each game activity was reviewed to verify whether it appropriately represented the stages of mathematical thinking, addressing the reviewer's concern regarding the lack of linkage between theory and design elements. The validation process was carried out by filling out the instrument and discussing it with the validators, and the results in the form of written feedback and comments were used as a basis for improving the product in the next stage of development, as shown in Table 6.

Table 6. Expert Validation Results

Assessment Aspect	V1	V2	V3	Average	Criteria
Content	4	4.71	5	4.57	Very Valid
Construct	4	4.71	4.71	4.47	Very Valid
ICT	3.86	4.71	5	4.52	Very Valid
Language	4.14	4.86	5	4.67	Very Valid
Overall Average				4.45	Very Valid

The validation results show that the developed Role-Playing Game (RPG) product meets the criteria of being very valid across all evaluated aspects. The content, construct, language, and ICT integration components each obtained average scores ranging from 4.47 to 4.67, indicating that the material accuracy, structural consistency, clarity of instructions, and technological functionality were assessed as highly appropriate. Overall, the average validation score of 4.45 confirms that the RPG is suitable to proceed to the next trial phase.

These results demonstrate strong coherence between the content and construct components, which reinforces the academic rigor of the media development process. In addition, the validation data provide initial empirical support, addressing the reviewer's recommendation prior to conducting larger-scale trials. The expert feedback also shows that the structure of tasks, the organization of the learning flow, and the interaction patterns within the game are consistent with the theoretical indicators of mathematical thinking, namely specializing, generalizing, conjecturing, and convincing. This consistency strengthens the connection between empirical findings and the underlying theory, as requested by the reviewer.

b. One to one

One-to-one trials were conducted with three tenth grade students who played the Role-Playing Game on arithmetic sequences through the itch.io platform. Students were asked to complete worksheets after finishing each problem so that their mathematical thinking processes could be observed in detail. During the trial, they also reported technical issues and difficulties in understanding certain questions. This stage provided early diagnostic evidence about students' ability to identify patterns, generalize results, and form conjectures, which helps address reviewer comments regarding the need for more concrete learning outcomes. Although no statistical analysis was applied at this stage,

the qualitative patterns that emerged indicated which tasks supported or hindered the development of mathematical thinking. The findings from this stage served as an initial basis for assessing practicality and guiding improvements to the product. The students' comments and the subsequent revisions can be seen in Table 7 below.

Table 7. Students' Responses during the Trial

No.	Students' Responses During the Trial	Revisions Implemented
1.	Add exit and return to main menu buttons during gameplay.	Added additional buttons to select exit or return to main menu.
2.	Provided a feature to repeat dialogues if missed.	Fixed, so that dialogues can be repeated.
3.	Accelerated character movement.	Movement speed has been enhanced.

These revisions reflect improvements that directly affect usability, which is essential for ensuring implementation validity as requested by the reviewer.

c. Small Group

A small group trial with six students in two groups was conducted to test the practicality of the revised RPG. Students played the arithmetic sequence game while completing a worksheet to trace their thinking and reported any content or technical difficulties. Afterward, they filled out a practicality questionnaire, summarized in Table 8. This stage also produced preliminary empirical indications regarding learning performance, such as the number of students who were able to determine the common difference and identify the nth-term pattern correctly. These observations serve as early evidence of learning outcomes, addressing the reviewer's concern about the lack of concrete examples of student achievement.

Table 8. Practicality Questionnaire Results for Arithmetic Adventure

Subject	Percentage (%)	Category
S1	75	Practical
S2	90	Very Practical
S3	82.5	Practical
S4	90	Very Practical
S5	97.5	Very Practical
S6	82.5	Practical
Average	86.25	Very Practical

Based on the data presented in Table 8, the average score given by students for the Role-Playing Game in Arithmetic Adventure was 86.25%, which falls into the very practical category. This result provides early quantitative evidence of the game's acceptability, while more detailed statistical analysis such as effect size and pretest–posttest gains will be discussed in the Implementation and Evaluation sections. The practicality score also indicates that students can operate the game effectively and that its design aligns well with the intended development objectives. Although the questionnaire results show that the game is considered very practical, a deeper understanding of the students' direct experiences while playing is still required. To address this need, researchers also collected comments and feedback from students to obtain a qualitative picture of the

strengths and challenges encountered during gameplay. A summary of the students' comments is presented in the following Table 9.

Table 9. Students' Responses during the Trial

No.	Students' Responses During the Trial	Revisions Implemented
1.	Provide question numbers according to the order in the LKPD; add variety to questions and other challenges; do not immediately move to another dialogue after interaction.	Question numbers are adjusted according to the LKPD; variety of questions is added; the dialogue system is improved so that it does not automatically move on
2.	Add more character options; improved character designs and names to make them easier to remember.	Added character options; bugs fixed; character designs and names explained at the beginning of the game.
3.	Expanding maps or play areas.	The play area is expanded by increasing the size of the maps used in the game.
4.	Provides more consistent arithmetic explanations; users can exit the explanation without having to repeat it.	Arithmetic explanations are clarified and standardized; an option to exit without repeating from the beginning has been added.

These comments reveal several advantages noted by students, along with challenges such as navigation difficulties, design clarity, and the need for certain content adjustments. These findings respond to the reviewer's observation regarding the absence of detailed descriptions of obstacles faced by students. The feedback helped guide revisions that refined both gameplay and the representation of mathematical tasks.

4. Implement

The implementation phase examined how the Arithmetic Adventure RPG supported the development of students' mathematical thinking abilities during classroom learning. The game was deployed using RPG Maker MV and accessed through a browser-based build hosted on itch.io, enabling students to play directly on their mobile phones. The learning activity involved 34 tenth-grade students who worked in pairs with their seat partners to accommodate device availability. The 90-minute session consisted of three components: RPG gameplay, worksheet-based reasoning activities, and a whole-class consolidation discussion led by the teacher. Within this structure, students progressed through a series of problem-solving stages embedded in the game narrative, each of which corresponded to the four indicators of mathematical thinking: specializing, generalizing, conjecturing, and convincing. The following subsections detail how each stage unfolded in practice and how students demonstrated learning through the tasks provided.

Stage 1: Specializing (Observation of Patterns)

In the specializing stage, students engaged with concrete examples presented early in the game narrative. They were introduced to the character Lyra, who prompted them to observe the number of trees arranged in several consecutive garden grids. Through careful inspection and recording of these values in their worksheets, students began identifying numerical

regularities, particularly the constant difference characteristic of arithmetic sequences. This activity required learners to focus on specific cases before attempting any abstraction, marking the emergence of initial mathematical thinking. Analysis of student worksheets indicated that 30 out of 34 students (88.2%) successfully recognized the constant difference, demonstrating strong early-stage pattern recognition that served as the foundation for subsequent reasoning, as shown in Figure 4.

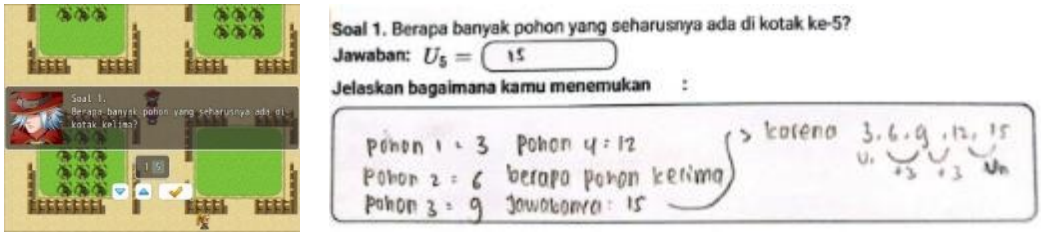


Figure 4. Specializing Stage Question and Student Response in the *Arithmetic Adventure*

Stage 2: Specializing (Observation of Patterns)

Building on the concrete observations from the previous stage, students transitioned into generalizing by comparing their recorded data and discussing patterns with their peers. The RPG presented tasks that required determining terms beyond those explicitly shown, guiding students to articulate the underlying relationship and derive the general form of an arithmetic sequence, $U_n = a + (n-1)b$. This process supported the shift from specific instances to broader structural understanding. Worksheet analysis showed that 25 students (73.5%) were able to formulate the general term correctly, although some struggled with identifying the initial value a. Collaborative pairs generally produced clearer reasoning, suggesting positive effects of peer interaction on conceptual generalization, as shown in Figure 5.

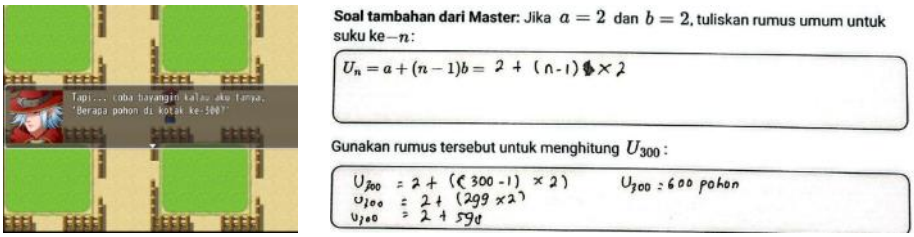


Figure 5. Generalizing Stage Question and Student Response in the *Arithmetic Adventure*

Stage 3: Conjecturing (Predicting and Testing Ideas)

The conjecturing stage challenged students to make predictions based on incomplete information. The game presented partially filled grids and required students to infer missing terms using the patterns they had previously identified. In their worksheets, students recorded both their conjectures and the reasoning behind them, encouraging not only prediction but also justification. This task emphasized the exploratory nature of mathematical thinking, where learners test hypotheses and evaluate their plausibility. A total of 27 students (79.4%) generated correct conjectures, demonstrating their ability to extend known patterns into new situations. Incorrect responses typically arose from inconsistent application of the constant difference, indicating areas where conceptual reinforcement was needed, as shown in Figure 6.

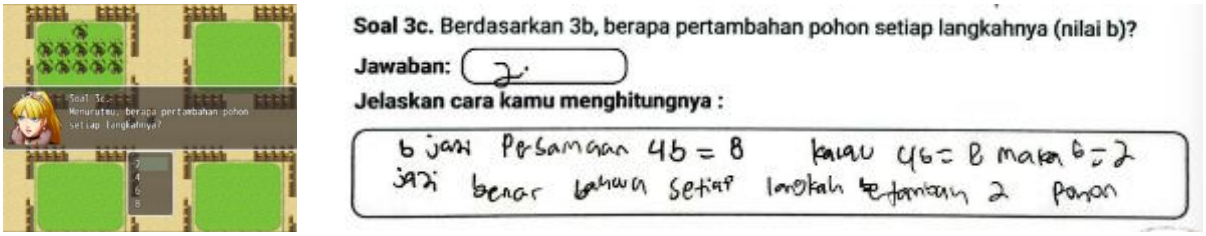


Figure 6. Conjecturing Stage Question and Student Response in the *Arithmetic Adventure*

Stage 4: Convincing (Justifying Mathematical Claims)

The final stage focused on convincing, where students constructed logical arguments to validate the formulas and predictions they had produced earlier. In the game, this was represented through an interaction with the character Lord Chaos, who required students to defend their reasoning. Learners compared alternative statements or formulas and identified which ones were valid, formalizing their arguments in writing. This stage demanded a higher level of reasoning, emphasizing coherence and justification rather than mere calculation. Rubric evaluations revealed that 22 students (64.7%) produced valid and logically structured explanations, while others provided partial justifications that lacked full argumentative clarity. This stage played a critical role in consolidating mathematical understanding and aligning students' intuitive reasoning with formal justification, as shown in Figure 7.

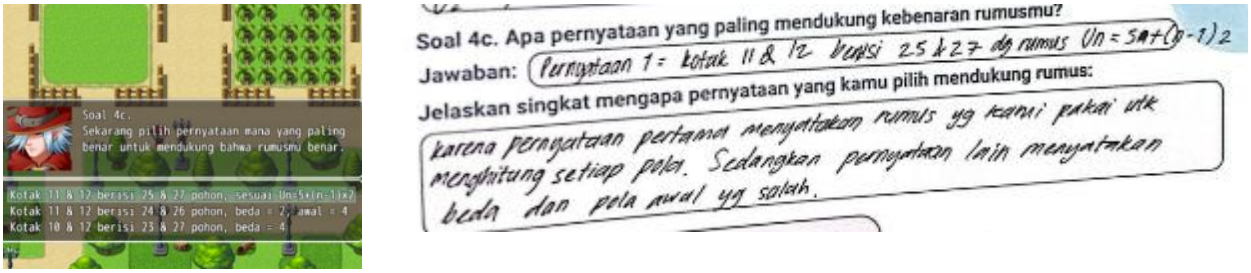


Figure 7. Conjecturing Stage Question and Student Response in the *Arithmetic Adventure*

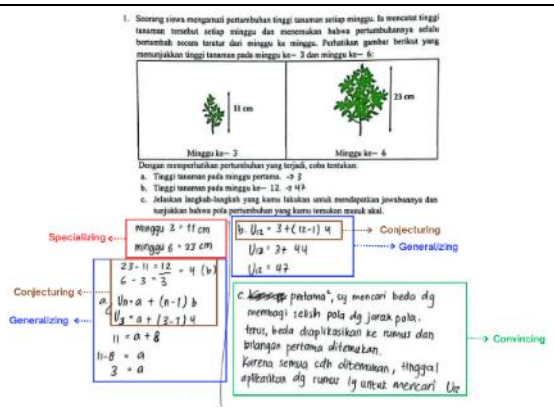
After completing all four stages, students participated in a whole-class discussion to synthesize their findings. They presented their reasoning, compared different solution approaches, and collaboratively refined the general formula and foundational concepts of arithmetic sequences. The teacher acted as a facilitator, posing guiding questions and supporting students who encountered conceptual or technical difficulties during gameplay. The session concluded with individual written reflections, in which students described what they had learned, challenges they faced, and which parts of the RPG most effectively supported their understanding. This reflective component strengthened metacognitive awareness and reinforced the progression from observation to generalization, hypothesis formation, and justification.

5. Evaluate

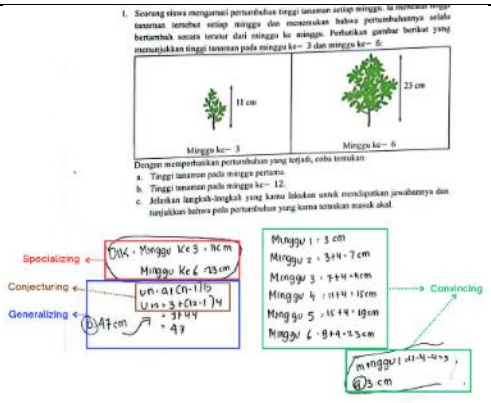
a. Mathematical Thinking Ability Test

After the implementation of Arithmetic Adventure, students completed a mathematical thinking ability test consisting of five items assessing four indicators: specializing, generalizing, conjecturing, and convincing. Figure 8 presents two representative

responses to the plant growth problem modelled as an arithmetic sequence, with given heights for Week 3 (11 cm) and Week 6 (23 cm). Students were asked to determine the height in Week 1, predict the height in Week 12, and justify their reasoning, as shown in Figure 8.



(a)



(b)

Problem (translated):

A student observed the growth of a plant each week. He recorded its height every week and found that the growth always increased regularly from week to week. The following picture shows the height of the plant in the 3rd week (11 cm) and the 6th week (23 cm).

Based on this information, answer the following:

a. What is the height of the plant in the 1st week?

b. What is the height of the plant in the 12th week?

c. Explain the steps you used to find the answers and show that the growth pattern you discovered makes sense.

Student's Answer (translated):

$Week\ 3\ =\ 11\ cm$
$Week\ 6\ =\ 23\ cm$

$23\ -\ 11\ =\ 12$
$6\ -\ 3\ =\ 3$
$b\ = \frac{12}{3}\ =\ 4$

$U_n = a + (n-1)b$
$U3 = a + (3-1)4$
$11 = a + 8$
$11 - 8 = a$
$3 = a$

$U_{12}\ =\ 3\ +\ (12\ -\ 1)4$
$U_{12}\ =\ 3\ +\ 44$
$U_{12}\ =\ 47$

First, I find the difference by dividing the pattern difference by the pattern distance. Then, the difference is applied to the formula and the first number is found. Since everything has been found, just apply the formula again to find U_{12}.

Student's Answer (translated):

$Week\ 3\ =\ 11\ cm$
$Week\ 6\ =\ 23\ cm$

$U_n = a + (n-1)b$
$U_{12} = 3 + (12-1)4$
$= 3 + 44$
$= 47\ cm$

$Week\ 1\ =\ 11\ -\ 4\ -\ 4\ =\ 3\ cm$
$Week\ 2\ =\ 3\ +\ 4\ =\ 7\ cm$
$Week\ 3\ =\ 7\ +\ 4\ =\ 11\ cm$
$Week\ 4\ =\ 11\ +\ 4\ =\ 15\ cm$
$Week\ 5\ =\ 15\ +\ 4\ =19\ cm$
$Week\ 6\ =\ 19\ +\ 4\ =\ 23\ cm$

Figure 8. Students' Answers of Mathematical Thinking Ability Test

The responses reveal meaningful differences in the quality of students' mathematical thinking. Student A (Figure 8(a)) demonstrated adequate specializing by identifying the known values and applying the arithmetic sequence formula in the generalizing stage to derive $a = 3$ and $U_{12} = 47$. The student also made a conjecture by determining the constant difference $b = 4$. However, the convincing stage was limited; the justification was brief and did not articulate clear logical validation.

Student B (Figure 8(b)) showed a more complete and systematic reasoning process. The student correctly used the given values (specializing), recognized a constant increase of 4 cm per week (generalizing), applied the general formula $U_n = a + (n-1)b$ to predict future values (conjecturing), and verified the pattern by reconstructing plant height week-by-week (convincing). This explicit validation demonstrates stronger justification and deeper mathematical reasoning. Overall, both students reached the correct numeric answers, but Student B exhibited more complete thinking across all four indicators. These qualitative findings suggest that the RPG environment encouraged students to articulate reasoning rather than rely on procedural calculation.

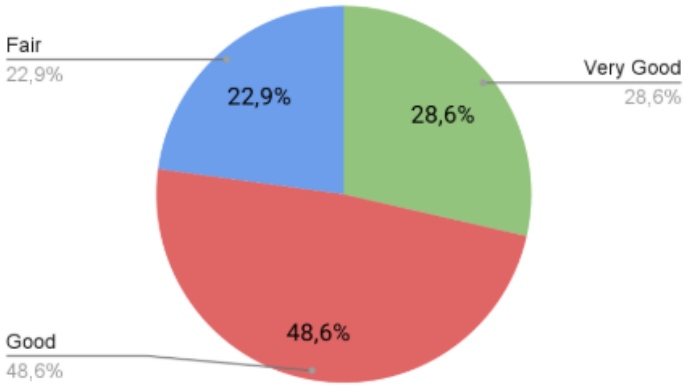


Figure 9. Mathematical Thinking Ability Test Results

Based on Figure 9, most students demonstrated "good" or higher performance, indicating consistent emergence of specializing, generalizing, conjecturing, and convincing during the posttest. To quantify the effectiveness of the RPG, a pretest–posttest written assessment was administered to 34 tenth-grade students, as shown in Table 10.

Table 10. Descriptive Statistics

Variable	N	Mean	SD	Minimum	Maximum
Pre-test	34	54,0588	9,55983	26	73
Post-test	34	73,6471	12,13031	44	94

The mean increase of 19.59 points indicates a substantial improvement in students' mathematical thinking ability after using the RPG based learning media. Observations during implementation also showed that students engaged systematically in all four indicators of mathematical thinking, namely specializing, generalizing, conjecturing, and convincing, which supports previous findings that game based environments promote

active reasoning and reflective learning (Adipat et al., 2021; Suryani et al., 2025)**.** Before conducting hypothesis testing, the Shapiro Wilk test confirmed that both the pretest score with $Sig. = 0.165$ and the posttest score with $Sig. = 0.402$ were normally distributed, thereby meeting the assumptions required for parametric analysis using a paired sample t test.

b. Formative Evaluation

The formative evaluation in this study refers to the previous stages. At that stage, the researchers obtained various inputs and responses that were useful for improving Arithmetic Adventure: The Mystery of Polaria Village. In addition, assessments were also carried out by experts and students. A summary of the formative evaluation results for the game developed is presented in Table 11.

Table 11. Recapitulation of the Evaluation of Role-Playing Game Development

Aspects	Result	Category
Validity	Based on expert assessments, the overall average validity reached 4.45, which means that the RPG on arithmetic sequences is valid in terms of content, construction, language, and ICT.	Very Valid
Practicality	The results of the practicality questionnaire given to students during one-to-one, small group, and field trials showed that the RPG achieved an average practicality percentage of 86.25%.	Very Practical
Potential Effects	The mathematical thinking ability test conducted after learning with the RPG showed that most students demonstrated improvements in specializing, generalizing, conjecturing, and convincing, which were in the good category.	Has a Potential Effect

These findings provide a strong basis for discussing the influence of RPGs on the development of students' mathematical thinking skills, the role of game-based media in supporting the learning process, and the limitations that still need to be considered.

6. Media Validity

Expert validation results show that RPG Arithmetic Adventure: The Mystery of Polaria Village obtained an average score of 4.55 in the highly valid category. The high scores in these four aspects confirm that the developed media has met quality standards and is suitable for use in mathematics learning. The content aspect scored 4.57, indicating that the arithmetic sequence material is packaged through a storyline and missions that are in line with the curriculum and logically structured, making it easier for students to understand number patterns (Daryanes et al., 2023). In the language aspect, the highest score of 4.67 indicates the use of clear and communicative language; this underlines that presentation in simple language can support students' conceptual understanding more deeply (Abadi et al., 2024). The integration of technology with a score of 4.52 also strengthens student engagement. The role of ICT in creating more active and meaningful learning interactions (Abdelrady & Akram, 2022; Alenezi et al., 2023).

Meanwhile, a construct score of 4.47 indicates that the designed pedagogical flow has been consistent with mathematical thinking indicators. These findings confirm that the development of mathematical thinking skills is not only in line with learning design, but also important as preparation for students to face various academic and daily life challenges (Farida et al., 2023). Academically, this validation confirms that the RPG integrates content, pedagogy, and technology in a coherent manner, which strengthens its theoretical contribution to educational design research by demonstrating how mathematical thinking indicators can be operationalized within a narrative-based digital environment. Thus, the validation results in the four aspects prove that this RPG is highly valid both substantively and pedagogically, and has strong potential in supporting the improvement of students' mathematical thinking abilities in arithmetic sequences.

7. Practicality of Use

The practicality test results show that RPG Arithmetic Adventure: The Mystery of Polaria Village scored an average of 86.25% in the very practical category. The majority of students rated this RPG as easy to use, interesting, and helpful in understanding arithmetic sequences. Game elements such as the visualization of tree patterns in the village garden, interactive missions, and character dialogues supported a fun learning experience that motivated students. Game-based learning can increase student participation and activity (Nadeem et al., 2023; Tavares, 2022).

Furthermore, small group trials showed that four students rated it as very practical (86%–97.5%) and two others rated it as practical (75%–82.5%). These results confirm the flexibility of RPGs, which allow students to learn through visual exploration, interaction with characters, and mission repetition, thereby accommodating diverse learning needs. This is consistent with Pan et al. (2021), who emphasize that game-based learning can create a learning experience that is enjoyable, interactive, challenging, contextual, and personalized to the needs of each student. The alignment between empirical practicality results and theoretical expectations indicates that the RPG design not only functions well in classroom settings but also embodies key principles of interactive and adaptive learning design, thereby reinforcing its pedagogical significance beyond practical usability alone. The consistency between the empirical results and theoretical foundations strengthens the conclusion that this RPG is not only practically feasible but also pedagogically aligned with principles of interactive and adaptive learning.

8. Potential Effects on Students' Mathematical Thinking Ability

The results of the potential effect test show that RPG Arithmetic Adventure: The Mystery of Polaria Village is able to support students' mathematical thinking ability. A total of 48.6% were in the good category, 28.6% in the very good category, and 22.9% in the fair category. This distribution shows that the majority of students displayed good mathematical thinking ability, particularly in the aspects of specializing, generalizing, conjecturing, and convincing, as shown in Table 12.

Table 12. Normality Test Results for Pretest and Posttest

Variable	N	Statistic	Sig.	Description
Pretest	34	0,954	0,165	Normally distributed
Posttest	34	0,968	0,402	Normally distributed

Following the normality results presented in the previous table, the Shapiro–Wilk test indicated that both the pretest ($p = 0.165$) and posttest ($p = 0.402$) scores met the assumption of normality, thereby allowing the use of parametric statistical procedures. Based on this prerequisite, a paired-sample t-test was conducted to compare students' performance before and after the intervention, as shown in Table 13.

Table 13. A Paired-Sample t-test

Variable	Mean Difference	SD	t_{test}	df	Sig (2-tailed)
Pretest-Posttest	-19,59	10,92	-10,461	33	0,000

Because the same group of students completed both assessments, the paired-sample t-test was the most appropriate method to detect changes attributable to the RPG intervention. The results indicate a statistically significant improvement in students' mathematical thinking scores after using the RPG ($p < 0.05$). To evaluate the magnitude of this improvement, the effect size was calculated using Cohen's d as:

$$d = \frac{M_{posttest} - M_{pretest}}{SD_{pooled}} = \frac{73.6471 - 54.0588}{16.1053} = 1.22 \quad (5)$$

Cohen's effect size was selected because it offers a straightforward and widely accepted measure for quantifying the magnitude of an intervention's impact in pretest–posttest designs. The resulting value ($d \approx 1.22$) indicates a large effect (Cohen, 1988), demonstrating that the RPG produced a substantial and meaningful improvement in students' mathematical thinking abilities. Reporting both statistical significance and effect size strengthens methodological rigor and reinforces the internal validity of these findings.

This improvement is supported by the learning experience created within the RPG. Students were invited to explore tree patterns in the village garden to identify numerical structures and formulate generalizations leading to arithmetic series formulas. Contextual learning has been shown to enhance motivation and achievement, strengthen critical thinking, and guide students in constructing meaningful concepts based on real experiences (Lestari et al., 2021). Visualization of tree patterns acted as a bridge between real-world contexts and abstract algebraic representations, helping students understand how observed patterns evolve into generalized rules. This aligns with the findings of Herrera et al. (2024) who emphasize that interactive visualization strengthens mathematical reasoning by making patterns more concrete and easier to manipulate. Additionally, the gradual structure of the RPG provided systematic scaffolding; as noted by Kim et al. (2022), structured learning environments guide students to test, evaluate, and refine their thinking strategies.

Taken together, the significant gain and large effect size support the argument that interactive RPG-based learning can enhance students' engagement, critical thinking, and ability to generalize mathematical patterns through contextual exploration. In this study, visualization

of tree patterns and interactive missions facilitated active participation, enabling students to construct mathematical ideas systematically and meaningfully.

9. Research Implications

The development of the RPG Arithmetic Adventure: The Mystery of Polaria Village has important implications for mathematics learning. The integration of real-world contexts in the form of tree patterns in the village garden helps students connect the abstract concept of arithmetic sequences with situations that are close to their daily lives, thereby supporting mathematical thinking processes. The development of games that combine theoretical content with contextual problem solving can provide learning experiences that are not only concept-based but also directly related to real situations that are relevant to students (Yadav & Oyelere, 2021). This contextual integration is particularly important because arithmetic sequences are often taught procedurally, and students rarely experience meaningful opportunities to engage in recognizing patterns, forming generalizations, or validating their reasoning through authentic tasks.

In addition, this RPG provides a flexible and interactive learning experience. Teachers can adjust the flow of the game, while students are involved in activities such as pattern observation, generalization, and reflection with characters. This is in line with the findings of Liline et al. (2024) that game-based learning can create an adaptive, collaborative environment and stimulate higher-order thinking skills through the active involvement of students in solving contextual challenges. The structured learning trajectory embedded in the game mechanics also enables a clearer emergence of mathematical thinking indicators (specializing, generalizing, conjecturing, convincing), which strengthens the theoretical alignment between the product design and the intended cognitive outcomes. Thus, this RPG has the potential to become an alternative medium in mathematics learning that is not only engaging but also supports students' mathematical thinking skills through a combination of real-world contexts, instructional flexibility, and interactive visualization. This research additionally contributes to the literature by demonstrating how RPG-based mechanics can be systematically aligned with mathematical thinking frameworks, offering a replicable model for future technology-enhanced learning designs.

10. Research and Product Limitations

This study has several limitations. First, the number of subjects is still limited to a small group, so the findings cannot be generalized (Muzari et al., 2022). The short implementation period also does not fully describe the long-term impact of RPG on students' mathematical thinking abilities, even though long-term implementation is necessary to see the sustainability of the learning media's effects. The limited sample characteristics also restrict the interpretation of effect size, meaning that future studies with larger and more diverse populations are needed to validate the consistency of learning gains.

From a technical aspect, bugs were still found, such as less-than-smooth transitions between maps, the absence of score displays and custom display names, and progress not being saved when the internet connection was interrupted. Although these bugs did not hinder understanding of the material, they did affect the students' learning experience. Therefore,

further development needs to focus on technical improvements, visual enhancements, mission variations, and an automatic feedback system. This is in line with Ahmad et al. (2022), who emphasize that technical quality and interactivity are important in the effectiveness of game-based learning. Further research is recommended to involve a larger sample, longer duration, and stronger experimental design to strengthen the findings. Future research should also incorporate a more rigorous experimental design, including control groups and extended pre–post measurements, to strengthen internal validity and allow deeper analysis of learning progression.

D. CONCLUSION AND SUGGESTIONS

The findings of this study indicate that Arithmetic Adventure: The Mystery of Polaria Village meets the criteria of validity, practicality, and potential effect in supporting students' mathematical thinking on arithmetic sequences. Expert validation showed a high level of validity (overall mean = 4.45), confirming that the game aligns with pedagogical, content, and ICT standards. Its usability was also strong, with a practicality score of 86.25%, demonstrating that students found the RPG easy to operate, engaging, and supportive of learning. Quantitative analysis further strengthens these results. Students' mathematical thinking scores increased significantly from the pretest ($M = 54.06$; $SD = 9.56$) to the posttest ($M = 73.65$; $SD = 12.13$). The Shapiro–Wilk test confirmed normality (pretest $Sig. = 0.165$; posttest $Sig. = 0.402$), allowing parametric testing. The paired-sample t-test indicated a significant improvement ($t = -10.461$, $p < 0.001$), and the effect size was large ($d = 1.79$), demonstrating substantial learning gains attributable to the RPG. This improvement reflects how the game's structure, progressing through specializing, generalizing, conjecturing, and convincing, effectively operationalized mathematical thinking theory within a narrative and contextual environment.

Beyond confirming the effectiveness of game-based learning, this study contributes theoretically by showing how a narrative-driven RPG can serve as a structured cognitive scaffold, extending existing models of mathematical thinking integration in digital environments. The novelty lies in embedding the complete sequence of mathematical thinking indicators into storyline-based tasks rather than isolated problem sets, offering a replicable design model for future educational games. Nevertheless, this study has limitations, including a small sample size, a short intervention period, and technical constraints in game performance. These factors restrict generalizability and call for further research involving larger samples, longer-term use, and more rigorous experimental designs. In sum, Arithmetic Adventure demonstrates that well-designed RPG environments can provide meaningful, contextualized experiences that strengthen students' mathematical thinking. The study contributes to mathematics education and learning technology by illustrating a practical and theoretically grounded approach to integrating narrative-based digital media with cognitive development goals.

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