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Air Temperature Prediction in Sleman Yogyakarta Using Fourier Series and Markov Switching

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ABSTRACT

Global warming increases the urgency of accurate local temperature forecasting, particularly in Sleman, Yogyakarta, a region characterized by diverse topography and high exposure to climate-related risks such as volcanic activity, agricultural vulnerability, and rapid urbanization. Such conditions increase the urgency for localized predictive models that can support agricultural planning, energy management, and disaster preparedness. This research used quantitative approach with a comparative predictive modelling design to predict the weekly average air temperature in Sleman by comparing two models: the Fourier Series regression and the Markov Switching Autoregressive (MSAR) model. The Fourier Series was selected for its ability to capture smooth seasonal and periodic behavior typical of climatological data, whereas the MSAR model was employed to accommodate regime shifts and nonlinear structural variations. The dataset comprises 127 weekly observations from January 2023 to June 2025 (BMKG), the data were split into 70% training and 30% testing. Model performance was assessed using GCV, MSE, MAE, MAPE, and residual diagnostics. Results show that the Fourier Series model performs substantially better, achieving lower GCV (0.3520), MSE (0.00415 training; 0.00114 testing), and MAE (0.34015 training; 0.12940 testing), as well as lower MAPE (1.26% training; 0.47% testing). In contrast, the MSAR model yields higher errors with GCV (0.5747), MSE (0.9113 training; 0.4686 testing), MAE (0.8005 training; 0.5512 testing), and MAPE (1.96% training; 1.34% testing). These results indicate that Sleman's temperature dynamics characterized by stable oscillatory patterns with minimal regime shifts are more effectively captured through harmonic decomposition. The study reinforces the importance of periodic modeling for mixed-topography regions like Sleman and recommends future research integrating additional climatic variables, hybrid statistical–machine-learning frameworks, and longer time spans to improve responsiveness to extreme events and nonlinear atmospheric behavior.

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A. INTRODUCTION

Climate change has emerged as a global emergency, manifesting in unprecedented environmental shifts including rising global temperatures, melting glaciers, frequent extreme weather events, and sea-level rise. In this context, air temperature stands out as a critical climatic indicator, serving as a primary variable in climate dynamics due to its wide-reaching influence on ecosystems, energy consumption, and public health (Lee & Dessler, 2023). Precision in air temperature prediction is becoming increasingly crucial for informing regional

policies, early warning systems, and sustainable development frameworks (Sevgin, 2025). However, modelling temperature data remains methodologically challenging due to its inherent nonlinearity, periodicity, and sensitivity to spatial heterogeneity (Sevgin, 2025).

Indonesia, an archipelagic nation straddling the equator, is highly vulnerable to climate variability. The country experiences a humid tropical climate with annual average temperatures ranging from 25–28°C, interspersed with seasonal monsoons. Studies show that the frequency of heatwaves and dry spells has increased in recent decades, affecting food security, health, and infrastructure resilience (Rahman et al., 2020; Christienova et al., 2018). Moreover, anthropogenic factors such as urbanization exacerbate local temperature variability through the Urban Heat Island (UHI) effect, especially in densely populated areas (Wong et al., 2016). This phenomenon is particularly pronounced in peri-urban areas like Sleman Regency, where land-use transformation, reduced vegetation cover, and heat-retaining infrastructure contribute to intensified microclimate shifts. Addressing these concerns demands location-specific modeling tools capable of reflecting fine-grained climatological dynamics and informing adaptive policy responses.

Air temperature is a fundamental climatic element that strongly influences human activities, agricultural productivity, water resources management, and overall environmental sustainability. Understanding its patterns and accurately predicting its future values are crucial for effective planning and for mitigating the adverse effects of extreme weather events, which are becoming more frequent due to global climate change. Rising air temperatures in Indonesia have significant implications for the country's demographic conditions. The relationship between rising air temperature and demographic patterns is complex and mutually reinforcing. Densely populated areas tend to exhibit elevated temperatures due to the Urban Heat Island (UHI) effect (Wong et al., 2016). This pattern is further exacerbated by the reduction of green open spaces as building development intensifies and heat-absorbing materials are widely used. Episodes of extreme heat can induce climate-related migration toward cooler regions, thereby altering the demographic composition of both origin and destination areas (Lee & Dessler, 2023). In addition, several studies indicate that excessively high temperatures may negatively affect fertility and reproductive health, for example through reduced sperm quality or pregnancy complications (Dastoorpoor et al., 2021; Lindell et al., 2022).

Sleman Regency in the Special Region of Yogyakarta has emerged as a critical zone in Indonesia's climate vulnerability map due to rapid urbanization and its proximity to Mount Merapi. The interplay between built-up land expansion and topographic diversity in Sleman creates microclimatic heterogeneity that amplifies the urban heat island (UHI) effect and affects regional temperature regimes. Empirical evidence shows that urbanization in Sleman contributes to localized warming, particularly in districts such as Depok and Ngaglik, where vegetation loss and impervious surfaces are prevalent. A study by sari utilized satellite-derived NDBI indices to show a consistent rise in land surface temperature (LST) in post-COVID periods due to land cover changes and reduced evapotranspiration rates (Sari, 2024). Another research by Arif & Toersilawati (2024) deployed simulation models to demonstrate how built-up area growth in Sleman intensifies UHI effects, leading to higher nocturnal temperatures that threaten public health and energy sustainability. These findings align with Fajary, who conducted a spatiotemporal assessment across Java and emphasized that Sleman shows

significantly elevated surface thermal conditions. Thus, localized temperature prediction models are indispensable not only for climate resilience but also for supporting the goals outlined in Indonesia's regional climate adaptation programs and urban sustainability plans (Fajary et al., 2024).

A variety of mathematical models have been employed to analyse and forecast meteorological time series. Traditional time series methods such as ARIMA (Autoregressive Integrated Moving Average) are frequently used, but they often struggle to capture the deterministic and periodic structures inherent in climate data (Box et al., 1994). Accordingly, for data with pronounced seasonal and cyclical characteristics, harmonic analysis based on Fourier series remains a powerful and intuitive tool. Fourier series decomposition excels at identifying and modelling periodic components by representing complex time series as sums of simple sine and cosine functions (Walker, 1988). This mathematical structure allows for precise representation of recurrent climatic phenomena, particularly those driven by Earth's orbital cycles, such as annual and diurnal temperature variations. Recent empirical applications have demonstrated the utility of this method in various tropical regions where periodicity is dominant, affirming its efficacy for both descriptive and predictive climatology (Taiwo et al., 2019; Anandari et al., 2022; Mardianto et al., 2019). Given its computational simplicity and interpretability, the Fourier approach is especially advantageous in forecasting local temperature dynamics in regions like Sleman that experience consistent seasonal rhythms but relatively low regime shifts.

The Markov Switching approach can also be used to estimate the average air temperature at Sleman, Yogyakarta. Because the model is particularly adept at capturing abrupt structural breaks and regime changes in time series data. It integrates discrete state-space modelling via Markov chains with traditional autoregression, enabling robust analysis of nonstationary and nonlinear phenomena (Hamilton, 1989). The MSAR model is advantageous in detecting latent regime shifts such as transitions between dry and wet seasons or heatwaves and normal conditions. Recent applications have demonstrated its effectiveness in environmental modelling, including high-frequency water temperatures (Spezia et al., 2023) and wind dynamics (Latimier et al., 2020). While MSAR is more complex in terms of computation, its ability to distinguish and forecast multiple behavioral regimes within a single time series renders it essential for climate modelling under uncertain or volatile conditions.

This research contributes to the expanding discourse on localized climate modelling by incorporating both harmonic regression techniques and regime-switching time series models. Through a comparative evaluation of the Fourier series and Markov Switching Autoregressive (MSAR) models, this study underscores the importance of methodological diversity in capturing the complex dynamics of air temperature ranging from periodic regularities to abrupt structural shifts. Such integrative approaches respond to the growing demand for adaptive forecasting models that are not only statistically sound but also contextually relevant to regions with nuanced climate behavior, such as Sleman. Recent studies have emphasized that combining deterministic and stochastic methods can enhance model flexibility and reduce forecasting errors in climatological time series (Mohammadi et al., 2021; Elseidi, 2023; Spezia et al., 2023). Moreover, methodological insights from this research are expected to inform interdisciplinary applications in environmental planning, disaster mitigation, and agricultural

risk modeling. In the future, there is strong potential to expand this framework by integrating machine learning techniques such as neural networks or ensemble models into hybrid architectures that unify statistical precision with pattern recognition capabilities (Neto et al., 2022). Such innovation is essential for developing climate-responsive decision support systems that align with national and regional sustainability goals.

The primary objective of this study is to develop and compare two forecasting models Fourier series and Markov Switching Autoregressive (MSAR) in predicting weekly average air temperature in Sleman Regency. By leveraging 127 observations from January 2023 to June 2025, the study aims to evaluate the efficacy of each model in capturing seasonal patterns and regime shifts in local climate data. This analysis is expected to yield predictive insights that are not only statistically robust but also relevant for practical applications in agricultural scheduling, energy management, and disaster preparedness in Sleman. Ultimately, this research aspires to contribute toward the development of adaptive climate models that support data-driven governance and sustainable urban planning at the regional level.

B. METHODS

This Study employs a quantitative approach with a comparative predictive modelling design. Two time series forecasting models, Fourier Series and the Markov Switching Autoregressive, are compared to determine which provides superior accuracy in forecasting weekly average air temperature in Sleman, Yogyakarta. The Fourier Series is applied to represent seasonal and periodic variations, while the Markov Switching model is utilized to capture regime shifts within the data. A comparative analysis of their predictive performance is conducted to identify which model provides more accurate results.

1. Type and Source of Data

This study employs secondary data obtained from the Meteorological, Climatological, and Geophysical Agency (BMKG) through its official online portal (Badan Meteorologi, Klimatologi, dan Geofisika (BMKG), 2025). The dataset consists of weekly observations of average air temperature in Sleman, Yogyakarta, covering the period from January 1, 2023 to June 1, 2025 with a total of approximately 127 observations. These data represent a univariate time series, where each observation is sequentially dependent on time.

2. Research Procedure

To achieve the research objectives, a series of procedures were carried out systematically. These procedures involve data collection, descriptive analysis, model construction, and evaluation, which are described in the following steps:

a. Collecting the Data
   The data used in this study are weekly climate observations obtained from the BMKG online portal for the period January 1, 2023 to June 1, 2025. The variables analyzed consist of average air temperature, recorded in Sleman, Yogyakarta.

b. Data Preprocessing
   Before conducting the analysis, the weekly temperature data from BMKG were checked to ensure completeness and consistency. Manual verification confirmed the absence of missing values and irregular timestamps. As the Fourier Series method is nonparametric

and does not require assumptions of stationarity, no transformation was applied. For the MSAR model, regime-switching inherently accommodates non-stationary patterns; therefore, formal stationarity testing was not required. Basic time series diagnostics, including visual inspection and autocorrelation patterns, were conducted to support appropriate model specification.

c. Analyzing Descriptive Statistics
   Descriptive statistical analysis was performed to provide an overview of the temperature data. This includes the calculation of measures such as mean, maximum, minimum, and standard deviation, as well as visualization through time series plots. This step aims to identify seasonal patterns, fluctuations, and general characteristics of the dataset.
d. Splitting into Training and Testing Datasets
   For model construction and evaluation, the dataset was partitioned into two subsets, identified as training data and testing data. Approximately 70% of the total observations were allocated to the training data for parameter estimation, while the remaining 30% were designated as testing data for assessing the predictive performance of the models.
e. Fourier Series Method
   The Fourier Series regression approach is applied to capture seasonal and periodic components in the air temperature data. This method approximates the observed series by a linear combination of sine and cosine functions with different frequencies, enabling the model to effectively represent oscillatory fluctuations (Zong-chang, 2013). The general form of the Fourier regression model is expressed as:

$$\hat{y}_r = \hat{\beta}_0 + \sum_{j=1}^{\lambda} \left[a_j \cos(2\pi j t_r) + b_j \sin(2\pi j t_r) \right] \quad (1)$$

where:
$\hat{y}_r$ = predicted value of the response variable at time index t_r
β_0 = intercept term
a_j, b_j = Fourier coefficients
t_r = time index of the observation (weekly in this study)
λ = number of harmonics retained in the model.

The coefficients $\hat{\beta}_0, a_j, b_j$ are estimated using the least squares method, ensuring that the fitted model minimizes the error between observed and predicted values (Anandari et al., 2022). The choice of the number of harmonics λ is crucial, since a small value may fail to capture important seasonal structures, while a large value increases the risk of overfitting (Mardianto et al., 2019). The optimal number of harmonics (λ) was selected using the minimum Generalized Cross Validation (GCV) score. GCV is preferred because it balances model smoothness and predictive accuracy, preventing overfitting in periodic time series (Eubank, 1999).

Fourier regression has been widely applied in climatological research, particularly for temperature and rainfall prediction, demonstrating its effectiveness in modeling time

series with strong seasonal cycles (Taiwo et al., 2019). Based on this foundation, the Fourier Series approach is considered suitable for forecasting weekly air temperature dynamics in Sleman, Yogyakarta.

f. Markov Switching Model

The Markov Switching Autoregressive (MSAR) model combines the concept of Markov chains with autoregressive processes to accommodate structural changes in data (Hamilton, 1989). These shifts in data structure are believed to be influenced by unobserved discrete random variables, denoted as s_t, which represent regimes or states. The advantage of applying Markov chains in this model is its ability to provide informative estimates before a change occurs, by considering the transition probability between states, for example from state 1 to state 2. Thus, this model explicitly includes the probability of remaining in a particular model or switching to another model. The MS(m)-AR(p) model is expressed as follow:

$$\left(y_t - \mu_{s_t}\right) = \sum_{i=1}^{P} \emptyset_i \left(y_{t-1} - \mu_{s_{t-1}}\right) + \varepsilon_t \tag{2}$$

The Markov State transition probabilities can also be used to understand switching models since the transition matrix's Markov chain shows that the current value is impacted by the value from the prior period (Kim & Nelson, 1999). It is possible to express the transition probability as follows if the likelihood of state s_t equals a certain value, while j for $j = 1,2, \dots, m$ which depends on the value of the previous period and just on the most recent value of s_{t-1}, then the transition probability can be written as follows.

$$p_{ij} = p[s_t = j | s_{t-1} = i, s_{t-2} = k, \dots] = p[s_t = j | s_{t-1} = i] \tag{3}$$

$$p_{ij} = p[s_t = 1 | s_{t-1} = j] = p_{i1};\ p[s_t = 2 | s_{t-1} = j] = p_{i2}; \dots;\ p[s_t = i | s_{t-1} = j] = p_{ij} \tag{4}$$

The following is an expression for the above equation as a transition probability matrix.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{i1} \\ p_{12} & p_{22} & \cdots & p_{i2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1j} & p_{2j} & \cdots & p_{ij} \end{bmatrix} \tag{5}$$

g. Model Evaluation

Model evaluation in this study is conducted to measure predictive accuracy and to compare the performance of the Fourier Series and Markov Switching models. The evaluation utilizes four commonly applied indicators: the Generalized Cross Validation (GCV), Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). Both criteria are widely applied in regression and forecasting studies to select the best model in terms of predictive performance.

GCV is a data-driven method for selecting optimal model parameters, particularly useful in regularization problems and smoothing parameter selection. The GCV criterion provides an estimate of the prediction error without requiring an independent validation dataset. For a given model with smoothing parameter c, the GCV is defined as (Ni'matuzzahroh & Dani, 2024):

$$GCV(c) = \frac{MSE(c)}{\left(n^{-1}tr(\boldsymbol{I}-\boldsymbol{A}(c))\right)^2} \tag{6}$$

where c is oscillation parameter, $\boldsymbol{I}$ is the identity matrix with size $n \times n, y_j$ is the response variable for the j^{th} observation, and $\hat{y}_j$ is the predicted value of the response variable for the j^{th} observation. The Mean Squared Error (MSE) is used to evaluate model performance by measuring the average squared difference between actual and predicted values. MSE emphasizes larger errors due to the squaring operation, making it useful for identifying models that produce substantial deviations (Hyndman & Koehler, 2006). The MSE is defined as:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 \tag{7}$$

The Mean Absolute Error (MAE) complements MSE by quantifying the average magnitude of forecast errors in absolute terms without penalizing large deviations excessively. MAE offers a more intuitive interpretation of typical prediction error and is less sensitive to outliers (Willmott & Matsuura, 2005). The MAE is formulated as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t| \tag{8}$$

Meanwhile, the Mean Absolute Percentage Error (MAPE) is employed to quantify forecast accuracy by comparing predicted values with actual observations (De Myttenaere et al., 2016). The formula for MAPE is given as:

$$MAPE = \frac{100\%}{\mathrm{n}}\sum_{t=1}^{n}\left|\frac{y_t-\hat{y}_t}{y_t}\right| \tag{9}$$

Where y_t is actual observation at time t, and $\hat{y}_t$ is the corresponding forecasted value. A smaller MAPE indicates a higher forecasting accuracy, with values below 10% considered highly accurate. All analyses were performed using software R supported by specialized packages for regression, time series analysis, and Markov switching models.

C. RESULT AND DISCUSSION

1. Descriptive Statistic

Air temperature data in Sleman Regency, Yogyakarta, was analyzed to provide an overview before the modeling process was carried out. The results of the descriptive analysis in this study are presented in Table 1.

Table 1. Statistics Descriptive

Variable	Mean	Minimum	Maximum	Standard Deviation
Air temperature	26.96	24.33	30.13	1.27

The descriptive statistics show that weekly temperatures in Sleman range from 24.33°C to 30.13°C, with the minimum temperature occurring in the fourth week of August 2023 and the maximum temperature recorded in the fifth week of October 2023. The mean temperature of 26.96°C and the relatively narrow variation reflect the stable thermal conditions characteristic of Yogyakarta's tropical monsoonal climate, where transitions between wet and dry seasons produce gradual rather than abrupt temperature changes. Local topographic influences, particularly from the Merapi volcanic region, also contribute to moderating short-term thermal fluctuations. These smooth and consistent oscillatory patterns further justify the use of periodic modelling approaches such as the Fourier series and MSAR in the subsequent analysis.

2. Predictions Analysis Using Fourier Series Method

The Fourier series method was applied to predict the weekly average air temperature in Sleman, Yogyakarta. The first step in constructing the model was to determine the optimal number of harmonics (λ), which was selected based on the minimum Generelized Cross Validation (GCV) value. The result of GCV calculation for several values of λ are shown in Table 2, where the smallest GCV was obtained at $\lambda = 14$. This value was therefore used in the final Fourier series model.

Table 2. GCV values for different numbers of harmonics

Lambda (λ)	GCV
1	1,2341
2	1,0222
3	0,6285
⋮	⋮
14	**0,3553**
15	0,3749
16	0,3756

After determining the optimal number of harmonics (λ), the next step is to estimate the Fourier coefficients. In this model, the parameters a_j represent the coefficients of the cosine functions, while b_j correspond to the coefficients of the sine functions. These parameters are obtained through the least squares estimation procedure and are used to construct the Fourier regression equation. The estimated coefficients are presented in Table 3.

Table 3. Estimated Fourier coefficients

Lambda (λ)	a_j	b_j
1	-0,5087	-1,3353
2	-0,3483	0,6023
3	-0,8613	-0,1088
⋮	⋮	⋮
12	0,1407	0,0820
13	-0,2590	0,1814
14	0,1762	0,0603

Referring to Equation (1) in the Methods section, the resulting Fourier regression model for the weekly average temperature can be written as follows:

$$\hat{y}_r = 27{,}06556 - 0{,}5087\cos(2\pi j t_r) - 1{,}3353\sin(2\pi j t_r) - 0{,}3483 \cos(2\pi j t_r) + 0{,}6023\sin(2\pi j t_r) - \cdots + 0{,}1762\cos(2\pi j t_r) + 0{,}0603\, sin(2\pi j t_r) \quad (8)$$

After obtaining the Fourier coefficients, these parameters were used to construct the regression equation, which represents the periodic variation in the temperature data. These coefficients determine the amplitude and phase of the periodic components that characterize the variation in the temperature data. The predictive performance of the Fourier Series model was then evaluated using both training and testing datasets, with the data split into 70% for training and 30% for testing. The evaluation metrics included the Generalized Cross Validation (GCV), Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). The result, as shown in Table 4 below.

Table 4. Evaluation metrics of the Fourier Series model

Dataset	**GCV**	**MSE**	**MAE**	**MAPE**
Training	0,3552561	0,00415	0,34015	1,25%
Testing		0,00114	0,12940	0,48%

The evaluation results indicate that the Fourier series model produced highly accurate predictions, with a MAPE value of 1.25% on the training data and 0.48% on the testing data. These low error values suggest that the model was able to capture the periodic variation of the weekly average temperature in Sleman effectively. The accuracy of the Fourier series model demonstrates its suitability for representing the seasonal pattern of climatological data. The close agreement between the predicted and actual values, particularly in the testing dataset, shows that the model generalizes well and does not exhibit signs of overfitting. Consequently, the Fourier series approach can be considered a reliable tool for forecasting temperature dynamics in Sleman, Yogyakarta.

3. Predictions Analysis Using Markov Switching Method

The average weekly air temperature in Sleman, Yogyakarta, may be predicted using the Markov switching method. Table 5 displays the findings of the MS(2) AR(2) parameter estimation for the average weekly air temperature in Sleman, Yogyakarta.

Table 5. The Parameter Estimation of the Markov Switching model

Description		Parameter	Estimation Score	*p-value*
AR(2) for the weekly average air temperature in Sleman, Yogyakarta	Regime 1	$\varphi_{0,1}$	-0.2526	0.3880
		$\varphi_{1,1}$	0.4358	0.0278
		$\varphi_{2,1}$	-1.3702	0.0025
	Regime 2	$\varphi_{0,2}$	-0.0238	0.7437
		$\varphi_{1,2}$	-0.3031	0.0161
		$\varphi_{2,2}$	0.1089	0.2983

The number of regimes and estimated values for each parameter of the MS(2) AR(2) model are displayed in Table 5. Since there are two regimes in the MS(2) AR(2) model for the weekly average air temperature in Sleman, Yogyakarta, the model employs the following equation.

$$\begin{cases} \hat{Y}_t = -0.2526 + 0.4358Y_{t-1} - 1.3702Y_{t-2} \\ \hat{Y}_t = -0.0238 - 0.3031Y_{t-1} - 0.1089Y_{t-2} \end{cases} \quad (9)$$

According to the equation above, the highest probability at time t will remain in regime 1 with a probability of 0.6216 if the average weekly air temperature difference in Sleman, Yogyakarta at periods t-1 and t-2 is in regime 1. The highest chance at time t will stay in regime 2 with a probability of 0.9436 if the average weekly air temperature difference in Sleman, Yogyakarta at times t-1 and t-2 is in regime 2. Thus, the MS(2) AR(2) model's transition matrix for Sleman, Yogyakarta's average weekly air temperature as follows.

$$\boldsymbol{P} = \begin{bmatrix} 0.6216 & 0.0564 \\ 0.3784 & 0.9436 \end{bmatrix} \quad (10)$$

The MS(2) AR(2) model's parameter estimate outcomes for the weekly average air temperature may be assessed using GCV, training MAPE, and test MAPE. These findings are shown in Table 6 as follows.

Table 6. Evaluation metrics of the Markov Switching model

Dataset	GCV	MSE	MAE	MAPE
Training	0.5747	0.9113	0.8005	1.9561%
Testing		0.4686	0.5512	1.3383%

The evaluation results indicate that the Markov Switching model produced highly accurate predictions, with a MAPE value of 1.96% on the training data and 1.33% on the testing data. These low error values suggest that the model was able to capture the periodic variation of the weekly average temperature in Sleman effectively. The accuracy of the Markov Switching model demonstrates its suitability for representing the seasonal pattern of climatological data. The close agreement between the predicted and actual values, particularly in the testing dataset, shows that the model generalizes well and does not exhibit signs of overfitting. Consequently, the Markov Switching approach can be considered a reliable tool for forecasting temperature dynamics in Sleman, Yogyakarta.

4. Selection of The Best Method

To evaluate the overall performance of the models, a comparison between the Fourier series method and the Markov Switching method was conducted. The comparison is based on the Generalized Cross Validation (GCV) value, Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE) for testing datasets. The results of the model comparison are summarized in Table 7.

Table 7. Comparison of Model Performance Metrics for Fourier Series and Markov Switching

Average of Temperature		
Metric	**Fourier Series Method**	**Markov Switching Method**
GCV	0.3520	0.5747
MSE Test	0,00114	0.4686
MAE Test	0,12940	0.5512
MAPE Test	0.4708	1.3383

As presented in Table 7, the Fourier Series method achieved lower values of GCV (0.3520), MSE (0,00114), MAE (0,12940), and MAPE for the testing data (0.4708) compared to the Markov Switching method. These results indicate that the Fourier series model provided better accuracy and generalization ability in predicting the weekly average temperature in Sleman. The comparison between the two models shows that the Fourier series approach performs better because Sleman's weekly temperature exhibits smooth and strongly periodic, making harmonic decomposition highly suitable for capturing seasonal patterns. The MSAR model, while flexible for data with abrupt changes or regime shifts, offers limited advantages in this context since the temperature series does not display significant structural transitions. As a result, Fourier aligns more closely with the underlying climatic dynamics of the region, whereas MSAR is better suited for time series characterized by nonlinear shifts rather than stable periodic oscillations.

The result of this study align with earlier studies employing harmonic or decomposition time-series methods for climatological forecasting. Zong-chang (2013) demonstrated that Fourier analysis can effectively capture the oscillatory behavior of air temperature and improve forecasting accuracy by representing its dominant periodic patterns. More recently, Utami et al. (2023) successfully applied nonparametric Fourier regression for climate-related downscaling in Indonesia, confirming the method's suitability for capturing smooth oscillatory patterns in tropical climates. In contrast, regime-switching approaches such as those evaluated by Bessac et al. (2016) have shown stronger performance when meteorological time series exhibit transitions between distinct dynamic states, as commonly observed in wind-field variability. Since our weekly temperature data for Sleman exhibit smooth periodic oscillations rather than frequent regime shifts, our finding that Fourier series outperforms MSAR is consistent with and extends these foundational studies. Based on this superiority, the Fourier series method was then employed to generate predictions of weekly average temperature for the next six periods beyond the observed data. The predicted values are presented in Table 8, which provides an overview of the expected short-term temperature dynamics in Sleman, Yogyakarta.

Table 8. Forecast of Weekly Average Temperature in Sleman Using the Fourier Series

Period	Forecasted Temperature (°C)
1	26.67
2	26.91
3	26.90
4	26.93
5	26.82
6	26.32

The forecasted weekly temperatures, which range only from 26.32°C to 26.93°C, indicate stable short-term conditions with minimal weekly variability. This narrow fluctuation range reflects the model's ability to capture smooth periodic patterns in the data, suggesting that near-term temperature levels in Sleman are expected to remain steady and within their typical climatological range.

D. CONCLUSION AND SUGGESTIONS

This study demonstrated the Fourier series method's superiority over Markov Switching Autoregressive for forecasting weekly air temperature in Sleman, Yogyakarta, using 127 observations from January 2023 to June 2025. The Fourier Series model achieved a minimum GCV value of 0.3520, with MAPE values of 1.26% for the training data and 0.47% for the testing data. In contrast, the Markov Switching model recorded higher error levels, with a GCV of 0.5747, MAPE of 1.96% on the training set, and 1.34% on the testing set. Conceptually, this superiority is consistent with the theoretical properties of Fourier Series, which is designed to capture smooth, continuous, and cyclic variations that dominate weekly temperature dynamics in Sleman. Because the underlying climatological patterns exhibit stable periodicity rather than abrupt regime shifts, Fourier-based decomposition aligns more closely with the intrinsic structure of the data, enabling more efficient representation of its dominant oscillatory components.

Several limitations should be noted. The analysis relied on a single climatological variable and a specific observation window, which may restrict the generalizability of the findings to other locations, seasons, or variables influenced by more complex atmospheric interactions. Additionally, the study does not incorporate potential external predictors such as humidity, wind speed, or precipitation, which may influence temperature variability and improve model performance.

Future research may explore hybrid forecasting approaches that combine Fourier-based seasonal components with nonlinear or machine-learning models to capture both periodic patterns and potential extreme fluctuations. Expanding the analysis by incorporating additional climatological variables such as humidity, precipitation, or radiation could also improve predictive robustness. Applying these models across broader temporal ranges and diverse regions would strengthen their external validity. Enhanced short-term temperature forecasting can further support institutional needs, including BMKG's early-warning services, local climate adaptation efforts, and mitigation planning in sectors such as agriculture, public health, and energy management.

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