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Mathematical Model of Joint Life Term Insurance Premiums under Inflation, Interest Rate, and Dependent Mortality

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ABSTRACT

Multilife insurance refers to a contract that covers two or more lives simultaneously, with joint life insurance representing a key form in which the benefit is paid upon the first death among the insured individuals. The lifetimes of insured individuals are typically not independent, as they may be influenced by shared environmental, health, or behavioral factors, leading to mortality dependence. Inflation and interest rates also play critical roles in determining the present value of benefits and premiums. However, most previous studies have examined either mortality dependence or macroeconomic effects in isolation. This study aims to develop a comprehensive mathematical model for determining joint life term insurance premiums that simultaneously incorporates mortality dependence through the Gumbel copula and interest rate and inflation through the Fisher equation. The model integrates demographic and economic risk components within a unified actuarial valuation framework, providing a more realistic representation of premium dynamics under varying financial conditions. Simulation results indicate that premiums incorporating inflation are consistently higher than those without inflation, whereas higher nominal interest rates result in lower premium levels. These findings reflect the theoretical relationship between inflation, real interest rates, and the time value of money. The study further introduces an elasticity-based analysis that quantifies the sensitivity of premiums to changes in inflation and interest rates, demonstrating nonlinear yet economically meaningful responses across different age structures of insured spouses. The results highlight the importance of jointly modeling mortality dependence and economic variables to enhance pricing accuracy and fairness in life insurance. The proposed model offers practical relevance for actuaries in premium determination, assists insurers in risk management and product design, and supports the development of resilient pricing strategies under inflationary and interest.

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A. INTRODUCTION

Term life joint life insurance represents a contract that provides financial protection for two or more insured individuals, where the benefit is paid upon the first death occurring within a specified period (Dębicka et al., 2023). In such contracts, the mortality risks of the insured lives are typically not independent, as they may be influenced by common factors such as health conditions, lifestyle, or shared environmental and familial characteristics (Gobbi et al., 2019). This interdependence adds a significant layer of complexity to actuarial modeling and premium determination compared to single-life insurance, which involves only one risk exposure

(Dufresne et al., 2018). As the demand for multi-life insurance products continues to increase, particularly in emerging markets such as Indonesia, the development of premium models that explicitly capture mortality dependence has become an increasingly important topic in actuarial research and practice (Zhang et al., 2025). Studies by Kara & Aslan (2025) have demonstrated that ignoring mortality dependence may lead to biases in the estimation of reserves and present values of benefits, which in turn can distort premium calculations. Therefore, explicitly incorporating this dependence in the modeling of life insurance contracts is essential to ensure actuarial fairness and accuracy in valuation.

Modeling approaches based on copulas, such as Archimedean or Gumbel families, have been demonstrated to effectively capture nonlinear dependencies between insured lives and enhance the accuracy of present-value and premium estimations (Kularatne et al., 2021). In the context of joint life insurance, the copula approach enables the dependence structure to be separated from the marginal lifetime distributions of individual lives, allowing for the relationship between lifetimes to be modelled flexibly without imposing a specific form of joint distribution (Li et al., 2020). This approach offers a significant advantage over conventional models, as it can capture tail dependence, the tendency of two extreme risks (such as the early deaths of both insured lives) to occur simultaneously (Frees et al., 1996).

In addition to mortality risk, the determination of life insurance premiums is strongly influenced by macroeconomic factors, particularly inflation and interest rates. Han and Hung (2017) found that inflation and nominal interest rates have a significant impact on the demand for life insurance, which in turn indirectly affects the level of premiums. (Flores et al., 2021; Han & Hung, 2017). Conversely, the interest rate plays a crucial role in determining the present value of benefits and premium payments; an increase in interest rates generally reduces the present value of insurance liabilities, as it raises the discount rate applied in present value calculations (Zhang et al., 2025). Fisher further established the theoretical linkage between inflation and interest rates through the well-known Fisher equation.

Previous studies have examined various factors influencing the determination of life insurance premiums; however, most have treated economic and demographic aspects separately. Jia et al. (2014) investigated the impact of stochastic interest rates on life insurance premiums without considering the effect of inflation. In contrast, Park et al. (2023) demonstrated that inflation risk has a significant influence on life insurance premiums and optimal insurance demand, as inflation uncertainty alters individuals' willingness to purchase and maintain coverage. Kaishev et al. (2007) utilized copula functions to model the joint distribution of survival times under competing risks, providing a flexible framework to capture dependency between multiple lifetimes. However, only a limited number of studies have integrated mortality dependence with the simultaneous effects of inflation and interest rates within the premium modeling of term joint life insurance.

This gap highlights the need for a comprehensive mathematical approach to capture the complex relationship between demographic and economic factors in premium determination. Mortality dependence causes the distribution of death times between insured lives to become non-independent, whereas inflation and interest rates influence the present value of benefits and premiums through the mechanisms of discounting and the real value of money (Dahl, 2004; Kaishev et al., 2007). Without a mathematical model capable of simultaneously accommodating both aspects, premium determination may become inaccurate, potentially leading to financial imbalances for insurance companies in the form of underpricing or overpricing. Therefore, an integrative mathematical model is required to link mortality dependence with macroeconomic fluctuations through the parameters of inflation and interest rates. Accordingly, the development of a mathematical model that integrates mortality dependence and macroeconomic factors has become both a scientific and practical imperative in modern actuarial science.

Building upon this urgency this study aims to develop a mathematical model for determining term joint life insurance premiums under mortality dependence using the Gumbel copula, while incorporating the effects of inflation and interest rates through the Fisher equation. Specifically, the objectives of this study are to: (1) develop a mathematical model for determining term life insurance premiums under a joint life framework with dependent mortality, explicitly incorporating the effects of interest rates and inflation; and (2) evaluate the impact of interest rate and inflation on joint life term insurance premiums across different age scenarios of the insure spouses.

Unlike previous studies that examined these aspects separately, this research develops a copula-based mathematical model that explicitly captures the joint influence of demographic and economic risks on premium determination. The primary purpose of this research is to provide both theoretical and practical insights for actuaries and insurers in accurately assessing and pricing joint life insurance policies under conditions of dependent mortality and fluctuating financial environments.

B. METHODS

This study is a quantitative theoretical study utilizing a mathematical modeling approach to determine joint life term insurance premiums under dependent mortality, the effects of inflation and interest rates. The dependence between lifetimes is modelled using a Gumbel copula, following the formalization of dependent risk structures proposed by Denuit et al. (2005), and its application to joint-life pricing contexts, as illustrated by Gobbi et al. (2019). Furthermore, financial and economic factors are incorporated by adjusting nominal interest rates for inflation using the Fisher relation Fisher (1930), consistent with the time-value-of-money framework discussed by Kellison (2009).

1. Data Description and Assumptions

The data used in this study are based on the Indonesian Mortality Table 2019 (TMI IV) (Asosiasi Asuransi Jiwa Indonesia (AAJI), 2019), which serves as the basis for calculating survival and death probabilities for males and females. The dataset reflects the national mortality rates and is widely used for actuarial computations in Indonesia. The interest rate (i) is set at 7%, reflecting the average BI rate from 2015 to 2024 (Badan Pusat Statistik Indonesia, 2025), with two additional constant rate scenarios considered, namely $i_1 < i$ dan $i_2 > i$ which is $i = 6\%$ and $i = 8\%$. The inflation rate is set at 3%, based on the average year-on-year inflation from 2015 to 2024 (Bank Indonesia, 2025), and is also assumed to be constant. The insurance term is $n = 10$ years. Three age scenarios for insured spouses, husband (x) and wife (y), are considered, as shown in Table 1.

Table 1. Scenarios of Insured Spouses' Ages

Scenario	Age Relationship of the Insured	Simulation Range (Ages)
1	Husband and wife of the same age	(55,55) to (65,65)
2	Husband is 5 years older than the wife's age	(55,50) to (65,60)
3	Wife is 3 years older than the husband's age	(55,58) to (65,68)

All data were processed using Python for modeling and numerical integration, and Microsoft Excel was employed for validation of calculations and sensitivity analysis.

2. Model Formulation

a. Single Life Distribution

The survival distribution for a husband aged (x) and a wife aged (y) is assumed to follow TMI IV. For *non-integer* ages, mortality rates are interpolated under the assumption of a *uniform distribution of deaths* (UDD).

1) Single-Life Survival Distribution under the UDD Assumption

Let (x) denote the husband aged x, and let T_x be the continuous type random variable representing the future lifetime of (x). The marginal survival distribution for a husband aged x can be expressed as the probability that (x) survives at least until age $x+n$, which can be written as

$$\begin{aligned} {}_np_x &= S_{T_x}(n) = \Pr(T_x > n) \\ &= \prod_{i=0}^{n-1} p_{x+i} = \prod_{i=0}^{n-1}(1-q_{x+i}) \end{aligned} \tag{1}$$

for all $n \geq 0$. Under the UDD assumption, if an individual aged x survives to age $x+n$, uq_{x+n} can be interpreted as the probability of failure during a fraction u of that year. Then, the fractional survival probability is $1-uq_{x+n}$. Hence, the probability that a husband aged x survives until age $x+t$, where t is non-integer, can be expressed as

$${}_tp_x = {}_np_x \cdot (1-uq_{x+n}), \qquad t = n+u, 0 \leq u < 1. \tag{2}$$

2) Cumulative Distribution Function

Let $F_{T_x}(t)$ denote the cumulative distribution function (c.d.f) of T_x and represents the probability that individual (x) does not survived beyond age $x+t$, and we refer F_{T_x} as the lifetime distribution from age x, and can be written as

$$\begin{aligned} F_{T_x}(t) &= {}_tq_x = \Pr(T_x \leq t) = 1 - {}_tp_x. \\ &= 1 - {}_np_x(1-uq_{x+n}) \end{aligned}$$

and $t = n+u$. Using the result in Equation (2), we obtain

$$F_{T_x}(t) = \textstyle\sum_{i=0}^{n-1}({}_ip_x q_{x+i}) + {}_np_x(uq_{x+n}). \tag{3}$$

b. Joint Life Distribution

1) Joint Survival Distribution under the Independence Assumption

   Let (x) and (y) denote individuals of age x and y. Define T_x and T_y as the continuous random variables representing the future lifetime of individuals (x) and (y). The joint survival distribution of (x) and (y) can be expressed as the probability that both individuals survive to at least age $x+t$ and $y+t$. This can be written as

$$S_{T_x,T_y}(t_1,t_2) = \Pr\left(T_x > t_1 \wedge T_y > t_2\right). \tag{4}$$

   If T_x and T_y are assumed to be independent, the joint survival distribution in Equation (4) can be expressed as

$$\begin{aligned} S_{T_x,T_y}(t_1,t_2) &= \Pr\left(T_x > t_1 \wedge T_y > t_2\right) \\ &= \Pr(T_x > t_1)\Pr\left(T_y > t_2\right) \\ &= S_{T_x}(t_1)S_{T_y}(t_2) \end{aligned}$$

   thus, if $t_1 = t_2 = t$, it can be written as

$$S_{T_x,T_y}(t,t) = {}_tp_x \cdot {}_tp_y. \tag{5}$$

2) Joint Life Survival Distribution under Dependence Assumption

   For dependent lifetimes of the husband (x) and wife (y), the joint survival probability can be expressed using the Gumbel copula as

$${}_tp_{xy} = S_{T_x,T_y}(t,t) = C_\theta\left({}_tp_x, {}_tp_y\right),$$

   The Gumbel copula $C_\theta(u,v)$ is defined as

$$C_\theta(u,v) = \exp\left[-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right], \quad \text{for } \theta \geq 1, \text{and } u,v \in (0,1],$$

   hence, the survival probability under dependent mortality is

$${}_tp_{xy} = \exp\left[-\left[(-\ln\ {}_tp_x)^\theta + \left(-\ln\ {}_tp_y\right)^\theta\right]^{\frac{1}{\theta}}\right], \tag{6}$$

   with ${}_tp_x$ and ${}_tp_y$ given by the UDD formulas.

3) Cumulative Distribution Function of Joint Life Status

   The probability that at least one of (x) and (y) dies within t years (first-to-die) is given by

$$F_{T_{xy}}(t) = {}_tq_{xy} = \Pr(\min[T(x), T(y)] \leq t),$$

and using a basic result in probability, we have

$$\begin{aligned} F_{T_{xy}}(t) = {}_tq_{xy} &= \Pr[T(x) \leq t] + \Pr[T(y) \leq t] - \Pr[T(x) \leq t \cap T(y) \leq t] \\ &= {}_tq_x + {}_tq_y - F_{T(x),T(y)}(t,t) \\ &= {}_tq_x + {}_tq_y - {}_tq_{\overline{xy}}. \end{aligned} \tag{7}$$

c. Dependence Modeling Using the Gumbel Copula

1) Model Justification

The Gumbel copula was selected based on the Akaike Information Criterion (AIC) value. Using the observed mortality data and the model assumptions, the Gumbel copula produced the smallest AIC value of –296.66, indicating the best fit to the empirical data. According to the AIC selection principle, a smaller AIC value implies a better balance between model fit and parsimony (Burnham & Anderson, 2002). Furthermore, the Gumbel copula is particularly suitable for modeling upper-tail dependence, which reflects the increased likelihood of simultaneous or closely occurring extreme mortality events within a spouses (Frees et al., 1996).

2) The Gumbel Copula

In this study, the mortality of the insured lives is assumed to be dependent. The Gumbel copula is applied to model this dependence structure. The Gumbel copula is a type of Archimedean copula with a generator function given by $\varphi(u) = (-\ln u)^{\theta}$. Using this generator, the corresponding bivariate copula can be expressed as

$$C_{\theta}(u,v) = \exp\left[-\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{1}{\theta}}\right], \text{ for } \theta \geq 1, \text{and } u, v \in (0,1]. \tag{8}$$

The parameter θ can be determined through the relationship between Kendall's τ and the parameter of an Archimedean copula. Let X and Y be continuous random variables with an Archimedean copula C generated by the function φ. Kendall's measure of dependence τ, is defined as follows:

$$\tau_{X,Y} = 1 + 4\int_0^1 \frac{\phi_{\theta}(t)}{\phi_{\theta}'(t)} dt.$$

By substituting the generator function of the Gumbel copula and its first derivative. Then, we have the relationship between Kendall's tau and the Gumbel parameter written as

$$\tau = 1 + 4\int_0^1 \frac{(-\ln u)^{\theta}}{\left(\frac{\theta(-\ln u)^{\theta-1}}{-u}\right)} du = \frac{\theta-1}{\theta}. \tag{9}$$

(Ghalibaf, 2020).

3) Joint Life Cumulative Distribution Function with the Gumbel Copula

Let F_{T_x,T_y} denote the joint cumulative distribution function of the random variables T_x and T_y, with F_{T_x} and F_{T_y} representing their respective marginal distribution functions. By Sklar's theorem, there exists a copula C such that, for all $t, t \in \mathbb{R}$:

$$F_{T_x,T_y}(t,t) = C_\theta\left(F_{T_x}(t), F_{T_y}(t)\right).$$

Based on Equation (8), the probability of failure of a spouses aged x and y within the next t years, denoted by $({}_tq_{\overline{xy}})$, under the assumption of mortality dependence, can be computed by letting u denote the probability that the husband aged x dies within t years $({}_tq_x)$ and v denote the probability that the wife aged y dies within t years $({}_tq_y)$, so that, we have

$${}_tq_{\overline{xy}} = C_c\left({}_tq_x, {}_tq_y; \theta\right) = \exp\left[-\left[(-\ln {}_tq_x)^\theta + \left(-\ln {}_tq_y\right)^\theta\right]^{\frac{1}{\theta}}\right]. \tag{10}$$

Using equations (7) and (10), the probability that at least one of (x) and (y) dies within t years (first-to-die), denoted by $\left({}_tq_{xy}\right)$ under the assumption of mortality dependence using the Gumbel copula, can be expressed as

$$\begin{aligned} {}_tq_{xy} &= {}_tq_x + {}_tq_y - \exp\left[-\left[(-\ln {}_tq_x)^\theta + \left(-\ln {}_tq_y\right)^\theta\right]^{\frac{1}{\theta}}\right] \\ &= \left(\sum_{i=0}^{n-1}({}_ip_x q_{x+i}) + {}_np_x(uq_{x+n})\right) + \left(\sum_{i=0}^{n-1}\left({}_ip_y q_{y+i}\right) + {}_np_y(uq_{x+n})\right) \\ &\quad - \exp\left(-\left(\left[-\ln F_{T_x}(t)\right]^\theta + \left[-\ln F_{T_y}(t)\right]^\theta\right)^{1/\theta}\right). \end{aligned} \tag{11}$$

d. Real Force of Interest

The nominal interest rate (i) is the rate set by financial institutions without accounting for inflation (π). In contrast, the interest rate adjusted for inflation, which reflects the growth of purchasing power, is referred to as the real interest rate (r). The relationship among these three variables was formulated by Fisher (Fisher, 1930) and is mathematically expressed as

$$1 + i = (1 + r)(1 + \pi) \Leftrightarrow r = \frac{1+i}{1+\pi} - 1. \tag{12}$$

In this study, the annual effective interest rate is assumed to be constant over time, so that the force of interest (δ) or discount rate can be determined as follows:

$$\delta = \ln(1 + i) \tag{13}$$

using Equations (12) and (13), the real force of interest (δ_r) is obtained as

$$\delta_r = \ln\left(\frac{1+i}{1+\pi}\right), \tag{14}$$

which reflects the instantaneous rate of return after adjusting for inflation.

e. Annual Life Term Insurance Premium

Consider a death benefit of 1 is payable immediately on the death of the first to die of husband (x) and wife (y) within n years. The actuarial present value of the term insurance benefit under joint life status with mortality dependence is expressed as

$$\begin{aligned} \bar{A}^{1}_{xy:\overline{n|}} &= \int_0^n e^{-\delta t} f_{T_{xy}}(t)\,dt = \int_0^n e^{-\delta t}\ {}_tp_{xy}\mu_{xy}(t)dt \\ &= \int_0^n e^{-\delta t}\left[\mu_x(t)\ {}_tp_x\left(1 - C_u\left({}_tq_x, {}_tq_y; \theta\right)\right) + \mu_y(t)\ {}_tp_y\left(1 - C_v\left({}_tq_x, {}_tq_y; \theta\right)\right)\right]dt, \end{aligned} \tag{15}$$

C_u, C_v denote the partial derivatives of the Gumbel copula with respect to u and v, respectively, while $\mu_x(t), \mu_y(t)$ represent the instantaneous force of mortality for the husband aged x and the wife aged y. The actuarial present value of an n-year temporary life annuity-due of 1 per payment under joint-life status with dependent mortality, denoted $\ddot{a}_{xy:\overline{n|}}$, can be expressed as

$$\ddot{a}_{xy:\overline{n|}} = \sum_{t=0}^{n-1} v^t\ {}_tp_{xy}, \tag{16}$$

where v^t is the annual discount factor. Under the equivalence principle, the annual level premium (P) paid in advance for an n-year joint-life term insurance with benefit 1 upon first death satisfies, we have:

$$P = \frac{\bar{A}^{1}_{xy:\overline{10|}}}{\ddot{a}_{xy:\overline{10|}}}$$

3. Simulation Procedure

All simulations, except for the annual premium calculations, were implemented in Python (version 3.12) using a combination of numerical and stochastic methods. The step-by-step simulation procedure is as follows:

a. Copula Model Fitting and Parameter Estimation

Kendall's tau and the copula dependence parameter were estimated using Python packages copulas and scipy.stats through maximum likelihood estimation (MLE) to capture the degree of dependency between the lifetimes of the spouses.

b. Simulation of Dependent Lifetimes

Dependent lifetime pairs were generated via the Gumbel copula using random uniform samples. This procedure was implemented using the numpy, pandas, mathplotlib.pyplot, scipy, and copulae libraries to simulate correlated survival times for both spouses.

c. Estimation of First-to-Die Probabilities
The probability of failure, defined as the first-to-die probability under the joint life framework, was calculated using Python libraries matplotlib.pyplot, scipy, and copulae. The computation was based on simulated dependent lifetime pairs obtained from the estimated Gumbel copula, allowing the model to account for dependence in mortality between spouses.

d. Calculating the force of interest and the real force of interest
The force of interest and the real force of interest were calculated using the Fisher equation, implemented through Python packages numpy and math.

e. Computation of Actuarial Present Values
Using numpy, pandas, math, and matplotlib.pyplot actuarial present values of benefits and joint-life annuities were computed numerically. Discounting was performed using both nominal and real interest rates derived from the Fisher-adjusted formula. Scenarios were evaluated under three nominal interest rates (6%, 7%, and 8%) and two economic settings (with and without inflation).

f. Premium and Elasticity Analysis
Annual premiums were calculated under the equivalence principle. Annual premiums were derived in Microsoft Excel using the simulated actuarial present values (APV) of benefit and APV of life annuity from Python. Premium elasticity with respect to interest and inflation rates was analyzed using regression-based sensitivity measures implemented in Python.

C. RESULT AND DISCUSSION

1. Simulation Result

a. Gumbel Copula Parameter Estimation
The dependence parameter θ was estimated from the Indonesian Mortality Table 2019 using Python with the copulae package. The computed Kendall's tau (τ) for the Gumbel copula model, it is 0.8901. Using Equation (9), we have

$$\tau = \frac{\theta - 1}{\theta}.$$

$$0.8901 = \frac{\theta - 1}{\theta}.$$

$$\theta - 0.8901\theta = 1 \rightarrow \theta = \frac{1}{0.1099} = 9.099 \approx 9.$$

b. Joint Life Death Probabilities

The computed probabilities of at least one death within the n-year period for a spouses aged x and y, under the assumption of dependent mortality, are presented in Figure 1. The results correspond to a 10-year joint life term insurance.

Figure 1. Joint life (first-to-die) probabilities for ages 55 to 65.

The simulation results indicate that the joint life probability of the first to die increases consistently as the spouses's age advances. The sharpest increment occurs when the husband's age progresses from 60 to 61 years. Among the three demographic scenarios, spouses in which the husband is five years older exhibit the lowest joint life probability of the first to die. In contrast, spouses in which the wife is three years older display the highest probability.

c. Results of Force of Interest Calculation

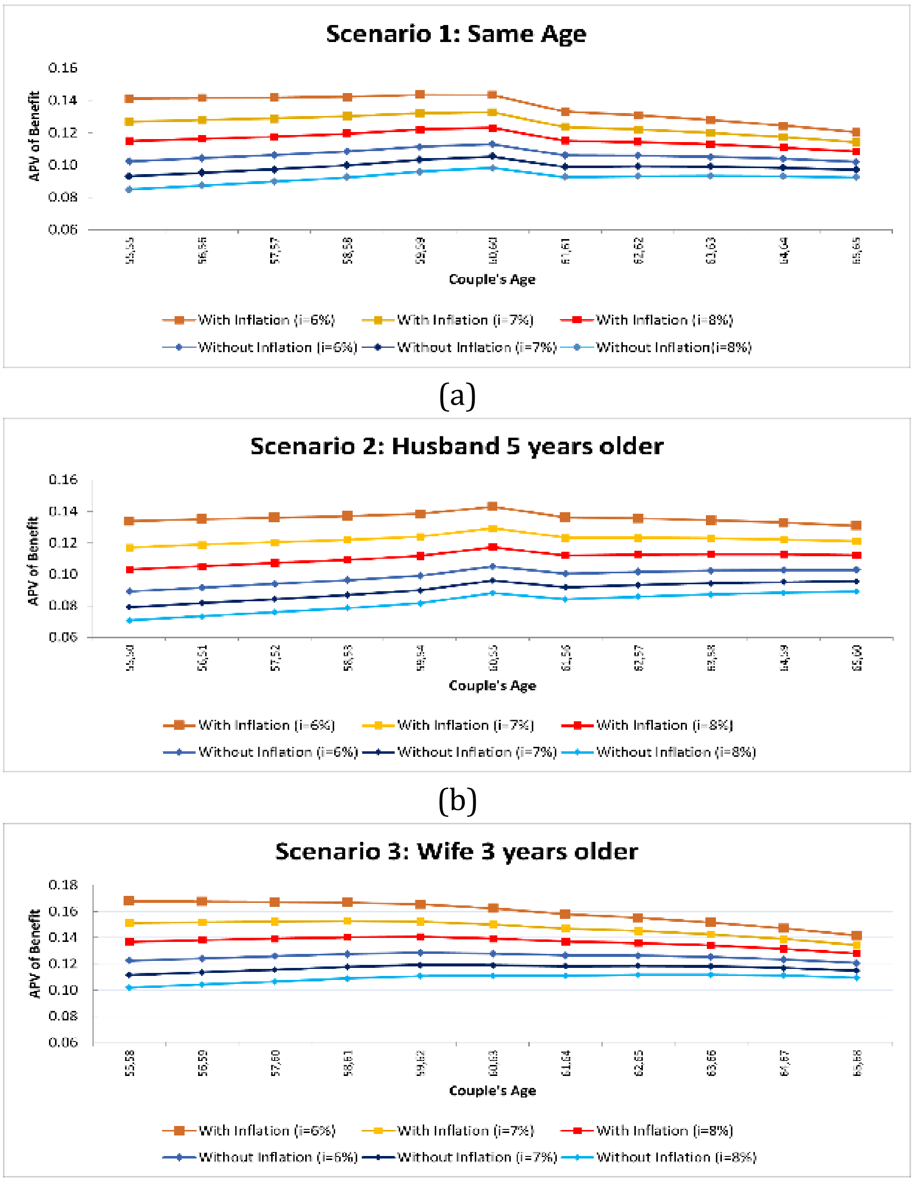
Fisher's Equation illustrates that an increase in inflation leads to a higher nominal interest rate when the real interest rate is held constant (Fisher, 1930; Mitchell-Innes et al., 2007). The calculations of r, and δ_r for nominal interest rates of 6%, 7%, and 8% with an inflation rate of 3% are presented in Table 2.

Table 2. Interest Rate(r), δ, and δ_r

i	δ	r	δ_r
6%	0.058	2.91%	0.029
7%	0.068	3.88%	0.038
8%	0.077	4.85%	0.047

d. Actuarial Present Value of Term Life Insurance Benefits under Joint Life Status

The simulation outcomes of the actuarial present value of the benefits for term life insurance with joint life status, considering mortality dependence, are illustrated in Figure 2.



(a)

(b)

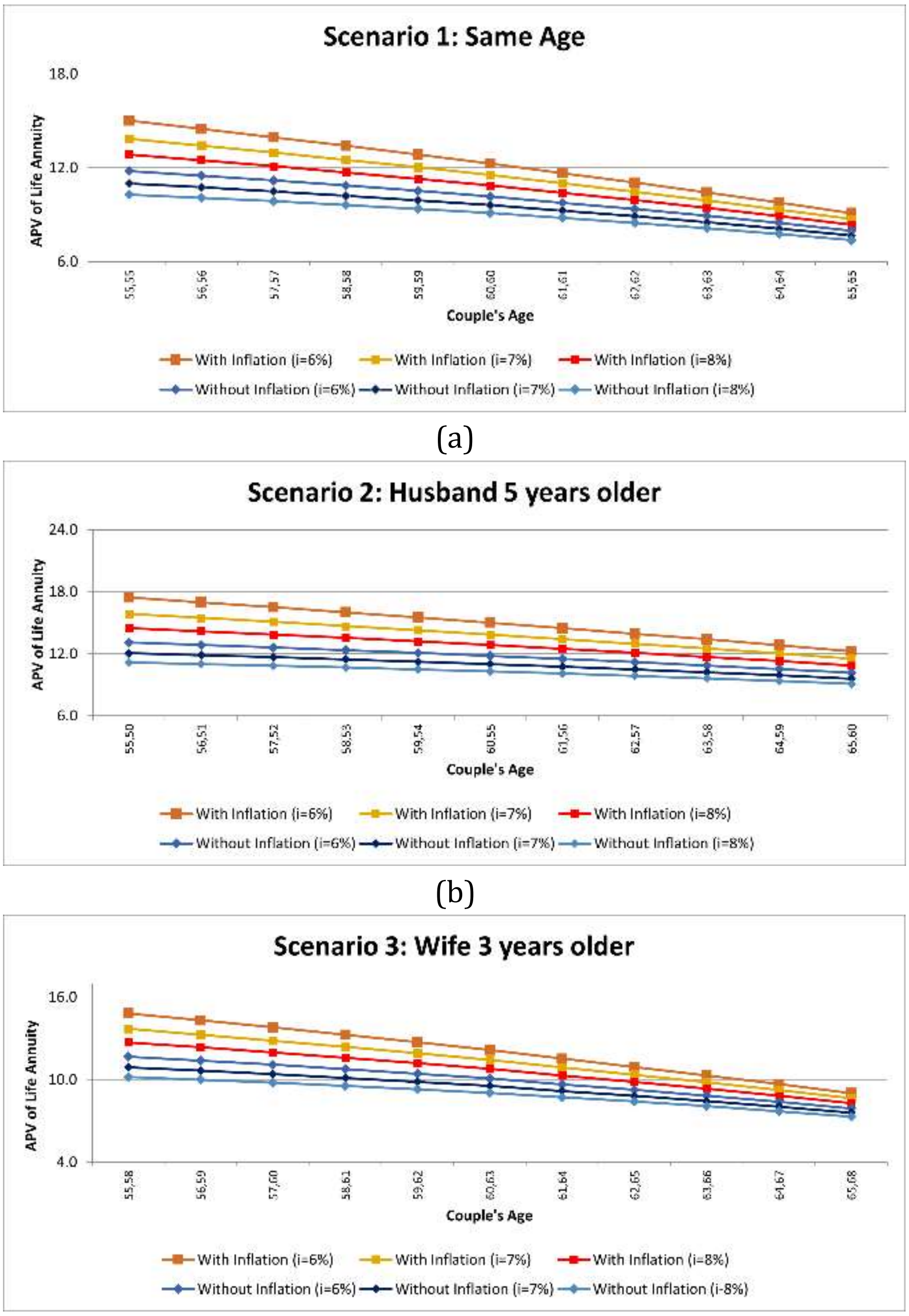
(c)

Figure 2. The Actuarial Present Value (APV) of joint life term life insurance benefits under under dependent mortality (a) scenario 1, (b) scenario 2, and (c) scenario 3

The simulation result in Figure 2 shows that the actuarial APV of benefits increases gradually up to the husband's age of 59–60 years, followed by a sharp decline after age 61. This decline is more pronounced under the inflation-adjusted condition than the non-inflation scenario. While the non-inflation curve displays a slight rebound after the initial drop, the inflation-adjusted APV of benefits remains consistently higher across all age groups, highlighting the amplifying effect of inflation on the nominal value of future claims. Furthermore, the results show that as the interest rate increases, the actuarial present value of benefits decreases. Among the three demographic scenarios, spouses where the husband is five years older record the lowest APV, whereas spouses where the wife is three years older show the highest APV.

e. Actuarial Present Value of Term Life Annuities under Joint Life Status

The calculation results of the actuarial present value of term life annuities with joint life status are presented in Figure 3.



(a)

(b)

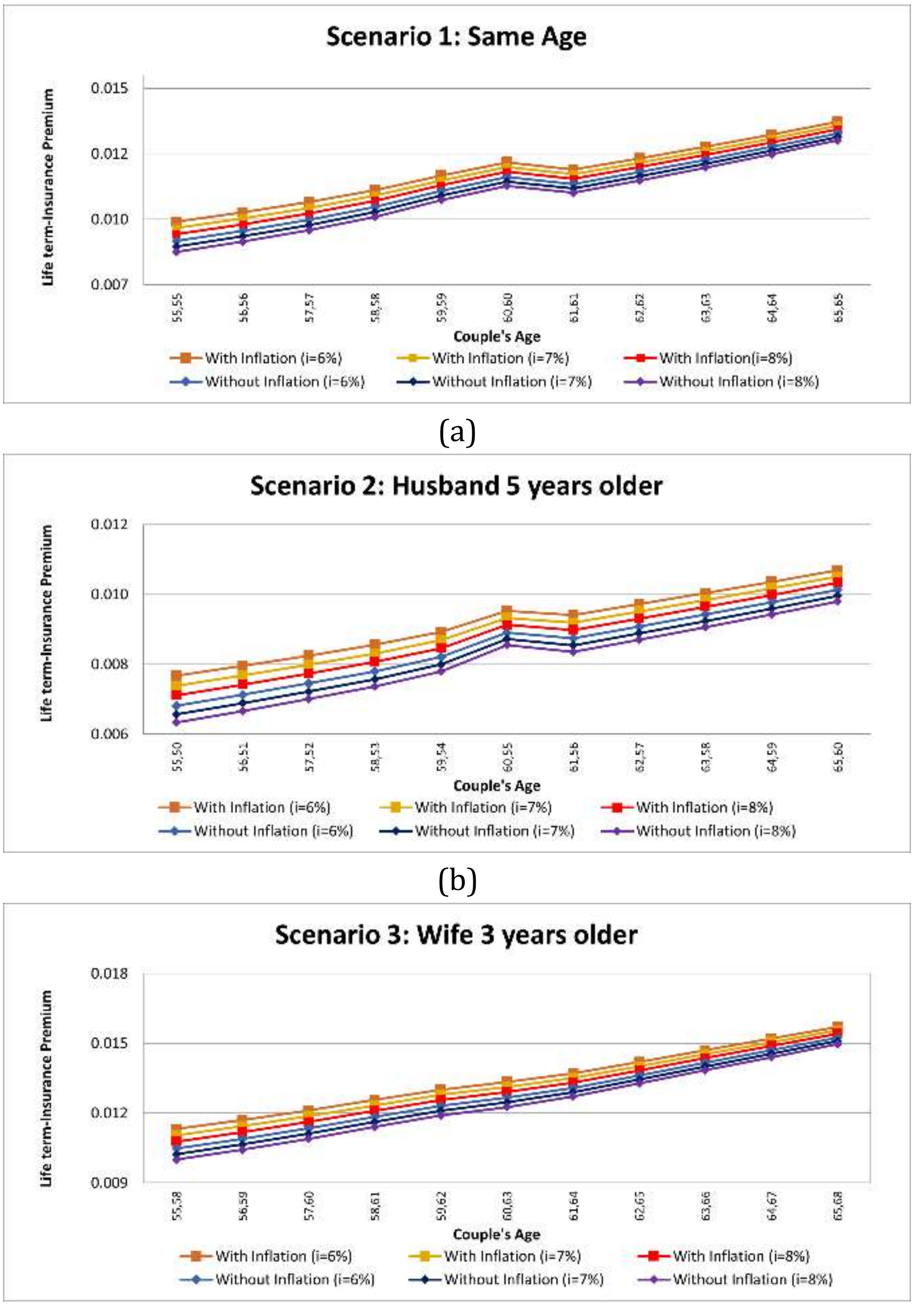
(c)

Figure 3. The Actuarial Present Value (APV) of a life annuity (a) scenario 1, (b) scenario 2, (c) scenario 3

According to Figure 3, the actuarial present value (APV) of the joint-life annuity due decreases as the initial age of the spouses increases. Spouses in which the husband is older exhibit the lowest annuity value compared to those of the same age, whereas spouses with an older wife show slightly higher values. The actuarial present value of the annuity with inflation adjustment is consistently higher than that without inflation across all age scenarios. Moreover, as the interest rate increases, the actuarial present value of the joint-life annuity declines for all demographic configurations.

f. Life Annual Premiums of Term Life Insurance under Joint Life Status

The premium represents the price that must be paid by the policyholder to obtain risk protection. Figure 4 presents the simulation results for the premium values of term life insurance under a joint life status with mortality dependence.



(a)

(b)

(c)

Figure 4. The premium of a joint life term insurance with dependent mortality
(a) scenario 1, (b) scenario 2, (c) scenario 3

According to Figure 4, the premium value increases with the age of the spouses at the time the contract is commenced. The scenario in which the husband is five years older yields the lowest premium, whereas the scenario with the wife three years older produces the highest premium. Premiums decrease as the interest rate increases and increase under inflation-adjusted conditions. For premiums without inflation, the values range from 0.00681 to 0.01524 at $i = 6\%$, 0.00657 to 0.01510 at $i = 7\%$, and 0.00634 to 0.01496 at $i = 8\%$. Under inflation-adjusted conditions, the premium values are consistently higher, ranging from 0.00767 to 0.01571 at $i = 6\%$, 0.00738 to 0.01556 at $i = 7\%$, and 0.00711 to 0.01541 at $i = 8\%$.

g. Elasticity Analysis

The impact of interest rate and inflation on the premium level of joint life term insurance, under mortality dependence, is assessed through an elasticity analysis. This evaluation quantifies the sensitivity of the premium to changes in macroeconomic parameters while accounting for the dependence structure between the lifetimes of the insured individuals. The results of this elasticity test are presented in Figure [number], illustrating how variations in interest and inflation rates influence the magnitude and stability of the calculated premiums

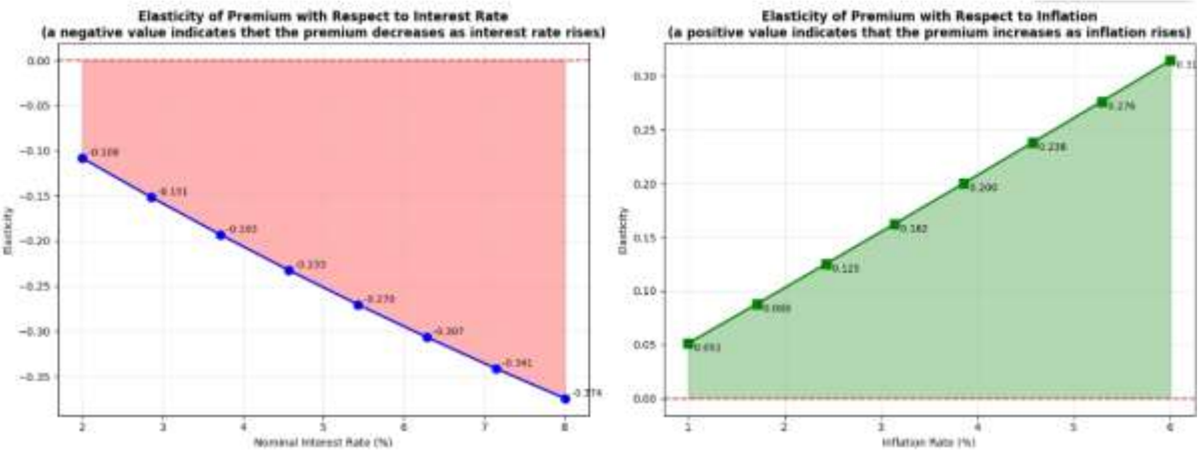


Figure 5. Elasticity Analysis

Figure 5 illustrates the elasticity of premiums with respect to interest rate and inflation. The elasticity of premiums with respect to the interest rate is negative, and the corresponding curve exhibits a downward trend, indicating that premiums decrease as the interest rate increases. Conversely, the elasticity of premiums with respect to inflation is also negative, but the curve displays an upward trend, suggesting that premiums increase as inflation rises.

2. Discussion

a. Mortality Dependence

The estimated Gumbel copula dependence parameter (θ = 9) reveals a significant dependence in the joint mortality of the insured lives. The corresponding Kendall's τ=0.8901 reflects a high degree of upper-tail dependence (Joe, 2014; Nelsen, 2006), This finding is consistent with Gobbi et al. (2019) and Dębicka et al. (2023), who emphasize that mortality independence models neglect the correlation effects between insured lives, resulting in underpricing and potential reserve inadequacy in multi-life insurance portfolios.

b. Demographic Patterns and Failure Probability

The increase in the joint life first-to-die probability with age aligns with mortality theory, which posits that the force of mortality rises exponentially with age (Dickson et al., 2013). The sharp surge observed between ages 60 and 61 reflects the accelerated phase of male mortality, consistent with empirical evidence that mortality intensifies more rapidly for men entering this age range (Li et al., 2013). From a demographic perspective, the age difference between spouses has a significant actuarial impact. When the husband is older, the probability of the first to die under the joint-life framework decreases because the wife, who biologically has a longer life expectancy (Austad & Fischer, 2016), reduces the likelihood of an early claim. Conversely, when the wife is older, the convergence of female mortality rates toward those of the younger male spouse accelerates the occurrence of the first death. This finding is consistent with Kaluszka & Okolewski (2014); Lu (2017), who demonstrated substantial spousal mortality dependence arising from heterogeneous survival dynamics between partners. Similarly, Ventura-Marco et al. (2023) reported that actuarial valuations of joint-life and spouses-based annuity products are highly sensitive to spousal age structures and the gender of the older partner. These results emphasize the actuarial sensitivity of the joint survival distribution to spousal age differences, with direct implications for product design and premium valuation in joint-life insurance.

c. Effect of Interest Rate on Premium

An increase in the constant interest rate amplifies the discount factor, thereby reducing both the APV of benefits and life annuities. Since the benefit is payable immediately upon the first death, the discounting effect remains dominant even for short-term contracts. Simulation results further indicate that the APV of benefits increases gradually up to around age 60, reflecting the dominance of the rising claim probability over discounting. Beyond age 61, the discounting effect becomes stronger, causing the APV to decline despite higher mortality. This turning point illustrates the interaction between age-dependent mortality acceleration and the time-value effect in present-value calculations. Elasticity analysis reinforces this pattern: the interest-rate elasticity of the pure annual premium remains negative (approximately −0.12 to −0.37), showing that a 1% relative rise in the nominal interest rate proportionally decreases the premium. The increasing magnitude of elasticity at higher rates indicates nonlinear sensitivity, confirming that discounting is the dominant actuarial mechanism shaping premium valuation under varying interest environments.

This phenomenon is consistent with the time value of money principle, which states that an increase in the interest rate decreases the present value of future liabilities (Kellison, 2009). In the context of joint-life term insurance, the exact mechanism applies to the valuation of death benefits, as higher interest rates reduce the discounted value of expected claims, thereby lowering the required premium for equivalence. Earlier actuarial studies have shown that joint-life contracts are susceptible to interest rate movements over medium- and long-term horizons. Recent empirical evidence by

Charupat et al. (2016) confirms that the pricing of life-contingent products responds sluggishly and asymmetrically to changes in interest rates, implying that market adjustments in premium levels occur gradually rather than instantaneously. Under a 10-year joint-life term insurance framework with benefits payable immediately upon the first death, the resulting premium reduction remains moderate yet actuarially significant, reflecting the non-linear impact of interest rate fluctuations on the joint-life present value of future benefits.

d. Effect of Inflation on Premium

The results indicate that inflation decreases the real effective interest rate while simultaneously increasing the nominal amount of future benefit payments. Consequently, the APV of benefits increases, as inflation-adjusted cash flows translate into higher nominal terms when discounted to the present. This outcome aligns with the classical Fisher effect, which posits that nominal interest rates adjust to preserve real returns under stable inflation expectations (Babbel, 1979; Fisher, 1930; Kellison, 2009). Although the APV of the life annuity (annuity due) also increases slightly owing to inflation's mitigating effect on real discounting of premiums paid at the beginning of each period, the magnitude of this increase is considerably smaller than that observed for the benefit component. As a result, the ratio between the APV of benefits and the APV of the life annuity rises, leading to an upward adjustment in the pure annual premium.

Empirically, premiums computed under inflationary conditions are consistently higher than those calculated without inflation, even when the nominal interest rate remains constant. This is because inflation erodes the real discounting power of the interest rate, amplifying the present value of future benefits more significantly than the corresponding increase in the present value of premiums. These findings are consistent with those of Kaluszka & Okolewski (2014) and recent empirical studies, which emphasise that inflation-indexed benefit structures elevate the present value of contingent claims in a nonlinear yet actuarially significant manner. Within a 10-year policy horizon, the inflationary effect remains moderate but meaningful, as the benefit is payable immediately upon the first death, accelerating the transmission of inflationary effects to the present value of claims.

e. Effect of Age Difference and Demographic Structure on Premium

The demographic structure of the insured spouses affects the premium through variations in joint survival and first-death probabilities. Under a dependent mortality framework, these probabilities cannot be treated as independent (Denuit et al., 2005). When the husband is older, the first-death probability is lower because the wife's longer life expectancy dominates the spouses' joint survival (Austad & Fischer, 2016). Consequently, the APV of benefits decreases, while the APV of the life annuity increases, resulting in a lower annual premium. Conversely, when the wife is older, the shorter joint lifetime increases the first-death probability, which raises the APV of benefits and reduces the APV of the annuity, leading to a higher annual premium (Dickson et al., 2013). This pattern is consistent with the findings of Dufresne et al. (2018), who demonstrated that larger spousal age gaps amplify the asymmetry of the first-death distribution within joint-life frameworks. Under dependent mortality, this asymmetry becomes more pronounced due to the contagion effect of mortality dependence, whereby an elevated death risk in one partner increases the short-term mortality probability of the other.

D. CONCLUSION AND SUGGESTIONS

This study demonstrates that both the interest rate and inflation have a significant impact on the premium valuation of term life insurance under the joint-life model with dependent mortality, as modelled using the Gumbel copula with a strong dependence parameter. An increase in the interest rate decreases the actuarial present value (APV) of benefits, resulting in a higher discounting effect and a corresponding reduction in the annual premium. Conversely, inflation increases the nominal value of future benefits and decreases the real interest rate, which raises both the APV and the premium level. This study contributes to the advancement of actuarial modeling by integrating mortality dependence and economic factors into the premium calculation framework for joint life insurance policies. The model provides a more realistic representation of risk correlation among insured lives, enhancing pricing precision and fairness in the insurance market. Methodologically, the study is limited by the assumption of constant interest and inflation rates, as well as a fixed dependence parameter. Future research could incorporate stochastic rate models or time-varying copulas to capture dynamic interactions more effectively. Strategically, the findings guide actuaries in pricing, assist insurers in product design and solvency management, and support regulators in evaluating the adequacy of risk-based premiums within a modern insurance environment.

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