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Bayesian Logistic Regression for Inhomogeneous Poisson Point Process: A Case Study of Post-Harvest Facilities in Sidenreng Rappang

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ABSTRACT

Understanding the spatial distribution of post-harvest infrastructure is crucial for improving the efficiency and resilience of agricultural supply chains, particularly in major food-producing regions. This study aims to extend the estimating equations based on the logistic regression likelihood within the Bayesian framework to model the spatial intensity of an Inhomogeneous Poisson Point Process (IPP). The proposed approach integrates prior information into the logistic regression likelihood by constructing posterior distributions, enabling a more comprehensive inference by quantifying parameter uncertainty. In contrast to conventional maximum likelihood (ML) estimation, which produces only point estimates, the Bayesian method provides a probabilistic characterization of parameter estimates using the Markov Chain Monte Carlo (MCMC) approach, specifically the Gibbs Sampling algorithm, to approximate posterior distributions. The methodological framework is applied to the spatial distribution of post-harvest rice facilities in Sidenreng Rappang Regency, Indonesia. The analysis is based on georeferenced observational data obtained from local goverment records and agricultural statistics, processed usign Geographic Information System (GIS) tools and statistical software. Spatial covariates include the proportion of paddy field area per village (Z_1), rice producing area (Z_2), and distance to the nearest Bulog warehouse (Z_3). The results indicate that Z_1 and Z_3 significantly affect the spatial intensity of post-harvest facilities, where areas with larger paddy field proportions are more likely to host such facilities, while increasing distance from Bulog reduces the likelihood of facility presence. The posterior trace and density plots demonstrate good convergence and mixing, confirming the reliability of the Gibbs Sampling procedure. Model comparison through the Akaike Information Criterion (AIC) and likelihood values shows that the Bayesian approach yields a substantially lower AIC, ten times smaller than the ML-based logistic regression, indicating superior model fit and computational efficiency. The findings suggest that integrating Bayesian inference into the IPP logistic framework enhances model interpretability and robustness, particularly in accounting for uncertainty and prior knowledge. The study underscores the practical importance of spatial modeling for agricultural infrastructure planning and offers a flexible computational framework applicable to other spatial point pattern analyses across diverse domains.

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A. INTRODUCTION

Spatial point pattern analysis has become increasingly important in various scientific domains, including ecology, facility location, epidemiology, infrastructure, and agricultural planning. For example, Baddeley investigated the spatial patterns of spines on a dendrite network (Baddeley, Jammalamadaka, et al., 2014), while Choiruddin analysed the effect of geological variables on earthquake cases in Sumatra by using a combination of elastic net and Poisson point process (Choiruddin et al., 2024). Panzeri recently studied the location arrangement of 5,176 road accidents in Bergamo, Italy, using perceptual semiparametric (Panzeri et al., 2025).

In many applications, spatial point pattern analysis aims to explain how the locations or events continue to exist and how environmental or covariates influence the distribution in space. Inhomogeneous Poisson Point Process (IPP) is one of the fundamental models in spatial point processes, used to describe point patterns whose intensity varies across the study region according to spatially dependent factors (Metrikasari & Choiruddin, 2021; Putri et al., 2024). IPP is the standard statistical framework to analyse the effect of covariates in the distribution of point patterns, where the intensity function is expressed as log-linear function of spatial covariates. The standard method to estimate the parameter of model is typically performed through likelihood, pseudo-likelihood, or palm-likelihood (Collins & Schliep, 2025; Lu & Friel, 2024). In addition, several methodological frameworks construct their likelihood functions by adopting either a Poisson or logistic regression model formulation, e.g. (Berman & Turner, 1992; Husain & Choiruddin, 2021; Putri et al., 2024).

Earlier methodological studies by Husain have compared Poisson and logistic regression approaches to estimate the intensity of spatial point processes (Husain & Choiruddin, 2021). Although Poisson regression is statistically efficient and widely adopted, it often requires many dummy points under the Berman–Turner approximation, resulting in a high computational cost (Turner et al., 2010). Logistic regression, on the other hand, achieves similar estimation performance with far fewer dummy points, leading to significant computational gains (Baddeley, Coeurjolly, et al., 2014). This efficiency makes logistic regression appealing for large-scale data sets or applications with critical computation time. Furthermore, in this study, we used logistic regression for IPP in the case of post-harvest rice facilities in Sidrap.

Despite the developments, most implementations of IPP models have been conducted under the frequentist framework (maximum likelihood), producing point estimates without fully characterizing parameter uncertainty, e.g., (Hasanah et al., 2022; Husain & Choiruddin, 2021; Møller & Rasmussen, 2022; Susanto et al., 2023). This limitation can hinder deeper inference, particularly in applied settings where prior knowledge exists or uncertainty quantification is essential for decision-making. Although logistic regression has been widely adopted as a computationally efficient approximation for IPP intensity estimation, its integration within a Bayesian framework remains relatively unexplored. The Bayesian method treats the regression parameter as a random variables, combining prior distributions with the likelihood to obtain posterior inference and credible intervals (Yasmirullah & Iriawan, 2019). In most IPP applications, posterior distributions are not available in closed form, necessitating simulation-based method such as Markov Chain Mote Carlo (MCMC) for estimation (Lukman et al., 2021). Addressing this gap, we propose a Bayesian logistic regression framework for IPP modelling than enables efficient estimation and full uncertainty quantification, illustrated through a post-harvest facility case study.

In the agricultural context, spatial point process modelling provides a powerful framework for analyzing the spatial organization of infrastructure supporting food production systems. Sidenreng Rappang Regency (Sidrap) in South Sulawesi is one of Indonesia's major rice-producing regions, characterized by extensive paddy cultivation and a dense network of post-harvest facilities that are critical to regional and national food supply chains. The spatial arrangement of rice milling units and warehouses directly affects post-harvest efficiency, transportation costs, accessibility for farmers, and storage capacity, making infrastructure placement a key determinant of supply chain performance. Understanding how these facilities are distributed in relation to agricultural production and logistical infrastructure is therefore essential for effective planning and policy intervention. In this study, the spatial intensity of post-harvest facilities is modelled using covariates that capture both production capacity and logistical accessibility, namely the proportion of paddy field area, harvested rice area, and distance to the nearest Bulog warehouse. Embedding these covariates within a Bayesian logistic regression framework enables robust uncertainty-aware inference, providing decision-relevant insights into how agricultural productivity and logistics jointly shape facility location patterns in Sidrap, with direct implications for infrastructure planning and food security policy.

This study extends IPP model estimation by adopting a Bayesian framework based on a logistic regression formulation. The logistic likelihood is combined with prior information to derive the posterior distribution of the IPP parameters, allowing parameter uncertainty to be explicitly accounted for in the analysis. The proposed approach is applied to model the spatial distribution of post-harvest rice facilities in Sidrap, using spatial covariates that include the proportion of paddy field area, rice-producing area, and the distance to the nearest Bulog warehouse.

B. METHODS

This study is quantitative, applied spatial statistical analysis that aims to model the spatial distribution of post-harvest rice facilities using a Bayesian Inhomogeneous Poisson point process framework. The analysis is based on observational spatial data and focuses on assessing the effects of agricultural and logistical covariates on facility location patterns. The primary research instruments consist of georeferenced spatial datasets of post-harvest facilities and associated covariates, which are processed using Geographic Information System (GIS) tools and statistical software. Spatial point process modeling and Bayesian inferences are implemented in R using the *spatstat* and *rstanarm* package, which serve as the main computational instruments for model estimation, posterior simulation, and spatial analysis.

1. Theoretical Background: Inhomogeneous Poisson Point Process

The spatial point pattern data is represented by $\boldsymbol{x} = \{x_1, x_2, \dots, x_n\}$, where $n \geq 0$, representing the set of locations of events observed in the observation window $\boldsymbol{W} \subset \mathbb{R}^2$ (Baddeley et al., 2016; Zhou & Wu, 2024). In our study, $\boldsymbol{x}$ denotes the location of post-harvest rice facilities such as warehouses and milling units. Furthermore, $\boldsymbol{x}$ is expressed as longitude and latitude. The underlying process generating such a point pattern is point process $\boldsymbol{X}$ in $\mathbb{R}^2$. Let $\boldsymbol{X} = \{X_1, X_2, \dots, X_n\}$ denote a spatial point process. It is assumed that $\boldsymbol{X}$ follows an

Inhomogeneous Poisson point Process (IPP), defined by an intensity function λ that varies across space according to spatial covariates (Waagepetersen, 2007).

The expected number of events within any subregion $\boldsymbol{A} \subseteq \boldsymbol{W}$ is defined as:

$$E[N(\boldsymbol{X} \cap \boldsymbol{A})] = \int_A \lambda(u)\, du, \quad u \in \boldsymbol{W}, \tag{1}$$

where $N(\boldsymbol{X} \cap \boldsymbol{A})$ represents the number of points processes located within subset $\boldsymbol{A} \subseteq \boldsymbol{W}$. The intensity of the process is expressed in a log-linear form.

$$\lambda(u) = \exp\big(\boldsymbol{\beta}\boldsymbol{Z}(u)\big), \tag{2}$$

where $\boldsymbol{Z}(u) = \big(Z_1(u), Z_2(u), \dots, Z_p(u)\big)^T$ is a vector of p spatial covariates and $\boldsymbol{\beta} = \big(\beta_0, \beta_1, \dots, \beta_p\big)$ are the corresponding regression coefficients.

2. Model Formulation

The spatial intensity function of the IPP is modeled using a log-linear specification that incorporates spatial covariates (Choiruddin, Aisah, et al., 2021; Coeurjolly & Lavancier, 2019):

$$\lambda(u) = \exp\big(\boldsymbol{\beta}\boldsymbol{Z}(u)\big), \tag{2}$$

where $\boldsymbol{Z}(u) = \big(Z_1(u), Z_2(u), \dots, Z_p(u)\big)^T$ is a vector of p spatial covariates evaluated at location u, and $\boldsymbol{\beta} = \big(\beta_0, \beta_1, \dots, \beta_p\big)$ are the corresponding regression coefficients. This formulation allows the spatial variation in location or event intensity to be explained by environmental, agricultural, and logistical factors. Parameter estimation for $\boldsymbol{\beta}$ is conducted through a logistic regression approximation and described in the following subsection.

3. Logistic Regression Representation of IPP

The parameter $\boldsymbol{\beta}$ in equation (2) is estimated using a logistic regression approximation. This approach introduces a set of random dummy points $\boldsymbol{d}$ generated from a point process $\boldsymbol{D}$ with a known intensity function $\delta(u)$. The dummy process $\boldsymbol{D}$ may follow a Poisson, binomial, or stratified binomial point process and is assumed to be independent from $\boldsymbol{X}$ (Baddeley et al., 2016; Baddeley, Coeurjolly, et al., 2014). The dummy point pattern is denoted by $\boldsymbol{d} = (d_1, d_2, \dots, d_m), m \geq n$.

Through this approach, the resulting estimation of the likelihood function can be expressed as follows:

$$\ell_{IPP}\big(\boldsymbol{\beta}|\boldsymbol{Z}(u)\big) = \sum_{u \in \boldsymbol{x}} \log\left(\frac{\lambda(u)}{\lambda(u) + \delta}\right) + \sum_{u \in \boldsymbol{d}} \log\left(\frac{\delta}{\lambda(u) + \delta}\right), \tag{3}$$

Equation (3) is equivalent to the log-likelihood of the logistic regression function with response variable $y_j = \mathbf{1}(u_j) \in \boldsymbol{x}$, an offset term $-\log(\delta)$, and the probability of success.

$$p(u_j) = \Pr(y_j = 1) = \frac{\lambda(u_j)}{\lambda(u_j) + \delta}.$$

The procedure for maximizing equations (3) can be achieved using an iteratively reweighted least squares (IRLS) procedure, yielding the approximation

$$\ell_{IPP}(\boldsymbol{\beta}|\boldsymbol{Z}(u)) \approx \sum_{j=1}^{n+m} \left(v_j \left(y_j^* - \boldsymbol{\beta}\boldsymbol{Z}(u_j)\right)^2\right), \tag{4}$$

where y_j^*, the working response is specified as follows

$$y_j^* = \widehat{\boldsymbol{\beta}}\boldsymbol{Z}(u_j) + \frac{1}{v_j}\left(y_j - \frac{\hat{\lambda}(u_j)}{\hat{\lambda}(u_j) + \delta}\right), \tag{5}$$

and

$$v_j = \hat{p}(u_j)\left(1 - \hat{p}(u_j)\right), \quad \hat{p}(u_j) = \frac{\hat{\lambda}(u_j)}{\hat{\lambda}(u_j) + \delta}. \tag{6}$$

4. The Bayesian Statistical Framework

This section briefly reviews the key components of Bayesian framework. Previous studies have extensively utilized and advanced this framework. For example, via (Kruschke, 2014; Muth et al., 2018). To account for parameter uncertainty, the logistic regression based IPP model is embedded within a Bayesian framework. The regression parameter $\boldsymbol{\beta}$ are treated as random variables, and inferece is based on their posterior distributions (McElreath, 2018). Given the likelihood function of logistic regression represented by Equation (4), the posterior distribution of $\boldsymbol{\beta}$ is obtained as:

$$\pi(\boldsymbol{\beta}|\boldsymbol{Z}(u)) \propto \ell_{IPP}(\boldsymbol{\beta}|\boldsymbol{Z}(u))\pi(\boldsymbol{\beta}), \tag{7}$$

where $\pi(\boldsymbol{\beta})$ represents the prior distribution. In this study, non-informative or weakly informative Gaussian priors are used (Fouskakis et al., 2015):

$$\boldsymbol{\beta} \sim N(\mathbf{0}, \sigma^2).,$$

where the hyperparameter σ^2 is unknown, then the hyperprior $\sigma^2 \sim Inv - Gamma(a_0, b_0)$ controls the prior variance of the parameter. So that the joint prior of $\boldsymbol{\beta}$ and the hyperprior is

$$\pi(\boldsymbol{\beta}) \propto \exp\left(-\frac{1}{2\sigma^2}||\boldsymbol{\beta}||_2^2\right), \tag{8}$$

$$\pi(\sigma^2) = \frac{b_0^{a_0}}{\Gamma(a_0)} (\sigma^2)^{-(a_0+1)} \exp\left(-\frac{b_0}{\sigma^2}\right). \tag{9}$$

For parameter $\boldsymbol{\beta}$, the posterior is formed by Equation (7), the posterior density is proportional to:

$$\pi(\boldsymbol{\beta}|\boldsymbol{Z}(u), \sigma^2) \propto \exp\left(-\frac{1}{2}\sum_{j=1}^{n+m}\left(v_j\left(y_j^* - \boldsymbol{\beta}\boldsymbol{Z}(u_j)\right)^2\right) - \frac{1}{2\sigma^2}||\boldsymbol{\beta}||_2^2\right), \tag{10}$$

and the posterior for the hyperparameter σ^2 is

$$\pi(\sigma^2|\boldsymbol{Z}(u), \boldsymbol{\beta}) \propto (\sigma^2)^{-(a_0+1)} \exp\left(-\frac{1}{2}\sum_{j=1}^{n+m}\left(v_j\left(y_j^* - \boldsymbol{\beta}\boldsymbol{Z}(u_j)\right)^2\right) - \frac{b_0}{\sigma^2}\right). \tag{11}$$

5. Estimation via Markov Chain Monte Carlo

The posterior function in Equation (10) and (11) is performed using Markov Chain Monte Carlo (MCMC) methods, as the posterior distribution arising from the logistic likelihood and hierarchical priors is not available in closed form. MCMC provides a simulation-based approach to approximate the joint posterior distribution of the model parameters and to quantify estimation uncertainty. In this study, the MCMC procedure is conducted following the Gibbs Sampling framework. Specifically, Gibbs sampling is employed due to its computational simplicity and effectiveness for hierarchical Bayesian models (Brooks, 1998; Ozgul et al., 2025; Walker & Soubeyrand, 2016). The algorithm to obtain the estimates of $\widehat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_p)$ is as follows the process of Gibbs Sampling (Vishnoi, 2021; Yuan et al., 2023).

Step 1 : Generate the initial parameter of $\boldsymbol{\beta}$ and the hyperparameter of σ^2 namely, $\boldsymbol{\theta}^{(0)} = \left(\beta_0^{(0)}, \beta_1^{(0)}, \dots, \beta_p^{(0)}, (\sigma^2)^{(0)}\right)$

Step 2 : For $t = 1,2,\dots,T$ repeat the following steps:
Assign $\boldsymbol{\theta} = \boldsymbol{\theta}^{(t-1)}$
Update $\boldsymbol{\theta}$ on the posterior in Equation (10) for the parameter $\boldsymbol{\beta}$ and Equation (11) for the hyperparameter σ^2. The complete process is as follows:
$\boldsymbol{\beta}^{(t)}$ from $\pi\left(\boldsymbol{\beta}|\boldsymbol{Z}(u), (\sigma^2)^{(t-1)}\right)$
$(\sigma^2)^{(t)}$ from $\pi(\sigma^2|\boldsymbol{Z}(u), \boldsymbol{\beta}^{(t)})$
Construct $\boldsymbol{\theta}^{(t)}$ and record it as the set of values produced during the $(t+1)$ th iteration of the algorithm.

Step 3 : To evaluate the estimation, check the convergence of $\boldsymbol{\theta}$ using trace or density plot.

6. Implementation and Model Evaluation

The Bayesian logistic regression model is implemented using the *stan_glm* function in the *rstanarm* package with a binomial logit link and sampling-based estimation. Logistic regression-based IPP likelihood are constructed using the *ppm* function in the *spatpatst* package with *method="logi"* (Baddeley et al., 2016).

For the model comparison and to evaluate the result of Gibbs Sampling, we perform the a Akaike Information Criteria (AIC) developed for a spatial point process defined by (Choiruddin, Coeurjolly, et al., 2021)

$$\mathrm{AIC}(\widehat{\boldsymbol{\beta}}) = -2\ell_{IPP}\big(\boldsymbol{\beta}|\boldsymbol{Z}(u)\big) + 2k, \tag{12}$$

where $\ell_{IPP}\big(\boldsymbol{\beta}|\boldsymbol{Z}(u)\big)$ is the maximum of the logistic regression likelihood and k is the size of $\widehat{\boldsymbol{\beta}}$. The likelihood in Gibbs sampling framework is produced by the average of $\widehat{\boldsymbol{\beta}}$ (Hug & Paciorek, 2021).

C. RESULTS AND DISCUSSION

1. Data Description

This study area is in Sidenreng Rappang (Sidrap) regency, Indonesia (Figure 1), encompassing information related to rice post-harvest facilities, the proportion of paddy field area by village, rice production levels in each village, and the distance between post-harvest facilities and Bulog storage centres (*Dinas Tanaman Pangan dan Hortikultura*, 2025). The post-harvest facilities dataset comprises two main types of infrastructure: rice warehouses and milling units, collectively classified as post-harvest facilities. A longitude and latitude represent each facility coordinate to facilitate spatial point pattern analysis. The spatial data were obtained through field surveys and regional agricultural records and validated through cross-referencing with Google Maps to ensure accuracy.

Figure 1 presents the locations of 146 post-harvest rice facilities in Sidrap ($\boldsymbol{W} = 1883\ km^2$), and Bulog storage locations in the same region. Each point in the dataset is associated with a processing capacity variable, classified into three categories: less than 2 tons per hour (represented by red dots), 2–3 tons per hour (green dots), and 3–4 tons per hour (blue dots). The spatial distribution depicted on the map shows that most facilities in Sidrap fall within the 2–3 tons per hour category, reflecting a predominance of medium-scale processing capacity typical of decentralized agricultural systems. High-capacity facilities (3–4 tons per hour) are relatively scarce but exhibit clustering patterns near major production areas and along primary road networks, particularly within the Maritengngae, Baranti, and Watang Sidenreng subdistricts. The fifteen Bulog facilities (represented by triangles) are concentrated in Watangpulu, Maritengngae, Watang Sidenreng, Baranti, and Duapitue. The spatial distribution of post-harvest facilities shows a tendency to cluster in proximity to Bulog locations. Accordingly, this study investigates the spatial relationship between post-harvest facility distribution and the positioning of Bulog centres.

Figure 2 presents the spatial covariates employed to model the distribution of post-harvest facilities in Sidrap. These three variables are transformed into pixel images by measuring of the proportion for Z_1 and Z_2 based on village and measuring the distance of each coordinate within post-harvest facilities to each of the nearest Bulog locations. Panel (a) illustrates the proportion of rice paddy area in each village (Z_1), reflecting the relative extent of rice cultivation across the region, with values ranging from low to high, represented by a blue to yellow colour gradient. Panel (b) displays the rice production area (Z_2) for each village, measured in hectares (Ha), emphasizing zones with higher levels of rice cultivation, where brighter colors denote larger production areas. Panel (c) depicts the distance to the nearest Bulog warehouse (Z_3), measured in meters, where the color gradient indicates proximity to Bulog infrastructure. Blue tones represent areas closer to Bulog, while pink tones indicate more distant locations. These covariates collectively elucidate the influence of agricultural and logistical factors on the spatial distribution of post-harvest infrastructure within the study region.

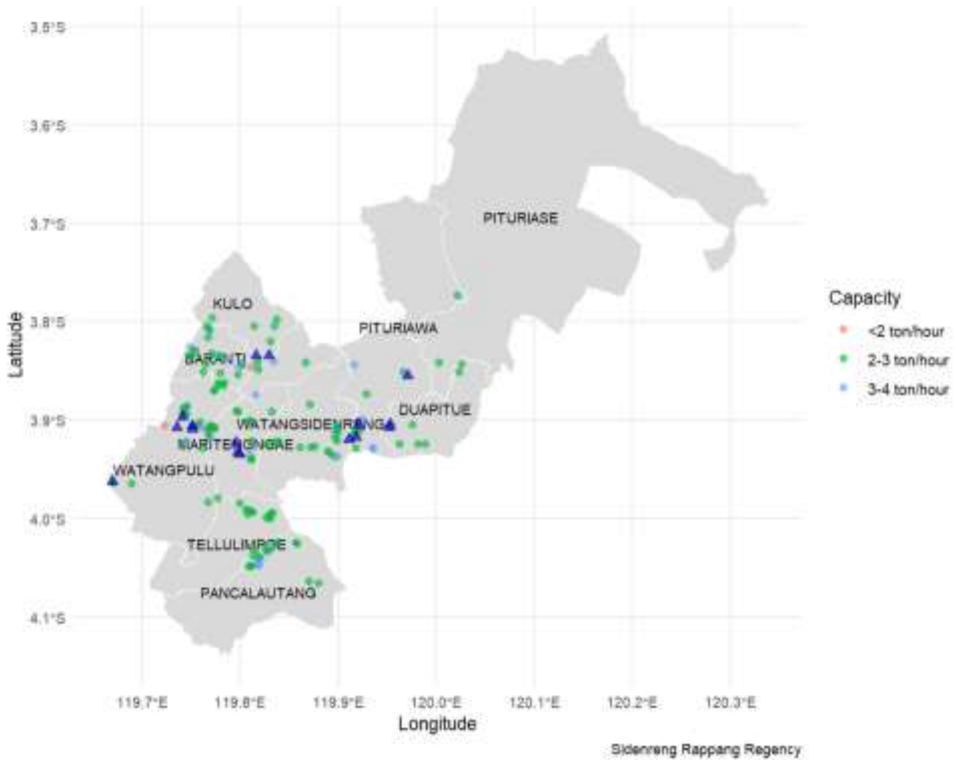

Figure 1. The distribution of locations of post-harvest facilities in Sidrap.
(Dot: rice warehouse and milling units, Triangle: Infrastructure of Bulog)

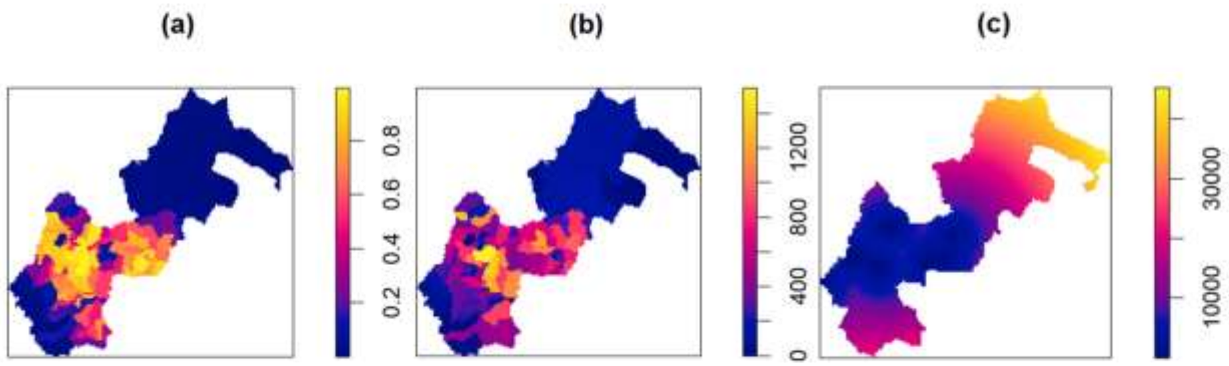

Figure 2. Spatial covariates: (a) proportion of paddy field area in each village (Z_1), (b) rice-producing area in each village (Ha) (Z_2), (c) post-harvest to Bulog distance (meter) (Z_3)

2. Logistic Regression Model

By assessing the spatial covariates (Figure 2) and the distribution of post-harvest facilities (Figure 1), several locations have a similar pattern. For example, the facilities in the middle of Sidrap such as Duapitue, Watangsindenreng, Maritengngae and Baranti, have a similar pattern with the proportion of paddy area (yellow indicates a high proportion of paddy fields) and distance to Bulog (dark blue indicates getting closer to Bulog). Furthermore, to examine the effects of spatial covariates, the logistic regression approach was applied within the framework of the Inhomogeneous Poisson Process (IPP). At the same time, Section C.3 incorporates a Gibbs Sampling method to enhance parameter inference.

The findings from the IPP model by maximum likelihood (ML) of logistic regression (Table 1) reveal that two of the three covariates significantly affect the spatial intensity of post-harvest rice facilities in Sidrap. In particular, the proportion of paddy field area per village (Z_1) exhibits a positive and statistically significant relationship (Estimate = 1.382; $|Z| = 3.239 > 1.96$), indicating that regions with a greater concentration of paddy fields are more likely to host post-harvest infrastructure. In contrast, the distance to the nearest Bulog warehouse (Z_3) demonstrates a negative and highly significant association with facility intensity (Estimate = -0.0001; $|Z| = 6.677 > 1.96$), indicating that areas farther from centralized storage facilities have a lower probability of accommodating post-harvest infrastructure.

Table 1. Parameter Estimation and Hypothesis Testing

	Estimate ($\hat{\beta}$)	**Standardized Estimate**	**$\vert Z_{value} \vert$**	**Significant**
Intercept	-1.5479	0.2581		
Z_1	1.3825	0.0427	3.239*	True
Z_2	-0.0004	0.0003	1.145	False
Z_3	-0.0001	0.00002	6.677*	True

*$Z_{table}(\alpha = 5\%)$=1.96

3. Model Comparison

To incorporate the element of uncertainty within the estimation procedure, we pay attention to integrating the Bayesian method (Gibbs Sampling algorithm) into the IPP by logistic regression. The comparison of two methods based on the Likelihood value (Equation 3) and the Akaike Information Criteria (AIC) value is given in Table 2. The results of the logistic regression using the maximum likelihood (ML) and Bayesian methods have been given. According to the AIC value, the Bayesian method provides ten times lower results than the logistic regression method. This suggests that the Bayesian method minimizes the information criteria more efficiently than the ML-based logistic regression. Overall, while both methods yield comparable results in terms of coefficient estimation, the lower AIC value for the Bayesian approach suggests that the Bayesian approach provides a more flexible framework for uncertainty-aware inference although direct comparison using AIC should be interpreted cautiously.

Furthermore, the sample of the posterior distribution of the regression coefficients can be seen by drawing the trace and density plot. The computational procedure employed in this study involved 10,000 iterations, producing 10,000 regression coefficient estimates for each parameter. The estimation outcomes are illustrated in Figure 3, which presents the corresponding trace and density plots.

To evaluate the process of Gibbs Sampling, we pay attention in the convergence distribution of the coefficient regression represented by the trace or density plot. Figure (a) presents the trace plots of the estimated parameters obtained through the Gibbs Sampling procedure for four parameters: the intercept, Z_1, Z_2, and Z_3. The trace plots exhibit stable fluctuations around a constant mean level without discernible trends, indicating that the Markov chains have reached their stationary distribution and display satisfactory mixing behavior. This suggests that the sampling process adequately explores the posterior space and supports reliable posterior inference. Figure (b) displays the corresponding density plots for each parameter, illustrating the estimated posterior distributions, with their spread reflecting the degree of uncertainty associated with each parameter estimate. Narrower densities indicate higher precision, while wider distributions suggest greater posterior variability. Although the diagnostic assessment in this study primarily relies on visual inspection of trace and density plots, the observed stability, mixing, and consistency of posterior summaries provide reasonable evidence that the MCMC procedure produces well-converged posterior estimates. These results support the robustness of the Bayesian inference while acknowledging that additional diagnostics, such as effective sample size or posterior predictive checks, could further strengthen the assessment in future studies.

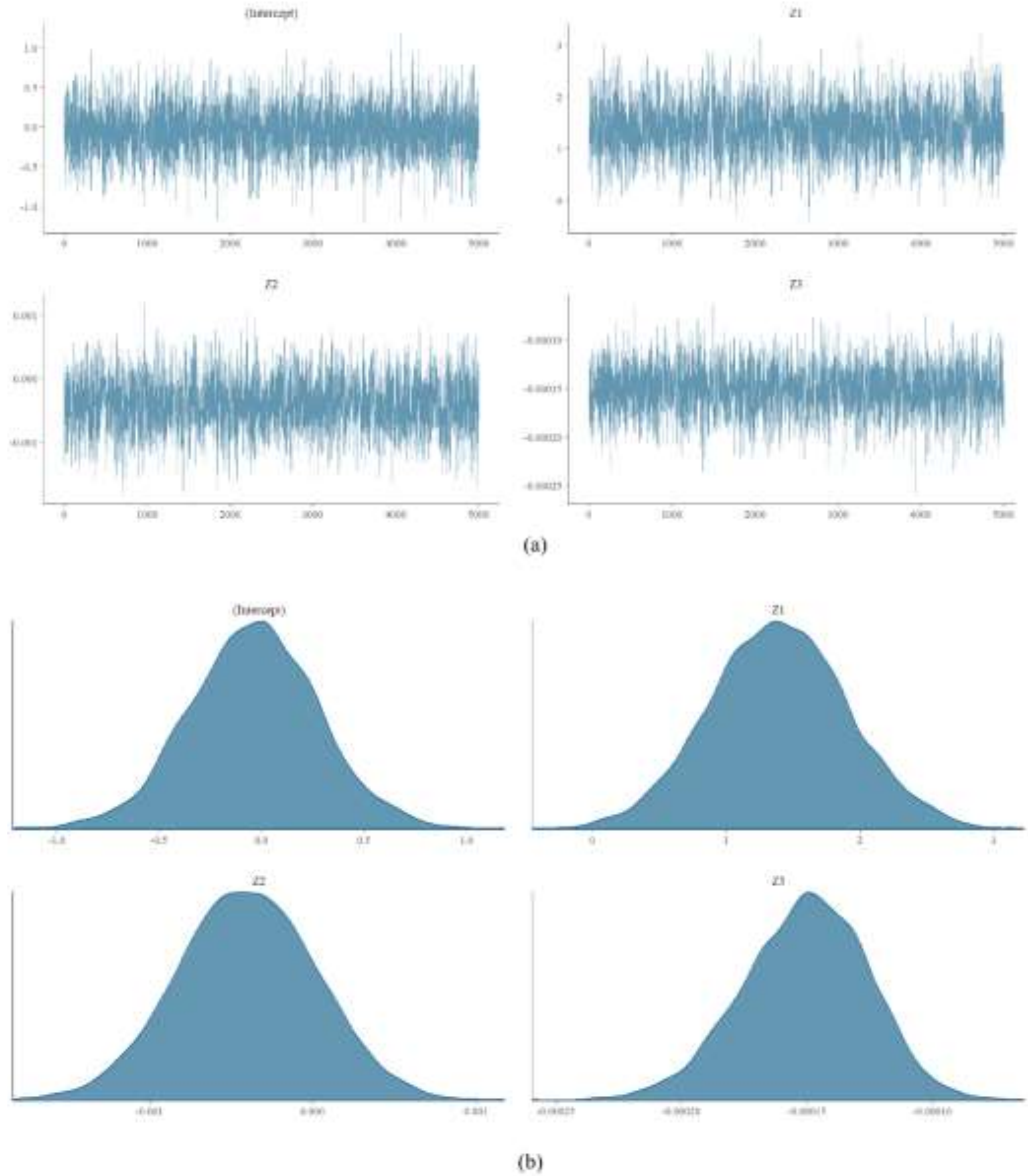

Figure 3. The Posterior plot: (a) Trace plot and (b) Density plot

In the Bayesian estimation results, the $q_{2.5\%}$ and $q_{97.5\%}$ values in Table 2 represent the lower and upper bounds of the 95% credible interval for each parameter, respectively. This interval provides a probabilistic interpretation of parameter uncertainty, indicating that there is a 95% probability that the parameter's actual value lies within these bounds, given the observed data and prior assumptions. In significance assessment, a parameter has a significant effect if its 95% credible interval does not include zero, suggesting that the parameter's posterior distribution is consistently positive or negative.

Based on the results, variables Z_1 ($q_{2.5\%} = 0.3602$; $q_{97.5\%} = 2.4553$) and Z_3 ($q_{2.5\%} = -0.0002$; $q_{97.5\%} = -0.0001$) exhibit credible intervals that exclude zero, indicating statistically significant effects of the proportion of paddy field area in each village and the distance of post-harvest facilities to Bulog on the distribution of post-harvest rice facilities in Sidrap. In contrast, the credible intervals for the intercept and Z_2 both contain zero, implying that their effects are not statistically significant under the Bayesian framework. Based on this criterion, the proportion of paddy field area (Z_1) exhibits a positive and significant effect, indicating that areas with larger proportions of paddy fields are more likely to host post-harvest facilities. This finding aligns with agricultural infrastructure theory, which suggests that facility placement tends to follow production intensity. Conversely, the distance to the nearest Bulog warehouse (Z_3) shows a significant negative effect, implying that post-harvest facilities are more likely to be located closer to Bulog infrastructure, reflecting the importance of logistical accessibility in post-harvest operations, as shown in Table 2.

Table 2. Comparison of Maksimum Likelihood and Bayesian Method

	Parameters	
	ML ($\hat{\beta}$)	**Bayesian** ($q_{2.5\%}$; $q_{97.5\%}$)
Intercept	-1.5479	(-0.6584 ; 0,5968)
Z_1	1.3825	(0.3602 ; 2.4553)
Z_2	-0.0004	(-0.0013 ; 0.0004)
Z_3	-0.0001	(-0.0002 ; -0.0001)
Likelihood	-2484	-232.09
AIC	4974	470.19

In contrast, the rice-producing area (Z_2) does not exhibit a statistically significant effect under the Bayesian framework. This result suggests that facility placement may be more strongly influenced by land-use structure and logistical connectivity than by harvested area alone. Such findings complement earlier studies on spatial infrastructure distribution, which highlight the combined role of production capacity and accessibility in shaping spatial patterns (Hasanah et al., 2022). From a policy perspective, these results underscore the importance of integrating agricultural land-use information and logistical networks in planning post-harvest infrastructure. The Bayesian framework employed in this study provides decision-relevant insights by quantifying uncertainty in covariate effects, thereby supporting more informed spatial planning and resource allocation in agricultural regions such as Sidenreng Rappang.

D. CONCLUSION AND SUGGESTIONS

This study was motivated by the need to develop an uncertainty-aware and computationally efficient framework for modelling inhomogeneous Poisson point processes using a Bayesian paradigm. The estimation procedure combines the likelihood of logistic regression studied by Husain & Choiruddin, 2021 in the Bayesian framework. To adress this objective , we integrated the logistic regression likelihood commonly used in IPP estimation with Bayesian inference and Markov Chain Monte Carlo (MCMC) sampling, and applied the resulting framework to analyze the spatial distribution of post-harvest rice facilities in Sidrap.

The empirical results demonstrate that the proposed Bayesian logistic IPP model provides interpretable posterior inference while maintaining competitive model fit. The analysis identifies two covariates with substantively meaningful effects on facility location: the proportion of paddy field area (Z_1), which exhibits a positive association with spatial intensity, and the distance to the nearest Bulog warehouse (Z_3), which shows a negative association. In contrast, harvested rice area (Z_2) does not display a statistically meaningful effect under the Bayesian framework. MCMC diagnostics, including trace and density plots, indicate stable and well-mixed chains, supporting the reliability of the posterior estimates and reinforcing the validity of the inferential conclusions. These findings have direct implications for agricultural infrastructure planning. The results suggest that post-harvest facilities tend to concentrate in areas with sustained cultivation capacity and strong logistical connectivity, rather than being driven solely by harvested output. From a policy perspective, this implies that spatial planning strategies should prioritize locations with high paddy field coverage and improved access to Bulog infrastructure to enhance post-harvest efficiency and supply chain performance.

Several directions for future research emerge from this study. First, incorporating additional spatial covariates such as road quality, terrain characteristics, and market accessibility could provide a more comprehensive representation of logistical and environmental constraints. Second, extending the model to hierarchical or multilevel structures would allow heterogeneity across sub districts or administrative units to be explicitly captured. Third, robustness analyses using alternative prior specifications or sparsity-inducing penalties could further assess the sensitivity of covariate effects. Finally, extending the framework to spatio-temporal point processes and adopting Bayesian model comparison criteria such as WAIC or leave-one-out cross-validation would strengthen inference and improve its relevance for dynamic agricultural systems and policy evaluation.

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