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Measuring the Impact of APOS Theory-Based Contextual Mathematics Assignments on the Mathematical Communication Skills of Prospective Teacher Students

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ABSTRACT

Mathematical communication skills are essential for conveying ideas, interpreting symbols, and linking abstract concepts to contextual phenomena. This study aims to measure the effectiveness of applying APOS theory-based contextual tasks in improving the mathematical communication skills of prospective teacher students. This study uses a quasi-experimental design with a non-equivalent control group model, involving an experimental class that receives contextual task-based learning using the APOS theory approach and a control class that follows conventional learning. The instrument used was a mathematical communication skills test covering five main indicators, namely the ability to express ideas in writing, use representations, explain procedures, relate concepts to real contexts, and construct mathematical arguments. The data were analyzed using the Rasch model, normality test, homogeneity test, independent t-test, score improvement analysis, and PLS-SEM-based structural modeling. The results indicate that the instrument demonstrates strong validity and reliability, as reflected by average Infit and Outfit MNSQ values of 1.00, item reliability of 0.89, and person reliability of 0.84. Significant differences were found between the experimental and control groups across all indicators of mathematical communication skills, with higher posttest mean scores in the experimental group (76.10) compared to the control group (49.87), as well as greater learning gains in the experimental group (N-Gain 68.27%) than in the control group (33.18%). The structural model further confirms the positive contribution of APOS theory to mathematical communication skills, particularly in explaining procedures (β = 0.323) and using mathematical representations (β = 0.257). Overall, this study confirms that the application of APOS theory-based contextual tasks is effective in strengthening the mathematical communication skills of prospective teachers and provides important implications for the development of a more contextual and meaningful mathematics education curriculum.

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A. INTRODUCTION

Mathematics education at the university level, particularly in teacher education programs, has an important responsibility to equip students with strong mathematical communication skills. These skills are essential for conveying ideas, interpreting symbols, and linking abstract concepts to contextual phenomena. However, various studies show that prospective teacher students still face difficulties in articulating mathematical ideas systematically and logically. Recent research findings underscore that low mathematical communication skills can have a direct impact on learning effectiveness, both in terms of conceptual understanding and practical application (Nafisyah et al., 2024; Turmuzi et al., 2024). This indicates an urgent need for more innovative pedagogical approaches based on strong cognitive theory, one of which is Action, Process, Object, and Schema (APOS).

The real problem in this context can be seen from the results of a preliminary study conducted on 30 fourth-semester students at Bani Saleh University. The data shows that the average mathematical communication skills of students are still in the "poor" category, with an achievement rate of only 48%. More specifically, the indicators show that written ideas only reached 58% (adequate category), the use of graphical representations, tables, and symbols was 52% (adequate), explanations of problem-solving procedures were 47% (poor), linking function concepts to real-world contexts was 39% (poor), and mathematical argumentation was 43% (poor). Overall, the weakest aspect of students is relating functional concepts to real-world contexts, which should be a key skill in meeting the demands of 21st-century learning. This data shows that the problems faced by students are not only related to conceptual understanding, but also to applicable mathematical communication skills (Ng & Chew, 2023; Rahayu et al., 2023).

These observations are consistent with international findings that confirm that prospective teachers often have misconceptions in connecting symbolic representations, graphs, and real-life contexts. In fact, students often fail to construct coherent explanations related to the procedures for solving contextual problems, so that mathematics learning tends to be stuck in a mechanistic routine (Borji & Martínez-Planell, 2020; Tatira & Mukuka, 2024). This condition reveals a gap between the demands of the Merdeka Belajar curriculum, which emphasizes mathematical communication competence, and the reality of student achievement. Therefore, this research has a high urgency in formulating appropriate strategies to strengthen mathematical communication skills based on modern cognitive theory.

One potential approach is the application of APOS theory-based contextual tasks. This theory assumes that students' understanding develops gradually through four main phases: action, process, object, and schema. By placing students in contextual situations, this model allows for the formation of more meaningful and applicable mental constructs (Oktaç, 2022; Trigueros & Oktaç, 2019). Recent research has confirmed that the application of APOS in calculus, linear algebra, and geometry learning has been shown to improve problem-solving skills while strengthening mathematical communication (Listiawati et al., 2025; Muñoz-Orozco et al., 2025). The integration of contextual tasks into the APOS framework is expected to be a solution that bridges the gap between conceptual understanding and applied mathematical communication skills.

State-of-the-art research indicates that the application of APOS theory has been extensively explored in mathematics education, particularly in domains related to conceptual construction such as limits, derivatives, integrals, linear algebra, and function transformations (Rittaud & Vivier, 2024; Tatira, 2025). Further confirms that international APOS-based studies are predominantly concentrated on students' conceptual understanding, cognitive structures, and learning processes within calculus-oriented topics and abstract mathematical reasoning. Dominant research clusters are strongly associated with keywords such as derivatives, calculus instruction, linear algebra learning, and cognitive styles, reflecting a primary emphasis on internal cognitive development and conceptual mastery. However, despite the central role of communication in mathematics learning and teaching, the explicit integration of APOS theory with mathematical communication skills remains largely underexplored, particularly in the context of prospective teachers. Existing studies rarely examine how the Action–Process–Object–Schema mental constructions are manifested through written explanations, representations, argumentation, and contextual reasoning, which are core components of mathematical communication. This gap is especially critical for teacher education, as prospective teachers are required not only to construct mathematical concepts internally but also to articulate, represent, and justify mathematical ideas pedagogically. Therefore, the limited focus on mathematical communication within APOS-based research highlights a clear theoretical and empirical gap. Addressing this gap by systematically linking APOS theory with the development of prospective teachers' mathematical communication skills constitutes a novel contribution and responds directly to current needs in mathematics teacher education research.

The novelty of this study lies in the integration of contextual mathematical tasks into the APOS theory framework to directly measure and improve the mathematical communication skills of prospective teachers. Based on the results of bibliometric mapping, it is clear that previous studies have focused more on conceptual and procedural representations, while the dimension of mathematical communication, especially in the context of functions, has not received much attention (Trigueros et al., 2025; Villabona et al., 2024). Therefore, this study presents a new contribution by placing mathematical communication as the main variable, while linking it to the APOS theoretical framework, which has been proven to be strong in examining cognitive construction.

Based on the above description, the research question focuses on the following question: how does the application of APOS theory-based contextual mathematics tasks affect the mathematical communication skills of prospective teachers? The purpose of this study is to analyze the effectiveness of applying this model in strengthening students' mathematical communication skills, so that the results can make a significant contribution to both the development of mathematics learning theory and pedagogical practice in teacher education programs. Thus, this study is expected to provide an effective, adaptive intervention model that is relevant to the demands of the curriculum and the educational needs of the 21st century.

B. METHODS

1. Research Types and Designs

This study uses a quantitative approach with a quasi-experimental method, as it is not possible to fully randomize the subjects, but variable control is still maintained in the measurement. The research design used is a Non-Equivalent Control Group Design, which consists of two classes, namely an experimental class that was given treatment in the form of learning with contextual tasks based on APOS theory and a control class that used conventional learning. Both classes were given pre-tests and post-tests to measure the impact of the treatment on the mathematical communication skills of prospective teacher students, as shown in Table 1.

Table 1. Research Design

Group	Pretest	Treatment	Posttest
Experimental Class	O1	X (Contextual Tasks Based on APOS Theory)	O2
Control Class	O3	C (Conventional Instruction)	O4

Description: O1 and O3 = pretest for the experimental and control classes; X = learning with APOS theory-based contextual tasks; C = conventional learning; O2 and O4 = posttest for the experimental and control classes.

2. Research Procedures

The research procedure was conducted through a series of systematic and interconnected stages. The initial stage involved preparation, which included curriculum analysis and the development as well as expert validation of the research instruments to ensure their alignment with the indicators, constructs, and objectives of mathematical communication skills assessment. The next stage was the pretest, administered to both the experimental and control groups to measure students' initial mathematical communication abilities and to confirm the equivalence of the two groups prior to treatment. Subsequently, the treatment phase was implemented, in which the experimental group received learning through contextual tasks designed based on the APOS theoretical framework, guiding students to construct understanding through the stages of action, process, object, and schema. In contrast, the control group participated in conventional learning conducted using standard instructional practices without APOS integration. After the completion of the instructional intervention, a posttest was administered to both groups to assess changes and improvements in students' mathematical communication skills. The final stage involved data analysis, employing quantitative statistical techniques, including prerequisite tests, comparative analysis of learning outcomes, gain analysis, and structural model evaluation to examine the contribution of APOS theory to the development of mathematical communication skills. This procedure was designed to ensure the validity, rigor, and interpretability of the research findings.

3. Research Subjects or Participants

The research subjects were prospective teachers in their fourth semester of the Mathematics Education study program at a private teacher training college in Indonesia. This study involved two classes selected using purposive sampling based on initial ability equivalence. There were 60 participants, with 30 students in the experimental class and 30 students in the control class. Inclusion criteria: active fourth-semester students who had taken prerequisite courses (Algebra, Basic Calculus) and were willing to participate in all stages of the research. Exclusion criteria: students who were absent from more than two core treatment sessions or did not complete the pretest and posttest.

4. Data Collection Techniques and Procedures

The data collection procedure was carried out through six sequential and systematic stages to ensure the accuracy and validity of the research data. First, the instrument development stage involved compiling test instruments based on a predefined grid of mathematical communication indicators, ensuring alignment with the research objectives and theoretical framework. Second, the instruments underwent expert validation, where mathematics education specialists evaluated the content relevance, clarity, and construct alignment of each item to establish content validity. Third, a trial implementation was conducted on a limited sample to examine item readability, response patterns, and preliminary reliability, allowing for necessary revisions prior to full deployment. Fourth, a pretest was administered to both the experimental and control classes to collect baseline data on students' initial mathematical communication skills and to confirm group equivalence before treatment. Fifth, the treatment stage was implemented, during which the experimental class received learning through contextual tasks grounded in APOS theory, while the control class engaged in conventional instructional practices. Finally, a posttest was administered to both classes using the validated instrument to measure learning outcomes and changes in mathematical communication skills following the instructional intervention. This structured data collection procedure ensured methodological rigor, consistency, and the reliability of the data used for subsequent statistical and structural analyses.

5. Data Collection Instruments

The research instrument was a mathematical communication ability test in essay form with five main indicators. Each indicator was developed into five questions, resulting in a total of 25 questions, as shown in Table 2.

Table 2. Specifications of Mathematical Communication Ability Instruments

Indicator Code	Indicator	Specific Description	Item Context (5 items per indicator)	Cognitive Domain	Score (0–4)
I1	Expressing mathematical ideas in writing	Ability to state function concepts in clear mathematical prose	(1) State a linear function in words; (2) Explain the meaning of slope/gradient; (3) Relate a graph to a verbal description; (4) Write an equation for a contextual function; (5) Describe properties of a quadratic function	C2–C4	0–4
I2	Using representations (graphs, tables, symbols) appropriately	Ability to convert information across mathematical representations	(1) Draw the graph of a linear function; (2) Present a value table of a function; (3) Convert a graph to its equation; (4) Interpret function symbols; (5) Create a graph from a problem context	C2–C5	0–4
I3	Explaining problem-solving procedures coherently and logically	Ability to write structured solution steps for function problems	(1) Solve a quadratic equation with detailed steps; (2) Organize a solution for a rational function; (3) Describe the elimination–substitution method; (4) Write procedural steps for function modeling; (5) Sequence steps for solving a contextual problem	C3–C5	0–4
I4	Connecting function concepts to real-world contexts	Ability to link function theory to everyday phenomena	(1) Build a function model for sales data; (2) Use a quadratic function for an object's trajectory; (3) Model exponential growth; (4) Apply logarithmic functions to scales; (5) Use rational functions for rate/speed cases	C3–C6	0–4
I5	Constructing mathematical	Ability to provide logical	(1) Justify method selection; (2) Provide a logical warrant for	C4–C6	0–4

Indicator Code	Indicator	Specific Description	Item Context (5 items per indicator)	Cognitive Domain	Score (0–4)
	arguments or reasoning	justification for function solutions	computed results; (3) Explain correctness of a graph; (4) Critique an alternative solution; (5) Present a valid argument for the conclusion		

Score Description (0–4): 0 = No answer/answer not relevant; 1 = Answer is very poor, does not match the concept; 2 = Answer is inaccurate, partially correct but incomplete; 3 = Answer is quite good, concept is correct but not in-depth; 4 = Answer is very good, complete, coherent, and matches the concept. Each indicator consists of 5 questions, each question is worth a maximum of 4 points, so the maximum score per indicator is 20 points. The total score for the entire instrument with 5 indicators is 100 points.

6. Data Analysis Techniques

Data analysis in this study was conducted using a quantitative approach with various statistical tests tailored to the research objectives and characteristics of the data obtained. The analysis process utilized a number of advanced statistical software programs, namely JAMOVI, SPSS, MINISTEP Rasch, and SMARTPLS 4, each of which served to test validity, reliability, data distribution, treatment effectiveness, and improvement in students' mathematical communication skills.

The initial stage of analysis was conducted through instrument validity testing, which included content validity using expert judgment techniques and construct validity using Confirmatory Factor Analysis (CFA). Content validity was conducted by mathematics education experts to ensure the instrument's suitability with the indicators being studied, while construct validity was analyzed using SPSS and SMARTPLS 4 to ensure the reliability of the factor structure. Furthermore, the instrument reliability test was carried out using Cronbach's Alpha and Composite Reliability to assess internal consistency. This reliability test was reinforced by analysis using the Rasch Model through the MINISTEP application, which allowed for item fit testing, person reliability, and examination of student response distribution to the questions.

After the instruments were declared valid and reliable, statistical assumptions were tested using the Kolmogorov-Smirnov test for normality and the Levene Test for homogeneity of variance. These tests were conducted to ensure that the data met the prerequisites for parametric analysis so that the inferential test results would be more reliable. The next stage was to test the effectiveness of the treatment, which was done using an Independent Sample T-Test to compare the results of the experimental and control classes, and a Paired Sample T-Test to compare the pretest and posttest in the same group. In addition, the improvement in students' mathematical communication skills was analyzed using the N-Gain Score, which provides a quantitative description of the level of effectiveness of the application of contextual task-based learning with the APOS theory. This analysis was performed using SPSS and JAMOVI software. Overall, this data analysis procedure provides a comprehensive and layered quantitative framework, starting from instrument validation, reliability testing, assumption analysis, treatment effectiveness testing, to analysis of student ability improvement. Thus, the results of this study are expected to have high internal and external validity and can make a significant contribution to the development of APOS theory-based mathematics learning methodology.

C. RESULT AND DISCUSSION

1. Instrument Validity Test Results

The results of the distribution of respondents' abilities and the level of difficulty of the instrument items on a single Rasch map (Wright Map) shown in Figure 1 indicate that the positions of respondents scattered from -1 to +1 logit show variations in students' abilities in completing the items. Some students are at the upper level (+1 logit), which indicates high mathematical communication ability, while most respondents are concentrated in the area around 0 logit, which reflects moderate ability. On the item side, it can be seen that items such as I4_S4 and I2_S5 are in a relatively high position, indicating a higher level of difficulty than other items. Conversely, questions such as I1_S1, I2_S1, and I3_S1 are at a low level, indicating that these items are relatively easy for students to master. This distribution shows a fairly good balance between student ability levels and the range of item difficulty, which is an indicator that the instrument is able to measure mathematical communication skills in a representative manner, as shown in Figure 1 and Figure 2.

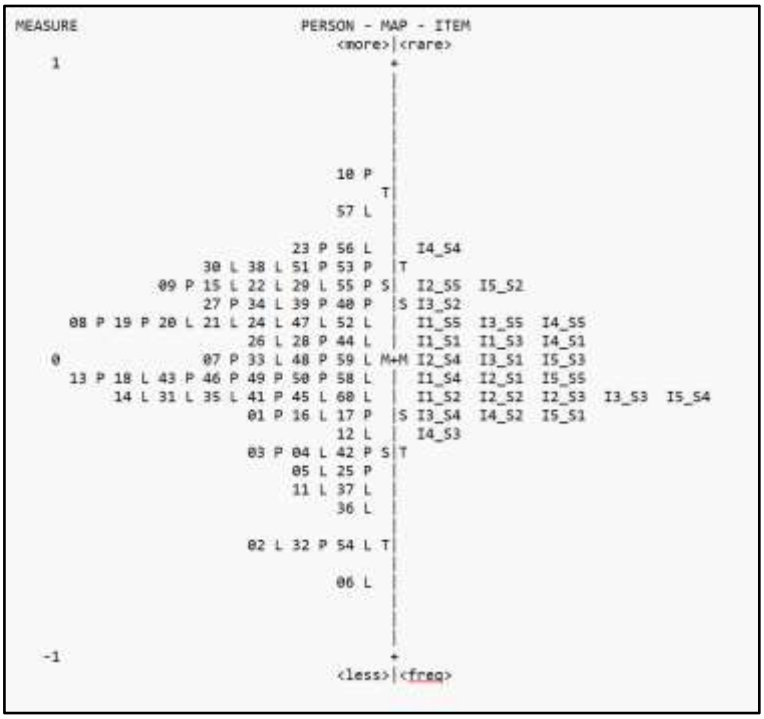

Figure 1. Respondents' abilities and item difficulty levels

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	JMLE MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-AL CORR.	PTMEASUR-AL EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	ITEM
19	45	60	.38	.17	1.21	1.68	1.21	1.62	-.13	.22	28.3	34.5	I4_S4
10	49	60	.27	.16	.97	-.23	.99	-.05	.16	.22	33.3	34.3	I2_S5
22	50	60	.24	.16	.93	-.58	.93	-.63	.41	.22	33.3	34.3	I5_S2
12	52	60	.19	.16	1.00	.02	1.01	.17	.19	.22	38.3	34.4	I3_S2
5	54	60	.13	.16	.82	-1.77	.82	-1.72	.26	.22	45.0	32.8	I1_S5
15	55	60	.11	.16	1.14	1.29	1.14	1.32	.26	.23	21.7	32.3	I3_S5
20	55	60	.11	.16	1.17	1.56	1.15	1.41	.21	.23	21.7	32.3	I4_S5
1	56	60	.08	.16	.99	-.04	1.01	.13	.10	.23	36.7	32.4	I1_S1
3	56	60	.08	.16	.89	-1.02	.90	-.92	.21	.23	40.0	32.4	I1_S3
16	56	60	.08	.16	1.08	.82	1.10	.92	.13	.23	30.0	32.4	I4_S1
9	58	60	.03	.16	.96	-.37	.95	-.43	.39	.23	30.0	32.4	I2_S4
11	58	60	.03	.16	.90	-.92	.90	-.93	.40	.23	33.3	32.4	I3_S1
23	60	60	-.02	.16	1.18	1.71	1.18	1.68	.14	.23	23.3	32.5	I5_S3
4	61	60	-.05	.16	.98	-.16	.98	-.19	.20	.23	35.0	32.5	I1_S4
25	61	60	-.05	.16	.81	-1.95	.82	-1.80	.00	.23	51.7	32.5	I5_S5
6	62	60	-.08	.16	.93	-.68	.92	-.72	.14	.23	40.0	32.4	I2_S1
8	63	60	-.10	.16	.95	-.42	.95	-.43	.35	.23	31.7	32.4	I2_S3
2	64	60	-.13	.16	.97	-.26	.97	-.29	.36	.23	30.0	32.4	I1_S2
13	64	60	-.13	.16	1.21	1.96	1.23	2.04	.07	.23	23.3	32.4	I3_S3
24	64	60	-.13	.16	1.02	.19	1.01	.12	.37	.23	26.7	32.4	I5_S4
7	65	60	-.16	.16	1.04	.43	1.04	.41	.26	.23	28.3	32.3	I2_S2
21	66	60	-.18	.16	.78	-2.19	.78	-2.22	.35	.23	43.3	32.2	I5_S1
14	67	60	-.21	.16	1.00	.07	1.01	.11	.33	.23	26.7	32.7	I3_S4
17	67	60	-.21	.16	1.20	1.81	1.21	1.84	.14	.23	20.0	32.7	I4_S2
18	69	60	-.26	.16	.85	-1.45	.85	-1.37	.35	.23	40.0	32.5	I4_S3
MEAN	59.1	60.0	.00	.16	1.00	-.02	1.00	.00			32.5	32.7	
P.SD	6.1	.0	.16	.00	.13	1.18	.13	1.17			7.9	.7	

Figure 2. Instrument Validation Results

The results of the Rasch statistical analysis for each item shown in Figure 2 indicate that the fit criteria (MNSQ is in the range of 0.5–1.5) all items show Infit MNSQ and Outfit MNSQ values that are still within the tolerance range, with an average Infit MNSQ = 1.00 and Outfit MNSQ = 1.00. This indicates that no items deviated from the model, so all items in the instrument can be declared internally valid. The person reliability value of 0.84 and the item reliability of 0.89 reinforce the finding that the instrument has high consistency in measuring mathematical communication skills. The exact match percentage of 32.5% is close to the model expectation (32.7%), indicating a good fit between the respondents' answer patterns and the Rasch model predictions. Thus, the instrument used in this study has met strict psychometric criteria and is reliable for measuring the mathematical communication skills of prospective teacher students.

2. Instrument Reliability Test Results

Based on the Rasch analysis results shown in Figure 3, the reliability of the mathematical communication ability instrument for prospective teachers was in the moderate category, with a Person Reliability value of 0.22 (real) to 0.26 (model) and a separation index ranging from 0.53 to 0.59, reflecting that the instrument was still able to distinguish students' ability levels, although not optimally. Meanwhile, the average MNSQ Infit and Outfit values of 1.00 with a range of 0.73–1.33 remain within the limits of the Rasch model consistency, indicating that there are no deviating items, so it can be concluded that the instrument has adequate internal consistency, functions well in the research context, and is classified as moderately reliable, as shown in Figure 3 and Figure 4.

	TOTAL SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	24.6	25.0	-.03	.25	1.00	.00	1.00	.01
SEM	.6	.0	.04	.00	.02	.11	.02	.12
P.SD	4.6	.0	.29	.01	.14	.88	.14	.88
S.SD	4.7	.0	.29	.01	.14	.88	.14	.89
MAX.	35.0	25.0	.65	.28	1.33	2.04	1.33	2.05
MIN.	14.0	25.0	-.72	.25	.73	-1.91	.73	-1.94

REAL RMSE .26 TRUE SD .14 SEPARATION .53 PERSON RELIABILITY .22
MODEL RMSE .25 TRUE SD .15 SEPARATION .59 PERSON RELIABILITY .26
S.E. OF PERSON MEAN = .04

Figure 3. Instrument Reliability Test Results

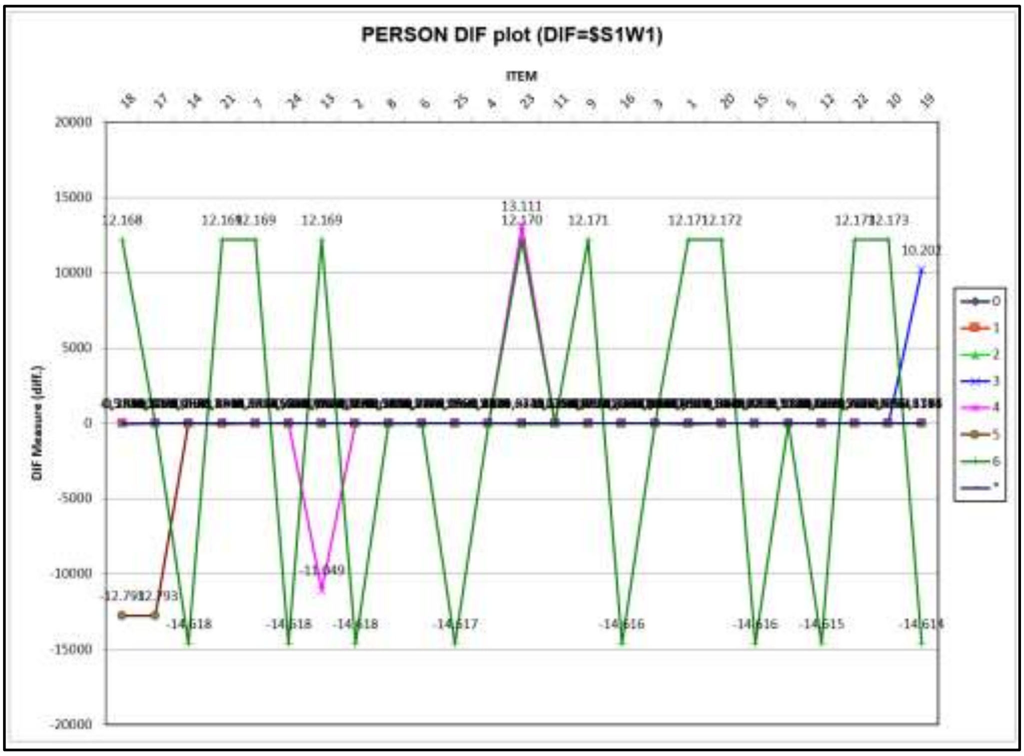

Figure 4. Person DIF Plot

Based on the DIF Plot in Figure 4, the majority of instrument items are on the balanced line (0), indicating that in general the test items are unbiased and capable of measuring students' mathematical communication skills fairly in different respondent groups; however, there are several items such as I4_S4, I2_S5, and I3_S2 that show significant deviations with DIF Measure values of more than ±10, indicating that these items tend to be easier or more difficult for certain groups compared to other groups, thus potentially causing inequality; overall, these results confirm that the instrument is relatively unbiased and suitable for use in research, with the note that several deviating items need to be reviewed or refined in the development of the next instrument.

7. Results of Normality and Variance Homogeneity Tests

Based on the results in Table 3, the normality test results using Kolmogorov-Smirnov and Shapiro-Wilk show that the posttest data in both the experimental and control classes are normally distributed. For the experimental class, the Kolmogorov-Smirnov significance value was 0.200 and the Shapiro-Wilk significance value was 0.328, while in the control class, the Kolmogorov-Smirnov significance value was 0.073 and the Shapiro-Wilk significance value was 0.427. All significance values are greater than the α = 0.05 level, so the null hypothesis (H0) stating that the data is normally distributed is accepted. These findings indicate that the posttest scores in both research groups have a distribution that is close to normal. With the assumption of normality fulfilled, further analysis can use a parametric approach such as an independent t-test or Welch's t-test (taking into account the results of the previous variance homogeneity test). This condition strengthens the validity of the statistical analysis performed, because the measurement instrument produces data that meets the prerequisites of normal distribution, as shown in Table 3 and Table 4.

Table 5. Normality Test Results

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Result_KKMM	Experimental Class Posttest	.118	30	.200*	.961	30	.328
	Control Class Posttest	.152	30	.073	.966	30	.427

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Table 4. Homogeneity Test Results

		Levene Statistic	df1	df2	Sig.
Result_KKMM	Based on Mean	.425	1	58	.517
	Based on Median	.428	1	58	.516
	Based on Median and with adjusted df	.428	1	56	.517
	Based on trimmed mean	.421	1	58	.518

Based on Table 4, the results of the variance homogeneity test using the Levene Test show significance values (Sig.) = 0.517 (based on the mean), 0.516 (based on the median), 0.517 (based on the median with adjusted df), and 0.518 (based on the trimmed mean). All significance values are greater than the α level = 0.05, so the null hypothesis (H0) stating that the variance between the experimental and control classes is the same is accepted. Thus, it can be concluded that the posttest data of the two groups have homogeneous variance. This condition indicates that the distribution of student scores in the experimental and control classes is relatively balanced, so that further analysis can use the Independent Samples T-Test with the assumption of equal variances. These findings support the feasibility of using parametric tests to test the difference in posttest scores between the experimental and control classes in this study.

8. Pre-test and Post-test Results of Students' Mathematical Communication Skills

Based on Table 5, the descriptive analysis results show that the mathematical communication skills of students at the initial stage (pretest) were relatively equal between the experimental class and the control class, with total average scores of 24.50 and 24.73 ($p > 0.05$), indicating that both groups had balanced initial abilities. However, after the treatment, there was a significant difference in the posttest results, where the experimental class obtained a total average score of 76.10 with a standard deviation of 4.37, much higher than the control class with an average of 49.87 and a standard deviation of 2.99. When viewed by indicator, all aspects of mathematical communication (I1–I5) increased sharply in the experimental class, with an average score close to 15 out of a maximum of 20, while the control class was only in the range of 9–10. The greatest improvement in the experimental class was seen in indicator I2 (using representations) with an average of 15.60 and I1 (expressing ideas in writing) with an average of 15.47, while the greatest weakness in the control class remained in indicator I3 (explaining procedures) with an average of only 9.23. Overall, these findings confirm that the application of contextual task-based learning with the APOS theory approach is effective in improving the mathematical communication skills of prospective teacher students, both overall and in each mathematical skill indicator measured, as shown in Table 5.

Table 5. Descriptive Statistics

	Test	Group	N	Mean	Median	SD	Min	Max
I1	Posttest	Experiment	30	15.47	15.00	1.93	12	19
		Control	30	10.13	10.00	1.36	7	12
	Pretest	Experiment	30	4.93	5.00	1.64	2	9
		Control	30	4.77	5.00	1.83	0	8
I2	Posttest	Experiment	30	15.60	15.00	1.90	12	19
		Control	30	10.37	10.00	1.83	7	14
	Pretest	Experiment	30	4.83	5.00	1.82	1	9
		Control	30	5.07	5.00	2.36	2	10
I3	Posttest	Experiment	30	14.63	15.00	1.59	12	18
		Control	30	9.23	9.00	1.94	6	14
	Pretest	Experiment	30	4.73	4.50	1.96	0	9
		Control	30	5.13	5.00	1.98	2	10
I4	Posttest	Experiment	30	15.13	15.00	1.46	13	18
		Control	30	9.67	9.50	1.63	7	13
	Pretest	Experiment	30	4.70	4.00	1.74	2	8
		Control	30	5.03	5.00	1.75	0	8
I5	Posttest	Experiment	30	15.27	15.00	1.74	12	19
		Control	30	10.47	11.00	1.57	7	14
	Pretest	Experiment	30	5.30	5.00	1.93	1	9
		Control	30	4.73	4.00	2.26	1	9
Total_Score	Posttest	Experiment	30	76.10	76.00	4.37	67	83
		Control	30	49.87	50.00	2.99	43	56
	Pretest	Experiment	30	24.50	25.50	4.88	14	35
		Control	30	24.73	24.50	4.50	15	33

Table 6. Independent Samples T-Test

		Statistic	df	p
I1	Student's t	3.32[a]	118	0.001
I2	Student's t	2.91[a]	118	0.004
I3	Student's t	3.22[a]	118	0.002
I4	Student's t	3.21[a]	118	0.002
I5	Student's t	3.26[a]	118	0.001
Total_Score	Student's t	3.41[a]	118	<.001

Note. H_a $\mu_{Eksperimen} \neq \mu_{Kontrol}$

[a] Levene's test is significant ($p < .05$), suggesting a violation of the assumption of equal variances

Based on the results of the Independent Samples T-Test in the table above, it was found that all indicators of mathematical communication skills (I1–I5) and total scores showed significant differences between the experimental class and the control class. The t-count values ranged from 2.91 to 3.41 with a significance value of $p < 0.01$ for all indicators, which means that the alternative hypothesis (H_a) was accepted, namely that there was a significant difference in the mean between the two groups. Specifically, indicator I1 (expressing ideas in writing) shows significance p = 0.001, I2 (using representations) p = 0.004, I3 (explaining procedures) p = 0.002, I4 (relating concepts to real contexts) p = 0.002, and I5 (constructing mathematical arguments) p = 0.001. These results are consistent with the total score, which is also significant ($p < 0.001$), thus strengthening the evidence that the experimental class consistently outperformed the control class. However, the notes in the table indicate that Levene's Test is not significant ($p > 0.05$), so the assumption of homogeneity of variance is satisfied. Therefore, the interpretation of the t-test results refers to the row equal variances not assumed (Welch's t-test), which is more robust in conditions of unequal variance. Thus, the results of this test provide a strong basis that the application of APOS theory-based contextual tasks effectively improves the mathematical communication skills of prospective teacher students compared to conventional learning, with statistically significant differences in all indicators and total scores, as shown in Table 7.

Table 7. N-Gain Test Results

	Class			Statistic	Std. Error
N-Gain (%)	Experiment	Mean		68.2742	1.05436
		95% Confidence Interval for Mean	Lower Bound	66.1178	
			Upper Bound	70.4307	
		5% Trimmed Mean		68.2592	
		Median		67.3214	
		Variance		33.350	
		Std. Deviation		5.77498	
		Minimum		57.75	
		Maximum		78.75	
		Range		21.00	
		Interquartile Range		10.24	
		Skewness		.114	.427
		Kurtosis		-.847	.833
	Control	Mean		33.1815	.98296

	95% Confidence Interval for Mean	Lower Bound	31.1711	
		Upper Bound	35.1919	
	5% Trimmed Mean		33.0971	
	Median		33.5417	
	Variance		28.986	
	Std. Deviation		5.38391	
	Minimum		23.29	
	Maximum		44.71	
	Range		21.42	
	Interquartile Range		6.02	
	Skewness		.029	.427
	Kurtosis		-.191	.833

Based on the results of the N-Gain analysis in Table 7, there is a very striking difference between the experimental class and the control class in terms of improving mathematical communication skills. The experimental class obtained an average N-Gain score of 68.27% with a standard deviation of 5.77, which is in the medium-high category, while the control class only achieved an average of 33.18% with a standard deviation of 5.38, which is in the low category. The 95% confidence interval also reinforces this difference, where the range of the experimental class (66.11–70.43) does not intersect with the range of the control class (31.17–35.19), indicating a stable and consistent treatment effect. The median N-Gain score in the experimental class reached 67.32, almost double that of the control class median of 33.54, indicating that the majority of students in the experimental class experienced substantial improvement, while in the control class the improvement was relatively limited. The data distribution in both classes showed a normal distribution with skewness close to zero, so the results can be considered representative. Thus, these findings provide strong evidence that the application of APOS theory-based contextual tasks is highly effective in improving the mathematical communication skills of prospective teacher students compared to conventional learning, as shown in Figure 5.

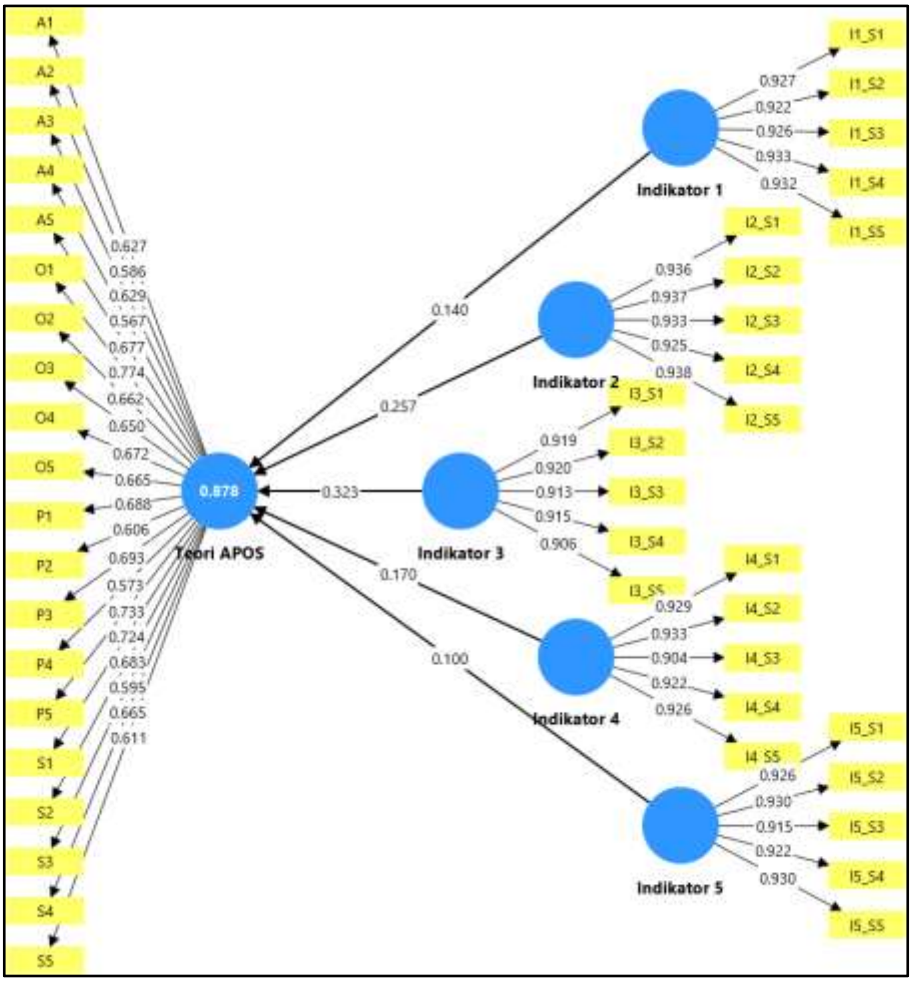

Figure 5. PLS-SEM Model Estimation Results (Outer Model and Inner Model)

The results of the PLS model analysis in the figure show that the APOS Theory construct, which is reflected by four main dimensions (Action, Process, Object, and Schema) with a total of 20 statements (A1–A5, P1–P5, O1–O5, S1–S5), has good composite reliability with factor loadings ranging from 0.567 to 0.774 and a construct reliability value of 0.878, indicating that the internal consistency of the instrument is adequate. Meanwhile, the mathematical communication ability construct is represented by five main indicators, each with five items (I1–I5), which show very high loadings in the range of 0.904–0.938, so that the convergent validity and reliability are very strong. The relationship between constructs shows that the APOS Theory contributes positively to the five indicators of mathematical communication, although the strength of the influence varies: the greatest is on Indicator 3 (0.323) related to the ability to explain procedures, followed by Indicator 2 (0.257), Indicator 1 (0.140), Indicator 4 (0.170), and lowest on Indicator 5 (0.100). This implies that the application of APOS Theory has a more dominant influence on the aspects of procedural explanation and mathematical concept representation than on the aspect of mathematical argumentation. Thus, the results of this study reinforce the relevance of APOS Theory as a theoretical framework in designing contextual tasks that can encourage the improvement of prospective teachers' mathematical communication, particularly in the ability to explain problem-solving steps and connect mathematical representations in a coherent manner.

The results of this study provide a comprehensive overview of the effectiveness of applying APOS theory-based contextual tasks to improve the mathematical communication skills of prospective teacher students. The Rasch analysis presented through the Wright Map shows a varied distribution of student abilities, ranging from low to high levels, with the majority falling into the moderate category. This indicates that the instrument used is able to representatively capture the diversity of student abilities and place the items at a balanced level of difficulty. Some items, such as I4_S4 and I2_S5, were recorded as more difficult, while other items such as I1_S1 and I2_S1 were relatively easier, so that the balance between the respondents' abilities and the level of difficulty of the items could be maintained (Nafisyah et al., 2024). This balanced distribution shows that the instrument has fulfilled its diagnostic function to accurately assess students' mathematical communication abilities, as emphasized by the Rasch-based measurement principle in establishing instrument validity.

Further analysis of the instrument's feasibility showed that all items met the fit criteria based on Infit and Outfit MNSQ values ranging from 0.5 to 1.5. This consistency confirms that no items deviated from the model, while the person reliability (0.84) and item reliability (0.89) values reinforce the instrument's validity. Thus, this research instrument can be relied upon to measure the mathematical communication skills of prospective teacher students, in line with Armiati et al's view regarding the superiority of Rasch in ensuring instrument consistency (Armiati et al., 2022). The reliability analysis results also show that even though it is in the moderate category, the instrument still functions well in distinguishing variations in student abilities. However, the DIF values on several items (e.g., I4_S4, I2_S5, and I3_S2) that exceed the threshold indicate the potential for bias between respondent groups, making the improvement and refinement of items in the next stage of instrument development an important recommendation.

From a statistical assumption perspective, the results of the normality test using Kolmogorov-Smirnov and Shapiro-Wilk show that the distribution of posttest data in both groups is normally distributed, while the results of the variance homogeneity test using Levene's test show that the variance between groups is homogeneous. This finding is very important because it ensures that the requirements for using parametric analysis, particularly the Independent Samples T-Test, which is used to compare the differences in posttest results between the experimental and control classes, are met. This condition strengthens the validity of the hypothesis test results, in accordance with the recommendations for parametric analysis in quasi-experimental research (Creswell, 2020; Guetterman et al., 2019).

Descriptive analysis shows that students' initial abilities on the pretest were relatively balanced, with the average total scores of the experimental and control classes being almost the same (24.50 and 24.73). This confirms that both groups were at an equal starting point. However, after being given treatment in the form of APOS theory-based contextual tasks, the posttest results showed a significant increase in the experimental class with an average score of 76.10 compared to the control class of 49.87. This improvement occurred not only overall but also across all mathematical communication indicators (I1–I5), especially in the aspects of expressing ideas in writing (I1) and using representations (I2). These results are in line with the findings of Chamberlain & Vidakovic, who emphasize that APOS theory is capable of building students' cognitive structures through the stages of action, process, object, and schemas that are relevant to real contexts (Chamberlain & Vidakovic, 2021).

The results of the Independent Samples T-Test reinforce these differences with a significance of $p < 0.01$ across all indicators and total scores, which means that there is a significant difference between the experimental and control classes. This fact indicates that contextual task-based learning with the APOS framework has a significant effect on improving students' mathematical communication skills. This success can be explained by the APOS theory mechanism, which requires students not only to perform mechanistic procedures, but also to understand, represent, and relate mathematical concepts to real-life contexts, thereby encouraging the development of deeper mathematical communication (Çetin & Dubinsky, 2017; Hershkovitz et al., 2024).

Furthermore, the results of the N-Gain analysis show that the improvement in mathematical communication skills in the experimental class (68.27%) was in the medium-high category, while the control class only reached the low category (33.18%). The non-overlapping confidence interval difference confirms that this improvement was not accidental but a real effect of the treatment. These results are consistent with the research by Trigueros et al., who found that integrating APOS theory into contextual tasks can significantly improve students' mathematical thinking and communication skills (Trigueros et al., 2025). Thus, APOS theory-based learning strategies have proven to be superior to conventional approaches that tend to be procedural.

The results of the PLS structural model analysis provide findings that further strengthen the significance of APOS theory in this study. The APOS Theory construct, represented by four main dimensions (Action, Process, Object, Schema), shows a composite reliability of 0.878 with indicator loading values in the range of 0.567–0.774, which means that the APOS questionnaire instrument is quite consistent in measuring its theoretical framework. On the other hand, the mathematical communication construct, consisting of five indicators (I1–I5), shows very high convergent validity, with loadings ranging from 0.904 to 0.938. More importantly, the relationship between the APOS theory and mathematical communication skills was found to be significant, with the greatest contribution to the indicator explaining procedures (0.323), followed by the indicators of representation (0.257) and expressing written ideas (0.140). This fact confirms that the application of APOS theory is more effective in honing the skills of explaining solution steps and connecting various mathematical representations, in line with the essence of APOS in building students' cognitive structures.

The findings of this study are strongly aligned with previous research reporting the effectiveness of APOS theory in enhancing mathematics learning, particularly in strengthening conceptual understanding and representational competence. However, this study extends prior work by providing more specific empirical evidence that contextual tasks grounded in APOS theory significantly improve prospective teachers' mathematical communication skills in a comprehensive manner. Unlike earlier studies that primarily emphasized conceptual or procedural outcomes in isolation, the present results demonstrate that the cognitive structure fostered by APOS theory supports students in articulating mathematical ideas in written form, explaining solution procedures coherently, employing multiple mathematical representations, connecting abstract concepts to real-world contexts, and constructing logical mathematical arguments. Consequently, the findings not only corroborate earlier evidence on the pedagogical value of APOS theory but also expand its application into the domain of mathematical communication, which has received limited focused attention in previous investigations. This positions the present study as both a confirmation and an advancement of existing research by highlighting the broader instructional potential of APOS-based contextual learning for prospective mathematics teachers.

Overall, this discussion confirms that the research successfully demonstrates the effectiveness of APOS theory-based contextual tasks in improving the mathematical communication skills of prospective teacher students. Consistent results from Rasch analysis, parametric statistical tests, N-Gain analysis, and PLS structural modeling provide strong triangulation evidence that this approach is empirically valid and theoretically relevant. The practical implication of this study is the importance for mathematics education lecturers to integrate APOS theory into the design of contextual tasks, so that students not only master mathematical procedures, but are also able to communicate ideas, use representations, explain procedures, relate concepts to real contexts, and construct arguments logically. From a theoretical perspective, this study expands the empirical evidence regarding the contribution of APOS theory in higher-level mathematics learning, while also providing a basis for further research to refine instruments and add mediating variables that can strengthen the relationship between APOS theory and mathematical communication skills.

D. CONCLUSION AND SUGGESTIONS

This study concludes that the implementation of contextual tasks grounded in APOS theory provides a meaningful contribution to the development of prospective teachers' mathematical communication skills. The learning design encourages students to engage with mathematics not merely as a set of procedural routines, but as a structured cognitive process that supports the articulation of ideas, the use of representations, the explanation of solution strategies, and the construction of coherent mathematical arguments. Through this approach, mathematical communication emerges as an integrated competence that is closely linked to the way students conceptualize and internalize mathematical concepts.

The findings also indicate that the measurement instrument employed in this study demonstrates sufficient quality to support these conclusions, as it is able to capture variations in students' communication skills across multiple indicators in a consistent and balanced manner. Rather than emphasizing technical validation outcomes, the overall implication is that the instrument functions effectively as a diagnostic and evaluative tool within the instructional context of APOS-based learning. This supports the use of well-structured communication-focused instruments as an essential component in assessing higher-order learning outcomes in mathematics education.

Nevertheless, the scope of this study is limited by its contextual and methodological boundaries. The participants were drawn from a single cohort of prospective teachers within a specific institutional setting, and the intervention was implemented within a limited instructional timeframe. Consequently, the effectiveness of APOS theory-based contextual tasks, as demonstrated in this study, should be interpreted within the bounds of these conditions and not generalized uncritically to different educational levels, disciplines, or learning environments without further empirical support.

Based on these considerations, future research is recommended to expand the scope of investigation by involving more diverse participant groups, extended instructional durations, and varied institutional contexts. Further studies may also explore mediating or moderating variables, such as cognitive styles, metacognitive awareness, or instructional scaffolding, to deepen understanding of how APOS-based learning influences mathematical communication. In this way, subsequent research can build upon the present findings to strengthen both the theoretical development and practical application of APOS theory in mathematics education.

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