JTAM (Jurnal Teori dan Aplikasi Matematika)
http://journal.ummat.ac.id/index.php/jtam
p-ISSN 2597-7512 | e-ISSN 2614-1175
Vol. 10, No. 2, April 2026, pp. 696-710

Trajectories of Cannibalism Interaction with Holling Type II and Monod–Haldane Functional Responses

Rian Ade Pratama[1]*, Dessy Rizki Suryani[1], Maria F. V. Ruslau[1], Etriana Meirista[1]
[1]Department of Mathematics Education, Musamus University, Indonesia
pratama@unmus.ac.id

Article History:
Received : 22-11-2025
Revised : 17-01-2026
Accepted : 19-01-2026
Online : 01-04-2026

Keywords:
Cannibalisme;
Holling type II;
Monod-Haldane;
Nonlinear differential equations;
Predator-prey dynamics.

ABSTRACT

The stability and equilibrium behavior of predator–prey systems involving cannibalistic interactions is crucial for explaining the long-term sustainability of ecological communities. This study aims to analyze the dynamics of a modified predator–prey model by incorporating cannibalism in predators as a self-regulating mechanism influencing population control. This study is a literature-based research, and the instruments employed are non-physical in nature, including a mathematical model, mathematical analysis tools, and numerical computation frameworks. The research methodology employs literature review and analysis of a model formulated as a system of nonlinear differential equations. This system describes the population dynamics of two prey species and one predator species exhibiting cannibalistic tendencies. Analytical and numerical approaches are utilized to determine equilibrium points, evaluate local stability, and assess the effects of density-dependent mortality and cannibalistic behavior on ecosystem balance. The results show that the proposed predator–prey model admits one trivial equilibrium, five semi-trivial equilibrium, and one coexistence equilibrium. The coexistence equilibrium is locally asymptotically stable and satisfies the Routh–Hurwitz stability criterion. Simulation numeric the cannibalism parameter and density-dependent mortality rates play a significant role in stabilizing the predator population dynamics. When the mortality coefficient increases, the predator population decreases toward a lower equilibrium point, while the prey population slightly increases due to reduced predation pressure. Eigenvalue analysis reinforces these findings by confirming the system's compliance with the Routh–Hurwitz stability conditions. Ecological implications, these findings suggest that cannibalistic behavior in predators acts as a natural feedback mechanism to regulate population density, enhance ecosystem stability, and support the long-term sustainability of predator–prey interactions. The cannibalistic character of the predator species does not necessarily lead to species extinction, but can instead facilitate a sustainable and balanced coexistence within the ecosystem.

https://doi.org/10.31764/jtam.v10i2.36659

—————————— ◆ ——————————

A. INTRODUCTION

Predator-prey system dynamics constitute a fundamental element in population ecology, as they dictate the distribution and abundance of various species within an ecosystem and provide a basis for forecasting population growth or extinction events. These interactions are paramount to the sustainability of natural ecosystems, a process significantly influenced and supported by diverse environmental factors. Interactions between different species are broadly classified into forms such as competition and mutualism. Competition arises when multiple species utilize the same limited resources or inflict mutual harm while vying for those resources.

Conversely, mutualism represents a long-term, intimate relationship where both participating species derive a net benefit. The Lotka-Volterra system stands as the foundational model in this field and has been extensively developed by modern research, particularly in describing the complex multi-species interactions within ecological settings. Integral to these dynamic systems is the incorporation of functional response models. A functional response is mathematically defined as the per-predator predation rate per unit time, expressed as a function of the prey density. Early development of predator-prey models primarily recognized three types of functional responses: Holling Type I, Holling Type II, and Holling Type III. However, the form of this response function has been continuously refined to more adequately represent the complexities of predation in diverse ecosystems. This ongoing development has led to the introduction of various other functional responses, including but not limited to Holling Type IV, Monod-Haldane, Beddington-DeAngelis, and Crowley-Martin.

Functional responses play a fundamental role in predator–prey modeling as they describe how the predation rate varies with prey density. The simplest formulation, known as Holling type I, assumes a linear increase in predation with prey abundance and neglects any saturation effects. Mathematically the model is $f(N) = aN$, where (N) is the population density and (a) is the attack rate or predation efficiency of the predator. Although this assumption is idealized, it provides a baseline description of systems in which predators consume prey in direct proportion to their availability (Alemu et al., 2025). To incorporate biological constraints, Holling type II introduces a handling time, resulting in a saturating functional response. The corresponding mathematical model is $(N) = \frac{aN}{1+ahN}$, where (h) is the handling time, which is the time needed for a predator to catch, consume, and digest one individual prey. In this case, the predation rate increases at low prey densities but levels off as prey becomes abundant, reflecting the limited processing capacity of predators. This saturation mechanism is known to promote the stabilization of predator–prey dynamics under certain parameter regimes (Shao, 2022).

The same functional response Holling type II, A further refinement is provided by Holling type III, $f(N) = \frac{aN^2}{1+ahN^2}$ which exhibits a sigmoidal response. This formulation accounts for reduced predation efficiency at low prey densities due to prey refuges or learning behavior, followed by a rapid increase at intermediate densities. Such a mechanism has been shown to enhance ecosystem stability by protecting rare prey from extinction (Aguegboh et al., 2025; Madhusudanan et al., 2022; Pratama et al., 2025). At very high prey densities, Holling type IV, also referred to as an inhibition response, captures the decline in predation efficiency caused by predator confusion or interference. Mathematically, the general form of the Holling type IV model is $f(N) = \frac{aN}{1+bN+cN^2}$. This response characterizes extreme saturation effects, where an excess of prey can paradoxically reduce the overall feeding rate of predators (Castillo-Alvino & Marvá, 2022). In addition to Holling-type responses, the Monod–Haldane functional response incorporates inhibitory effects at high prey densities, leading to a decrease in feeding efficiency when prey becomes overly abundant. Mathematically the model is $f(N) = \frac{aN}{b+N+cN^2}$. This formulation is particularly relevant for ecological systems in which predator performance deteriorates due to overexploitation or environmental constraints (Khajanchi, 2017; Zhang & Yuan, 2021).

To explicitly account for predator interference, the Beddington–DeAngelis functional response $f(N,P) = \frac{aN}{1+ahN+cP}$, considers the impact of predator density on feeding rates, showing that per capita predation decreases as predator abundance increases. This approach has been widely adopted to describe competitive interactions among predators (Luo & Wang, 2021; Zhou et al., 2021). Similarly, the Crowley–Martin functional response: $f(N,P) = \frac{aN}{(1+ahN)(1+cP)}$ extends this framework by allowing predator interference to influence both prey searching efficiency and handling time, offering a more realistic representation of predator–predator interactions in socially structured populations (Kumar & Pramanick, 2022). Finally, the ratio-dependent functional response: $f(N,P) = \frac{a\frac{N}{P}}{\left(1+ah\frac{N}{P}\right)}$, emphasizes that predation rates depend on the prey–predator ratio rather than prey density alone. This formulation is particularly suitable for densely populated systems and highlights the role of predator competition in shaping population dynamics (Chowdhury et al., 2022).

The functional response is one of the most critical components in predator–prey dynamics. In this proposed research, the Monod–Haldane and Holling Type II functional response models were selected because they most accurately represent the nature and behavioral characteristics of the assumed predator species. The primary distinction between these types of functional responses lies in their dependency behavior. Research conducted by Alemu et al. (2025) examined Holling Types I, II, and III, which are prey-dependent response functions. Meanwhile, Holling Type IV, as well as the Crowley–Martin and Beddington–DeAngelis models, are classified as predator-dependent functional responses. In natural ecosystems, every species has developed its own strategies for survival. Some species live solitarily, while others thrive in colonies, groups, or herds. For certain species, long-term survival is best achieved through living with or near conspecifics. The need for cooperation is fulfilled when individual members' needs are met, and forming groups or teams becomes a fundamental mechanism that enables members to consistently obtain resources and ensure survival.

Team formation provides two major advantages: improved foraging efficiency and reduced predation risk. Studies conducted by Pratama (2022) and (Purnomo et al. (2025) also confirmed that foraging in groups is more efficient than foraging individually, while simultaneously minimizing exposure to predators. Predators themselves also exhibit intraspecific predation, in which individuals prey upon members of their own species. One significant form of intraspecific predation is cannibalism (Mayntz & Toft, 2006), an ecological phenomenon that plays a crucial role in predator–prey population dynamics. Cannibalism often emerges as an adaptive mechanism triggered by the scarcity of primary prey. It can also function as a competitive advantage among predators. Interestingly, some forms of cannibalistic predation are not motivated by nutritional gain but rather by the elimination of potential competitors. Studies by Cavassa et al. (2022) and Li et al. (2022), for example, investigated wolf spiders, revealing that cannibalism among them primarily serves to reduce competition from future rivals. Similarly, Bose et al. 2016 (2019) reported that male midshipman fish exhibited high levels of egg cannibalism in nests taken over from other males "nest take-over" compared to their own nests. Another example is Aidablennius sphynx, which displays total filial cannibalism as a survival strategy, as shown in the study by (Takegaki et al., 2023). Cannibalistic behavior is widespread and can be realistically incorporated into predator–prey models. Functionally, cannibalism serves as a self-limiting factor for predator populations (Li & Yan, 2022). While it provides nutritional benefits, it simultaneously reduces population density, particularly through the predation of juvenile or weaker individuals (Chowdhury et al., 2022). Moreover, cannibalism can stabilize system dynamics by preventing predator extinction when prey populations drastically decline. In this study, we analyzed the dynamics of a one predator and two prey model that incorporates two different functional responses (Li et al., 2018).

The first prey is assumed to be strong, while the second is considered weak or vulnerable. The predation processes of both prey groups are treated independently: the Monod–Haldane functional response is applied to the first prey, while the Holling Type II functional response is applied to the second. A key consideration in this model is the inclusion of cannibalism within the predator population. Previous studies have not examined cannibalism in predators that engage in dual predation involving two prey species simultaneously. Therefore, incorporating cannibalism as a dynamic variable represents both a novel contribution and an identified research gap in existing studies.

B. METHODS

This research employs an analytical approach supported by a comprehensive literature review. The predator–prey model utilized in this study is adapted from the population dynamics framework proposed by (Alsakaji et al., 2021). The data analysis techniques in this study are mathematical model formulation, equilibrium point determination, local stability analysis, numerical simulation and ecological interpretation. Local stability analysis will be conducted using the Routh–Hurwitz criterion. Numerical simulations of the proposed predator–prey model are performed using Maple 2016. Based on the aforementioned assumptions, the following mathematical model is formulated:

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right) - \rho xy - \frac{\beta xz}{b+az^2} + mxyz \\
\frac{dy}{dt} &= ry\left(1-\frac{y}{k}\right) - \rho xy - \frac{\alpha yz}{c+y} + mxyz \\
\frac{dz}{dt} &= \frac{\mu\beta xz}{b+az^2} + \frac{\mu\alpha yz}{c+y} - sz^2 - \delta z
\end{aligned} \tag{1}$$

where $x(t)$, $y(t)$ and $z(t)$ represent the population densities of the first prey, second prey, and predator, respectively. Each population inhabits the same ecosystem with an identical carrying capacity. Predators interact with prey in a random and non-structured manner, reflecting natural ecological interactions. The predators exhibit non-selective feeding behavior, meaning they do not specialize on particular prey species and consume available prey indiscriminately. Each population satisfies non-negative initial conditions, such that $x_0 \geq 0$, $y_0 \geq 0$ and $z_0 \geq 0$. The descriptions of the variables and parameters used in the model are presented in the following Table 1.

Table 1. Description of Model Variables and Parameters

Parameters	Description
$x(t), y(t)$	Prey population (time dependent),
$z(t)$	Predator population (time dependent),
r	Prey intrinsic growth rate,
k	Prey environmental carrying capacity for x and y,
a	Inverse measure of inhibitory effect.
m	Rate of cooperation of the preys,
μ	An equal transformation rate of predator to prey,
δ	Death rate of the predator,
s	Laju interaksi cannibalisme predator,
b, c	Intra specific components of x and y,
β, α	Rate of predation,

C. RESULT AND DISCUSSION

1. Equilibrium Stability

Equilibrium analysis is carried out using a linearized differential equation model:

$$\begin{aligned} rx\left(1-\frac{x}{k}\right) &= \rho xy + \frac{\beta xz}{b+az^2} - mxyz \\ ry\left(1-\frac{y}{k}\right) &= \rho xy + \frac{\alpha yz}{c+y} - mxyz \\ sz^2 + \delta z &= \frac{\mu\beta xz}{b+az^2} + \frac{\mu\alpha yz}{c+y} \end{aligned} \tag{2}$$

Model (2) was analyzed to obtain the mathematical solutions for $x(t)$, $y(t)$, and $z(t)$. The initial conditions of Model (2) are fundamental to ensure that the model admits realistic (non-negative) solutions. The equilibrium set of Model (2) is defined in the non-negative cone, $R^3_+ = \{(x,y,z): x \geq 0, y \geq 0, z \geq 0\}$. Equilibrium analysis is conducted to examine the local dynamics of the system. The local stability analysis of the interior equilibrium reveals the parameter conditions that characterize the functions describing the interactions among the three species. Changes in the growth behavior of each species can also be mathematically inferred from the given parameters. Model (2) exhibits both boundary and interior equilibria. Here, we focus on the existence of the interior equilibrium $\varepsilon(x,y,z)$, which is the positive solution of the corresponding algebraic equations. Model (2) can be associated with the following linearized differential equations:

$$\begin{aligned} &\frac{\big(amyz^3 + az^2(-\rho y + r) + z(mby-\beta) + bk(-\rho y + r) - rx(az^2+b)\big)x}{k(b+az^2)} = 0 \\ &\frac{(((r+(mz-\rho)x)y + (r+(mz-\rho)x)c - z\alpha)k - yr(y+c))y}{k*(y+c)} = 0 \\ &\frac{(-as(y+c)z^3 + ((\alpha\mu-\delta_2)y - \delta_2 c)az^2 - bs(y+c)z + ((\alpha\mu-\delta_2)b + \mu\beta x)y - c(-\beta\mu x + b\delta_2))z}{(az^2+b)(y+c)} = 0 \end{aligned} \tag{3}$$

From Equation 3, the characteristic equation of Model (2) is obtained as a 13th-degree polynomial equation. Due to its complexity, an analytical solution is not feasible. Therefore, this study employs a numerical approach. Among the 13th-order solutions, there are seven equilibrium points associated with Model (2). The selection of these equilibrium points is constrained to non-negative values, in order to avoid assuming negative population growth, which falls outside the scope of this research. The equilibrium points can be categorized into four types:

a. Trivial $\varepsilon(0,0,0)$ represents the mathematically simplest solution. Ecologically, it corresponds to a state where no populations are growing or declining. Such conditions may occur in real ecosystems affected by severe natural disasters or widespread infectious diseases. Historically, this scenario could describe the growth dynamics of prehistoric species, such as dinosaurs.
b. Semi-trivial $(0, y, 0)$, $\varepsilon(x, 0, y)$, $\varepsilon(x, y, 0)$, $\varepsilon(0,0,0)$ and $\varepsilon(x, 0,0)$correspond to conditions in which one or two populations go extinct, while the remaining populations persist. These equilibria often arise due to environmental changes, dominant predation, or other ecological pressures.
c. Coexistence equilibrium $\varepsilon(x, y, z)$ represents a saddle point reflecting the stable and sustainable growth of all species in the system. Under this condition, all species exhibit significantly positive growth, indicating the absence of total extinction. This equilibrium is the focus of the analysis regarding species sustainability.

The solution set $\varepsilon(x, y, z)$ of the developed predator–prey model consists of 13 solution sets. Theoretically, seven possible equilibria are identified, as summarized in the following Table 2:

Table 2. Number of Root Pairs

No	Number of Real Roots	Sum of Complex Conjugate Roots
1	13	0
2	11	1 (2 complexes)
3	9	2 (4 complexes)
4	7	3 (6 complexes)
5	5	4 (8 complexes)
6	3	5 (10 size complexes)
7	1	6 (12 size complexes)

The equilibrium point $\varepsilon(x, y, z)$ will be analyzed using the Routh–Hurwitz criterion. Additionally, the point will be examined through Jacobian analysis and the eigenvalue theorem. This analysis aims to determine the number of real root pairs that can be used for stability assessment. The Jacobian matrix associated with the equilibrium point $\varepsilon(x, y, z)$is given as follows:

$$J(\varepsilon) = \begin{bmatrix} j\varepsilon_{11} & j\varepsilon_{12} & j\varepsilon_{13} \\ j\varepsilon_{21} & j\varepsilon_{22} & j\varepsilon_{23} \\ j\varepsilon_{31} & j\varepsilon_{32} & j\varepsilon_{33} \end{bmatrix} \quad (4)$$

where,

$$j\varepsilon_{11} = r\left(1 - \frac{x}{k}\right) - \frac{rx}{k} - \rho y - \frac{\beta z}{az^2 + b} + myz,$$

$j\varepsilon_{12} = mxz - \rho x,$
$j\varepsilon_{13} = -\frac{\beta x}{az^2 + b} + \frac{2x\beta az^2}{(az^2 + b)^2} + mxy,$
$j\varepsilon_{21} = myz - \rho y,$
$j\varepsilon_{22} = r\left(1 - \frac{y}{k}\right) - \frac{ry}{k} - \rho x - \frac{\alpha z}{y + c} + \frac{\alpha yz}{(y + c)^2} + mxz,$
$j\varepsilon_{23} = -\frac{\alpha y}{y + c} + mxy,$
$j\varepsilon_{31} = -\frac{\mu\beta z}{az^2 + b},$
$j\varepsilon_{32} = \frac{\mu\alpha z}{y + c} - \frac{\mu\alpha yz}{(y + c)^2},$
$j\varepsilon_{33} = \frac{\mu\beta x}{az^2 + b} - \frac{2xa\mu\beta az^2}{(az^2 + b)^2} + \frac{\mu\alpha y}{y + c} - 2sz - \delta,$

Characteristic equation of Jacobian matrix;

$$\lambda^3 + p\lambda^2 + r\lambda + s, \tag{5}$$

where:
$p = -yes_{11} - yes_{22} - yes_{33}$
$r = M$(major minor number)
$s = det(j\varepsilon)$ (matrix determinant).

The necessary conditions used are to meet$p > 0, r > 0, s > 0$, And $rp > s$. The characteristic equation that appears is in the form of a third-order or cubic polynomial. In this characteristic equation, there are three eigenvalues that appear, including $\lambda_1 < 0, \lambda_2 < 0$ and $\lambda_3 < 0$.

2. Numerical Simulation

In this section, the equilibrium point analysis is presented using specific parameter values. The parameters were selected based on relevant research and fundamental ecological assumptions. The chosen parameter values are as follows: $r = 1.5, k = 100, \delta = 0.25, \beta = 0.2, a = 0.5, c = 8, n = 0.6, \rho = 0.0004, b = 7, \alpha = 0.3, \sigma = 0.5, m = 0.0005, \mu = 0.32,$ dan $s = 0.2$. Model (2) with parameters;

$$\begin{aligned} \frac{dx}{dt} &= 1.5x\left(1 - \frac{x}{100}\right) - 0.004xy - \frac{0.2xz}{7 + 0.5z^2} + 0.0005xyz \\ \frac{dy}{dt} &= 1.5y\left(1 - \frac{y}{100}\right) - 0.004xy - \frac{0.3yz}{8 + y} + 0.0005xyz \\ \frac{dz}{dt} &= \frac{0.064xz}{7 + 0.5z^2} + \frac{0.096yz}{8 + y} - 0.2z^2 - 0.25z \end{aligned} \tag{6}$$

From model equation (6), the non-negative equilibrium point is obtained:

$$\varepsilon_0(0,0,0) \tag{7}$$

$$\varepsilon_1(78.94736842,78.94736842,0), \varepsilon_2(96.94954936,0,2.111490555), \varepsilon_3(0,100,0), \varepsilon_4(100,0,0) \tag{8}$$

$$\varepsilon_5(88089.66062, 88089.66062, 37.96606378),$$
$$\varepsilon_6(80.38134164, 83.55614705, 2.028162392) \quad (9)$$

All equilibrium points are categorized as trivial, semi-trivial, or coexistence. Further stability analysis is conducted only for the equilibrium point ε_6. The equilibrium ε_6 is analyzed using the Jacobian matrix and the Routh–Hurwitz criterion. At the equilibrium point ε_6, the Jacobian matrix is obtained as follows:

$$J(\varepsilon_6) = \begin{bmatrix} -1.205720124 & -0.2400121596 & 2.389323621 \\ -0.2494918706 & -1.247277255 & 3.084391024 \\ 0.01433216152 & 0.000185818491 & -0.6636199784 \end{bmatrix} \quad (10)$$

At the equilibrium point $\varepsilon_6$we obtain the form of the Jacobian matrix as follows;

$$\lambda^3 + 3.116617357\lambda^2 + 3.037026962\lambda + 0.9255758185, \quad (13)$$

a necessary and sufficient condition for the Routh-Hurwitz characteristic equation is that the eigenfactors are satisfied. $\lambda < 0$; $\lambda_1 = -1.51614797926081$, $\lambda_2 = -0.973142203387053$, and $\lambda_3 = -0.627327174752140$.

The eigenvalue factors satisfy the sufficiency conditions of the Routh–Hurwitz criterion, indicating local stability. The local stability at the equilibrium point ε_6 reflects the ecosystem's capacity for sustainable persistence. In this study, the analysis is limited to local stability. Population growth can already be inferred from this local stability analysis. Additionally, trajectory analysis is provided to illustrate the dynamics of population growth. The growth trajectories of predator and prey populations in Equation (2) are as shown in Figure 1, Figure 2, and Figure 3.

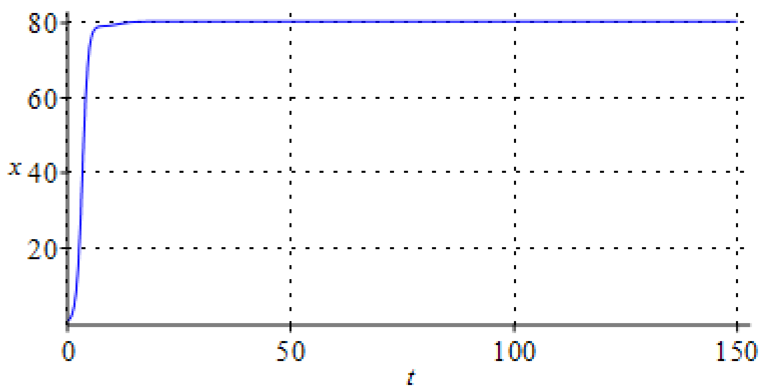

Figure 1. Trajectories species prey $x(t)$

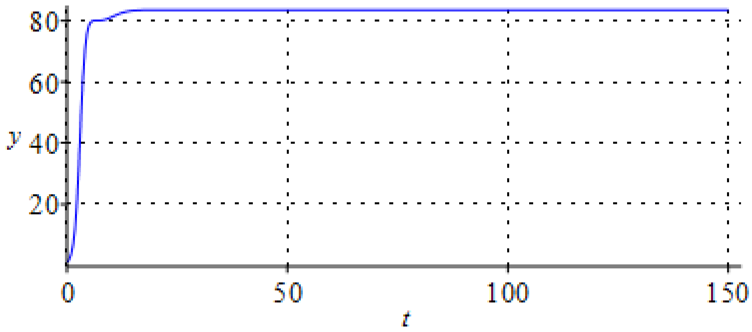

Figure 2. Trajectories species prey $y(t)$

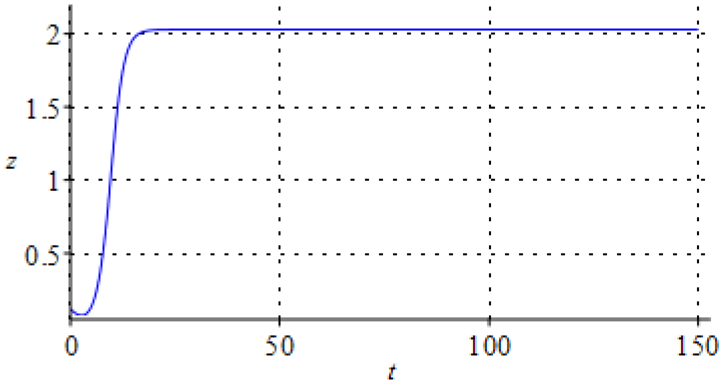

Figure 3. Trajectories species predator $z(t)$

The trajectories shown in Figure 1 illustrate the population growth pattern of the prey, which follows a logistic growth model. Mathematically, this dynamic is described by the equation $rx\left(1-\frac{x}{k}\right)$, where r is the intrinsic growth rate and k is the environmental carrying capacity. Initially, when the population is small $(x \leq k)$, the growth rate is approximately exponential due to abundant resources. As the population increases, competition for resources intensifies, slowing the growth rate until the population stabilizes at approximately $x \approx 80.38134164$. Ecologically, this reflects a scenario where the prey population grows rapidly in the early phase but gradually slows as resource limitations constrain further growth, ultimately reaching an ecological equilibrium where the birth rate balances the death rate. A similar pattern is observed in Figure 2, which shows the growth of the second prey population (y). The population exhibits a rapid transient behavior for $t \leq 10$. Mathematically, this curve resembles a first-order or higher-order overdamped response to a step input, where the differential equation $\frac{dy}{dt}$ is inversely proportional to the difference between the steady-state value and the current population. The growth rate is high when the population is far from its carrying capacity and slows as it approaches equilibrium. Ecologically, this curve indicates that the second prey population initially grows without constraint due to a large environmental carrying capacity. Overall, the Monod–Haldane functional response significantly influences the prey population dynamics, ensuring that growth slows as resource limits are approached. In contrast, the predator population (z) dynamics over time exhibit a different behaviour t. Initially, the predator population is very low and may experience a slight decline due to limited prey availability. As the prey population increases and provides sufficient energy resources, the predator population grows rapidly until it approaches the equilibrium value $z \approx 2.028162392$. Once this equilibrium is reached, growth ceases and the population stabilizes.

This indicates that the system has attained an ecological equilibrium, where predator birth rates balance mortality rates. Mathematically, this behavior is captured by the nonlinear predator–prey differential equations. The equilibrium point $\varepsilon_6(80.38134164, 83.55614705, 2.028162392)$ represents a stable fixed point of the three-species predator–prey system. This condition reflects a balance between the natural growth rate of the prey, which follows the logistic model with environmental limitations, and the losses due to predation, represented by terms such as ρxy and $\frac{\beta xz}{b+az^2}$. Meanwhile, predator growth depends on the availability of prey as an energy source but is also constrained by cannibalism-induced mortality (sz^2). Cannibalism has a significant influence on predator population dynamics. It acts as a density-dependent mortality factor (Li et al., 2018), contributing a nonlinear negative term to $\frac{dz}{dt}$. Consequently, (z) at higher predator densities, population reduction accelerates, as expressed by $\left(\partial \frac{(\frac{dz}{dt})}{\partial z}\right)$, which contains $-2sz - \delta +$ plus contributions from the functional response. This makes the Jacobian matrix more negative when s is large, potentially reducing predator equilibrium density. If s becomes too high, density-dependent mortality may exceed the benefits of predation, leading the predator population toward extinction. Conversely, lower values of s allow predators to achieve higher equilibrium levels, exerting stronger predation pressure on the prey. This finding has also been confirmed by (J. Li et al., 2020) dan (Jiang et al., 2022). The numerical results were obtained from steady-state simulations over $t \in [0,150]$ with initial conditions $x(0) = 1, y(0) = 1, z(0) = 0.1$. The results of these numerical simulations are presented as shown in Table 3.

Table 3. Changes in equilibrium due to cannibalism

Parameter (s)	**Equilibrium prey** (x)	**Equilibrium prey** (y)	**Equilibrium predator** (z)
0.05	111.29	113.68	4.696
0.10	106.39	109.09	3.534
0.20	102.57	105.26	2.465
0.50	99.48	101.42	1.301
1.00	98.39	99.56	0.707

The trajectories of each population are illustrated as shown in Figure 4 and Figure 5.

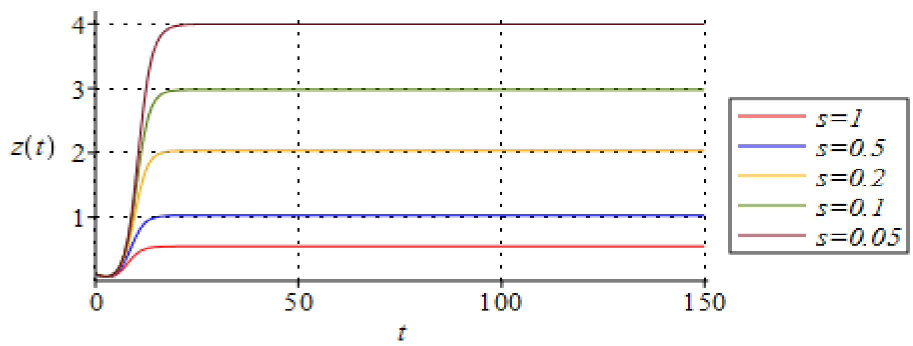

Figure 4. Predator species trajectories with respect to the cannibalism coefficient (s)

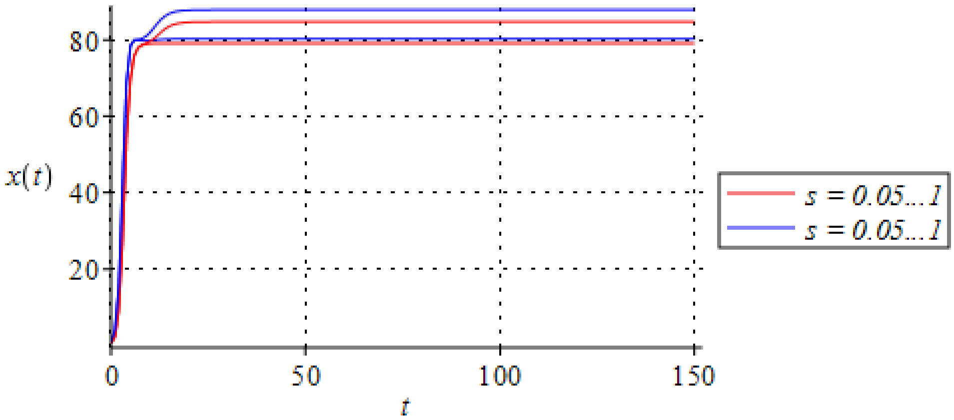

Figure 5. Prey species trajectories with respect to the cannibalism coefficient (s)

It is clearly observed from the numerical simulations that as s increases, the predator equilibrium (z) decreases monotonically. Higher density-dependent mortality results in a lower predator population at equilibrium. The decline in predator population (z) is accompanied by a slight increase in the prey populations (x, y), indicating that when s is large, predation pressure weakens. Changes in prey populations are relatively small compared to changes in predator populations; in other words, the prey populations remain close to their carrying capacities. All simulations with the given parameter values converge to a fixed point without exhibiting large oscillations or chaotic behavior. Cannibalism ($-sz^2$) acts as a nonlinear damping mechanism, helping the system reach steady state more quickly and preventing large predator overshoots. Ecologically, this effect constrains predator growth at high densities, thereby enhancing system stability. The energy transferred from prey to predator is not fully efficient, and combined with density-dependent mortality, it keeps predator numbers much lower than those of the prey. While this contrasts with the naturally voracious behavior of predators, cannibalism limits predator population size, balancing the system.

This study aimed to investigate the dynamics of predator populations exhibiting cannibalistic behavior. The analysis focused on determining the equilibrium points and local stability of the system. Through the predator–prey modeling framework, seven equilibrium points were identified, encompassing trivial, semi-trivial, and coexistence equilibria. Local stability was assessed using the Routh–Hurwitz criterion, based on the principle that species persistence in ecosystems must be maintained over long periods. The analysis indicated that the eigenvalues satisfied the stability conditions according to established principles. Numerical simulations showed that the cannibalism parameter s, appearing in the density-dependent mortality term ($-sz^2$), has a significant impact on the stability of the predator–prey system. As (s) increases, the predator population decreases sharply, reaching a lower equilibrium, while the prey population increases slightly due to reduced predation pressure. Mathematically, increasing (s) amplifies the predator's rate of decline at high densities, which can be interpreted ecologically as a natural regulatory mechanism arising from intraspecific competition, limited space, or elevated disease risk in dense predator populations. This results in a more stable system, with smaller oscillations and faster convergence to the equilibrium point. Consequently, the term ($-sz^2$) functions as a damping factor, enhancing long-term ecological stability. However, the predator–prey model employed in this study has certain limitations. The model is deterministic and does not account for stochastic factors such as environmental variability, spatial behaviors, or evolutionary adaptations between predator and prey, all of which can influence real population dynamics. Moreover, the assumption that predator–prey interactions depend solely on population density ignores time delays, age structure, and seasonal effects, which are important in natural ecosystems.

The findings of this study, which indicate that increasing the cannibalism parameter (s) reduces predator population levels while stabilizing system dynamics, are consistent with several recent studies emphasizing the regulatory role of cannibalism in predator–prey interactions. Cannibalism has been widely recognized as a density-dependent mechanism that limits predator overgrowth and mitigates population oscillations. Shishikura & Choh (2024) demonstrated that incorporating cannibalism into predator populations introduces additional nonlinear damping effects that significantly enhance system stability. Similarly, Dai et al. (2025) and Chathuranga et al. (2020) reported that predator cannibalism acts as an internal feedback mechanism that reduces predator density under overcrowded conditions, thereby preventing destabilizing fluctuations. These results strongly support the ecological interpretation of the mortality term $(-sz^2)$ in the proposed model as a stabilizing factor. Moreover, the observed acceleration of convergence toward equilibrium due to increasing cannibalism aligns with theoretical evidence reported in recent predator–prey modeling literature. Li et al. (2020) showed that predator cannibalism reduces excessive predator pressure and leads to more stable coexistence states. Zhao & Shen (2025) further confirmed that cannibalism-driven mortality suppresses oscillatory behavior and produces long-term stabilization of ecological systems. From an ecological perspective, this mechanism reflects intensified intraspecific competition, spatial limitation, and stress-induced mortality among predators at high densities. Therefore, the present results reinforce previous studies by demonstrating that predator cannibalism functions as a natural stabilizing force that regulates population imbalance and promotes sustainable coexistence within predator–prey communities. For future research, the model could be extended by incorporating more complex functional responses (e.g., Holling Type IV or Beddington–DeAngelis), time-delay components, and spatial diffusion approaches to better represent population dispersal. Integrating stochastic processes and empirical data would also enhance the model's realism, making it more representative of actual ecosystem dynamics and potentially applicable in natural resource management and species conservation.

D. CONCLUSION AND SUGGESTIONS

Based on the results of the analysis and simulations, it can be concluded that the predator population dynamics model incorporating cannibalism exhibits seven equilibrium points. These equilibrium points reflect various interspecies interaction conditions within the ecosystem. The three types of equilibria identified are trivial, semi-trivial, and coexistence equilibria. Further analysis was conducted specifically on the coexistence equilibrium $\varepsilon_6(80.38134164, 83.55614705, 2.028162392)$, which demonstrates how the system can converge toward stability. The stability of the system depends on growth parameters, predation rates, and the influence of cannibalism within the predator population. Stability tests using the Routh–Hurwitz criterion indicate that most equilibrium points are locally stable, meaning the system can return to equilibrium after small perturbations. Furthermore, The findings in this study include suggest that cannibalism in predator populations plays a crucial role in regulating population density and maintaining ecological balance. Ecologically, this mechanism prevents excessive predator population outbreaks and supports the long-term sustainability of the ecosystem. In summary, this study not only provides a theoretical understanding of predator–prey system stability with cannibalism but also establishes a foundation for the development of more complex models in the future, incorporating spatial dynamics, stochastic factors, and ecological adaptations.

This study aimed to investigate the dynamics of predator populations exhibiting cannibalistic behavior. The analysis focused on determining the equilibrium points and local stability of the system. Through the predator–prey modeling framework, seven equilibrium points were identified, encompassing trivial, semi-trivial, and coexistence equilibria. Local stability was assessed using the Routh–Hurwitz criterion, based on the principle that species persistence in ecosystems must be maintained over long periods. The analysis indicated that the eigenvalues satisfied the stability conditions according to established principles. Numerical simulations showed that the cannibalism parameter s, appearing in the density-dependent mortality term $(-sz^2)$, has a significant impact on the stability of the predator–prey system. As s increases, the predator population decreases sharply, reaching a lower equilibrium, while the prey population increases slightly due to reduced predation pressure. Mathematically, increasing s amplifies the predator's rate of decline at high densities, which can be interpreted ecologically as a natural regulatory mechanism arising from intraspecific competition, limited space, or elevated disease risk in dense predator populations.

This results in a more stable system, with smaller oscillations and faster convergence to the equilibrium point. Consequently, the term $(-sz^2)$ functions as a damping factor, enhancing long-term ecological stability. However, the predator–prey model employed in this study has certain limitations. The model is deterministic and does not account for stochastic factors such as environmental variability, spatial behaviors, or evolutionary adaptations between predator and prey, all of which can influence real population dynamics. These findings also constitute new research results. Moreover, the assumption that predator–prey interactions depend solely on population density ignores time delays, age structure, and seasonal effects, which are important in natural ecosystems. For future research, the model could be extended by incorporating more complex functional responses (e.g., Holling Type IV or Beddington–DeAngelis), time-delay components, and spatial diffusion approaches to better represent population dispersal. Integrating stochastic processes and empirical data would also enhance the model's realism, making it more representative of actual ecosystem dynamics and potentially applicable in natural resource management and species conservation.

REFERENCES

Aguegboh, N. S., Onyiaji, N., Okeke, C. A., Daniel, N. U., Walter, O., & Diallo, B. (2025). Analysis of a fractional-order prey-predator model with prey refuge and predator harvest using the consumption number: Holling type III functional response. *Computational and Mathematical Biophysics*, *13*(1), 20250023. https://doi.org/10.1515/cmb-2025-0023

Alemu, S. M., Dawed, M. Y., & Mamo, T. T. (2025). Existence of Hydra Effect in a Three-Species Food Chain Mathematical Model With General Holling Type Response Functions. *Natural Resource Modeling*, *38*(4), e70009. https://doi.org/10.1111/nrm.70009

Alsakaji, H. J., Kundu, S., & Rihan, F. A. (2021). Delay differential model of one-predator two-prey system with Monod-Haldane and holling type II functional responses. *Applied Mathematics and Computation*, *397*(5), 125919. https://doi.org/10.1016/j.amc.2020.125919

Bose, A. P. H., Lau, M. J., Cogliati, K. M., Neff, B., & Balshine, S. (2019). Cannibalism of young is related to low paternity and nest take-overs in an intertidal fish. *Animal Behaviour*, *153*(7), 41–48. https://doi.org/10.1016/j.anbehav.2019.04.018

Bose, A. P. H., McClelland, G. B., & Balshine, S. (2016). Cannibalism, competition, and costly care in the plainfin midshipman fish, Porichthys notatus. *Behavioral Ecology*, *27*(2), 628–636. https://doi.org/10.1093/beheco/arv203

Castillo-Alvino, H. H., & Marvá, M. (2022). Group defense promotes coexistence in interference competition: The Holling type IV competitive response. *Mathematics and Computers in Simulation*, *198*(8), 426–445. https://doi.org/10.1016/j.matcom.2022.02.031

Cavassa, D., Postiglioni, R., Aisenberg, A., & Defeo, O. (2022). Relationship between beach morphodynamics and body traits in a sand-dwelling wolf spider. *Acta Oecologica*, *114*(5), 103808. https://doi.org/10.1016/j.actao.2021.103808

Chathuranga, W. G. D., Karunaratne, S. H. P. P., & Silva, W. A. P. P. D. (2020). Predator–prey interactions and the cannibalism of larvae of Armigeres subalbatus (Diptera: Culicidae). *Journal of Asia-Pacific Entomology*, *23*(1), 124–131. https://doi.org/10.1016/j.aspen.2019.11.010

Chowdhury, P. R., Banerjee, M., & Petrovskii, S. (2022). Canards, relaxation oscillations, and pattern formation in a slow-fast ratio-dependent predator-prey system. *Applied Mathematical Modelling*, *109*(9), 519–535. https://doi.org/10.1016/j.apm.2022.04.022

Dai, W., Lu, L., Qiu, H., Zhong, J., Ling, S., Xu, J., Keller, L., & Yan, Z. (2025). Targeted cannibalism of queens mediated by worker signals in fire ants. *Current Biology*, *35*(24), 5931-5937.e4. https://doi.org/10.1016/j.cub.2025.10.044

Jiang, D., Wen, X., & Zhou, B. (2022). Stationary distribution and extinction of a stochastic two-stage model of social insects with egg cannibalism. *Applied Mathematics Letters*, *132*(10), 108100. https://doi.org/10.1016/j.aml.2022.108100

Khajanchi, S. (2017). Modeling the dynamics of stage-structure predator-prey system with Monod–Haldane type response function. *Applied Mathematics and Computation*, *302*(12), 122–143. https://doi.org/10.1016/j.amc.2017.01.019

Kumar, V., & Pramanick, B. (2022). Impact of nanoparticles on the dynamics of a Crowley–Martin type phytoplankton–zooplankton interaction model. *Results in Control and Optimization*, *8*(9), 100139. https://doi.org/10.1016/j.rico.2022.100139

Li, J., Li, F., Gao, H., Zhang, Y., & Liu, Z. (2022). Characterization of cuticular proteins in CPR family in the wolf spider, Pardosa pseudoannulata, and the response of one subfamily genes to environmental stresses. *Insect Biochemistry and Molecular Biology*, *150*(10), 103859. https://doi.org/10.1016/j.ibmb.2022.103859

Li, J., Zhu, X., Lin, X., & Li, J. (2020). Impact of cannibalism on dynamics of a structured predator–prey system. *Applied Mathematical Modelling*, *78*(2), 1–19. https://doi.org/10.1016/j.apm.2019.09.022

Li, N., & Yan, M. (2022). Bifurcation control of a delayed fractional-order prey-predator model with cannibalism and disease. *Physica A: Statistical Mechanics and Its Applications*, *600*(15), 127600. https://doi.org/10.1016/j.physa.2022.127600

Li, Y., Liu, H., & Yang, R. (2018). A delayed diffusive predator–prey system with predator cannibalism. *Computers & Mathematics with Applications*, *75*(5), 1355–1367. https://doi.org/10.1016/j.camwa.2017.11.006

Luo, D., & Wang, Q. (2021). Global dynamics of a Beddington–DeAngelis amensalism system with weak Allee effect on the first species. *Applied Mathematics and Computation*, *408*(11), 126368. https://doi.org/10.1016/j.amc.2021.126368

Madhusudanan, V., Srinivas, M. N., Nwokoye, C. H., Murthy, B. S. N., & Sridhar, S. (2022). HOPF-bifurcation analysis of delayed computer virus model with holling type iii incidence function and treatment. *Scientific African*, *15*(3), e01125. https://doi.org/10.1016/j.sciaf.2022.e01125

Mayntz, D., & Toft, S. (2006). Nutritional value of cannibalism and the role of starvation and nutrient imbalance for cannibalistic tendencies in a generalist predator. *Journal of Animal Ecology*, *75*(1), 288–297. https://doi.org/10.1111/j.1365-2656.2006.01046.x

Pratama, R. A. (2022). Impact Of Fear Behavior On Prey Population Growth Prey-Predator Interaction. *BAREKENG: Jurnal Ilmu Matematika Dan Terapan*, *16*(2), 371–378. https://doi.org/10.30598/barekengvol16iss2pp371-378

Pratama, R. A., Suryani, D. R., Ruslau, M. F. V., Meirista, E., & Nurhayati, N. (2025). Analysis Dynamics Model Predator-Prey with Holling Type III Response Function and Anti-Predator Behavior. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *9*(3), 919. https://doi.org/10.31764/jtam.v9i3.31533

Purnomo, A. S., Darti, I., Suryanto, A., & Kusumawinahyu, W. M. (2025). Dynamical Analysis of Discrete-Time Modified Leslie-Gower Predator-Prey with Fear Effect. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *9*(1), 24. https://doi.org/10.31764/jtam.v9i1.26515

Shao, Y. (2022). Global stability of a delayed predator–prey system with fear and Holling-type II functional response in deterministic and stochastic environments. *Mathematics and Computers in Simulation*, *200*(10), 65–77. https://doi.org/10.1016/j.matcom.2022.04.013

Shishikura, S., & Choh, Y. (2024). Adult females and larvae of the predatory mite Gynaeseius liturivorus avoid cannibalism via kin recognition. *Animal Behaviour*, *211*, 35–41. https://doi.org/10.1016/j.anbehav.2024.02.021

Takegaki, T., Nakatake, Y., Matsumoto, Y., Suga, K., & Amiya, N. (2023). Early Filial Cannibalism in Fish Revisited: Endocrinological Constraint, Costs of Parental Care, and Mating Possibility. *The American Naturalist*, *201*(6), 841–850. https://doi.org/10.1086/724284

Zhang, X., & Yuan, R. (2021). A stochastic chemostat model with mean-reverting Ornstein-Uhlenbeck process and Monod-Haldane response function. *Applied Mathematics and Computation*, *394*(8), 125833. https://doi.org/10.1016/j.amc.2020.125833

Zhao, Z., & Shen, Y. (2025). Dynamic complexity of Holling-Tanner predator–prey system with predator cannibalism. *Mathematics and Computers in Simulation*, *232*, 227–244. https://doi.org/10.1016/j.matcom.2024.12.025

Zhou, X., Zhang, L., Zheng, T., Li, H.-L., & Teng, Z. (2021). Global stability for a delayed HIV reactivation model with latent infection and Beddington–DeAngelis incidence. *Applied Mathematics Letters*, *117*(7), 107047. https://doi.org/10.1016/j.aml.2021.107047