JTAM (Jurnal Teori dan Aplikasi Matematika)
http://journal.ummat.ac.id/index.php/jtam
p-ISSN 2597-7512 | e-ISSN 2614-1175
Vol. 10, No. 2, April 2026, pp. 396-411

# Forecasting Indonesia's Export Revenue through a Vector Autoregressive Exogenous Approach

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**Article History:**
Received : 28-11-2025
Revised : 26-01-2026
Accepted : 27-01-2026
Online : 01-04-2026

**Keywords:**
Exchange Rate;
Export;
Forecasting;
Modeling;
VARX.

**ABSTRACT**

The Vector Autoregressive with Exogenous Variables (VARX) model extends the conventional VAR framework by explicitly incorporating external macroeconomic drivers, offering a more structurally informed approach to export forecasting. This study contributes to the literature by introducing a disaggregated modeling strategy that treats oil and gas exports and non-oil and gas exports as separate endogenous components, an aspect that has been largely overlooked in previous studies on Indonesia's export performance. By positioning VARX as a system-based forecasting tool rather than a purely statistical extension, this research provides an updated methodological perspective on export revenue analysis. Using monthly data from January 2015 to December 2024, this study evaluates several VARX specifications that integrate the rupiah–US dollar exchange rate and West Texas Intermediate (WTI) crude oil prices as exogenous variables. Model selection is conducted based on a combination of information criteria and forecasting performance indicators, leading to the identification of VARX(5,6) as the most suitable specification. The inclusion of exogenous variables is shown to substantially enhance predictive accuracy, confirming the relevance of external economic shocks in shaping Indonesia's export revenue dynamics. Empirical results indicate that WTI oil prices exert a significant causal influence on export revenue, while the exchange rate effect becomes meaningful when jointly evaluated with oil prices and endogenous export components. The selected VARX(5,6) model demonstrates strong forecasting performance, achieving a MAPE of 5.60% and an nRMSE of 6.40%. From a policy standpoint, these findings suggest that export planning and stabilization policies should explicitly account for global oil price volatility and exchange rate interactions. The proposed VARX framework can therefore serve as a practical analytical tool for policymakers to anticipate short-term export fluctuations and design responsive trade and macroeconomic strategies under external uncertainty.

https://doi.org/10.31764/jtam.v10i2.36760



## A. INTRODUCTION

Time series data refers to a sequence of observations collected at regular and consistent time intervals. Its distinctive feature lies in dependency between consecutive observations over time, which enables the identification of underlying patterns (Box, 2016), making it a fundamental tool for forecasting in finance and economics (Wei, 2006). A key element in developing a forecasting model for time series is the concept of stationarity. However, many real-world time series exhibit non-stationary trends, necessitating robust modelling approaches, one of which is the Autoregressive Integrated Moving Average (ARIMA) model (Xu, 2024; Jadhav, 2017). But, this models only uses information from its own past data (univariate), so it is less than optimal because it ignores external factors that influence real-world data. To overcome this, exogenous variables are added to obtain the Autoregressive Integrated Moving Average Exogenous (ARIMAX) model (Paul, 2015). An exogenous variable is a variable that influences another variable but is not influenced by other variables within the model. The role of this variable is to provide additional information to improve forecasting accuracy and has been proven by (Akanbi & Bello, 2024; Yulianti & Effendie, 2024).

However, ARIMAX model only capture one-way relationships from exogenous to endogenous variables (Haydier et al., 2023). In reality, especially in economics, variables often interact and influence each other simultaneously over time. For this reason, the Vector Autoregressive (VAR) model was developed as a multivariate approach that combines several autoregressive models of interrelated variables (Setiawan et al., 2024). VAR is effective in capturing correlations and mutual influences among variables, but it also has weaknesses because fully endogenous and closed system (all variables within the system affect one another, without considering external factors). In practice, however, many exogenous variables significantly influence endogenous variables without being affected in return. Neglecting these exogenous factors may lead to omitted variable bias, causing inaccuracies in parameter estimation and reducing the reliability of forecasts.

To address the limitations of VAR, exogenous variables were incorporated, resulting in the Vector Autoregressive Exogenous (VARX) model (Belomestny et al., 2021). By including external factors, VARX improves the ability to explain changes in endogenous variables and reduces the need for long lag structures, thus avoiding unnecessary complexity. However, VARX does not automatically ensure stationarity, so testing and, if necessary, data transformation or differencing must be conducted first. Empirical studies highlight the model's strong forecasting performance. For instance, Setiawan et al. (2024) found that VARX(1,1) yielded high accuracy in predicting money circulation, quasi-money, and securities, while Slatina et al. (2023) demonstrated its effectiveness for forecasting IHSG and JII by including Brent oil prices. These studies confirm that VARX is reliable for real-world applications and suitable for modeling multiple variables, making it highly relevant for forecasting Indonesia's export revenues.

Exports, as a form of international trade, involve delivering goods and services from one country to another and play a vital role in generating foreign exchange to support national income and economic growth (Ee, 2016). They are generally classified into oil and gas exports-such as crude oil and natural gas, and non-oil and gas exports, including coal, palm oil, and other commodities (Ministry of Trade of the Republic of Indonesia., 2025). Although exports play a vital role in the economy, their revenues often fluctuate due to external factors such as exchange rates and global crude oil prices (Nicholson, 2017). The exchange rate reflects the value of the domestic currency against foreign currencies, with the US dollar being the main medium of trade (Paul, 2015). Crude oil, as a major global commodity, is benchmarked mainly by Brent and West Texas Intermediate (WTI), where WTI significantly influences global oil prices and thereby impacts trade and export revenues (Filippidis et al., 2023).

Empirical studies show the effectiveness of the VARX model in forecasting exports, such as VARX(2,2), which produced accurate results (MAPE 5.94%) for Indonesia's total exports when including the Jakarta Composite Index (Belomestny et al., 2021). However, since oil and gas exports and non-oil and gas exports have different dynamics and determinants, separating them provides more accurate forecasts. Therefore, this study applies the VARX model by distinguishing these two categories while considering the exchange rate and WTI oil prices, with the goal of producing more precise forecasts and identifying key influencing factors.

## B. METHODS

This study uses secondary data from January 2015 to December 2024. The endogenous variables include Indonesia's oil and gas export revenues $\left(Z_{(1,t)}\right)$ and non-oil and gas exports $\left(Z_{(2,t)}\right)$, both sourced from the Ministry of Trade of the Republic of Indonesia in US$ millions. The exogenous variables include the rupiah exchange rate against the US dollar in USD/IDR $\left(X_{(1,t)}\right)$ (also sourced from the Ministry of Trade of the Republic of Indonesia) and the price of West Texas Intermediate (WTI) crude oil in dollars/barrel $\left(X_{(2,t)}\right)$ from the World Bank Group. The stages of research conducted in this article are as follows:

1. Collect data.
2. Splitting the training and test data with a 90:10 ratio from the total data obtained. Data from January 2015 to December 2023 served as training data, and data from January 2024 to December 2024 served as test data.
3. Explore data observation. By examining time series plots, one can identify important patterns such as trends, seasonality, and cyclical fluctuations.
4. Testing the level of correlation between exogenous variables using a multicollinearity test with the Variance Inflation Factor (VIF) test (Santi, 2023). If $VIF > 10$, the data has a very strong relationship between exogenous variables (high multicollinearity).
5. Testing and modifying stationarity with respect to variance using the Box-Cox transformation. If $\lambda \neq 1$, the data is not stationary in variance (Wei, 2006).
6. Testing and modifying stationarity with respect to mean using the Augmented Dicky Fuller (ADF) and modifying stationarity with differencing (if the data is not stationary). Hypothesis testing using the ADF test is (Belomestny et al., 2021), $H_0 : \gamma = 0$ (not stationer) versus $H_1 : \gamma \neq 0$ (stationer) . If $|ADF_{test}| > \left|t_{(0.05;n)}\right|$ , then $H_0$ rejected so it can be stated that the data is stationary.
7. Determining the optimal maximum lag length. According to Lutkepohl and Saikkonen (1999), this stage is optional with the aim of limiting the optimal lag search space and adjusting it to the amount of data, where n is total observation data (Alnashwan, 2017).
8. Model specification and VAR parameter estimation (Alnashwan, 2017), the VAR model is:

$$\boldsymbol{Z}_t = \boldsymbol{\beta} + \boldsymbol{\phi}_1 \boldsymbol{Z}_{t-1} + \boldsymbol{\phi}_2 \boldsymbol{Z}_{t-2} + \ldots + \boldsymbol{\phi}_p \boldsymbol{Z}_{t-p} + \boldsymbol{a}_t \quad (1)$$

where $Z_{j,t}$ is data for the $j$-th endogenous variable at time t with $j = 1,2,\ldots,k$ and $t = 1,2,..,n$ . $Z_{j,t-i}$ is data for the $j$-th endogenous variable at time t− with $j = 1,2,\ldots,k$; $t = 1,2,..,n$ and $i = 1,2,\ldots,p$. $\phi_{jj(i)}$ is parameter coefficients of the j-th endogenous variable equation for the $j$-th variable at lag $i$ with $i = 1,2,\ldots,p$. $a_{j,t}$ is error value at time t. The VAR parameter estimation in this study uses the ordinary least squares (OLS) method with the following equation:

$$\widehat{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\,\boldsymbol{X}\right)^{T}\boldsymbol{X}'\,\boldsymbol{Y} \tag{2}$$

9. Determining the optimal lag of the VAR model to obtain the best temporary model can be done using one of the model selection criteria, namely the Akaike Information Criterion (AIC) (Setiawan et al., 2024).
10. Conducting stability tests on the model obtained by utilizing the endogenous lag coefficient value. That value is substituted into the companion matrix by (Box, 2016):

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 & \cdots & \cdots & \boldsymbol{\phi}_p \\ \boldsymbol{I}_k & 0 & \cdots & \cdots & 0 \\ 0 & \boldsymbol{I}_k & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{I}_k & 0 \end{bmatrix} \tag{3}$$

once the matrix is determined, the next step is to calculate the determinant $|\boldsymbol{C} - \lambda\boldsymbol{I}| = 0$ to obtain the characteristic equation that will be solved to find the value of λ. If all $|\lambda_n| < 1$, then a model is said to be stable as a whole (Eshete, 2018).
11. Model residual fit testing, includes the following assumptions:
    a. White noise residual assumption using the Portmanteau Multivariate Test with the hypothesis (Sutthichaimethee, 2020), $H_0 : \rho_1 = \rho_2 = \cdots = \rho_h = 0$ (white noise) versus $H_1$ : Min $\exists\ \rho_k \neq 0$ with $k = 1,2,\ldots,h$ (not white noise) with test statistics as follows:

$$Q_h = n\sum_{j=1}^{h} tr\left(\widehat{\boldsymbol{C}}_j'\ \widehat{\boldsymbol{C}}_0^{-1}\widehat{\boldsymbol{C}}_j\,\widehat{\boldsymbol{C}}_0^{-1}\right) \tag{4}$$

    where $n$ is the number of samples, $h$ is the number of lags, $\widehat{\boldsymbol{C}}_j$ is the covariance matrix of the residual vector estimate at time $t$ ($\widehat{u_t}$), and tr( ) is the trace operator. If $Q_h \geq X^2_{K^2(h-p);a}$ then $H_0$ rejected so it can be stated residual isn't white noise.

    b. Multivariate normality assumption using the Q-Q plot correlation coefficient test White noise residual assumption using the Portmanteau Test with the hypothesis (Setiawan et al., 2024), $H_0$ : multivariate normally distributed residuals and $H_1$ : multivariate non-normally distributed residuals, with test statistics as follows:

$$r_Q = \frac{\sum_{t=1}^{n}(d_t^2 - \bar{d}^2)(q_t - \bar{q_t})}{\sqrt{\sum_{t=1}^{n}\left(d_t^2 - \bar{d}^2\right)^2 \sum_{t=1}^{n}(q_t - \bar{q_t})^2}} \tag{5}$$

where $r_Q$ is the correlation value between the chi square quantile values $\left(q_t = X^2_{k,\frac{n-t+0.5}{n}}\right)$ and the mahalanobis distance of each variable $\left(d_t^2 = \left(x_{t,j} - \overline{x}_j\right)^T \times \boldsymbol{S}^{-1} \times \left(x_{t,j} - \overline{x}_j\right)\right)$ with $x_{t,j}$ is the $t-th$ observation value for the $j-th$ variable $(j = 1,2,..,k)$ and $\boldsymbol{S}^{-1}$ is the inverse covariance matrix of the observed values. If $r_Q \geq r_{Q_{a;n}}$ then $H_1$ rejected so it can be stated residual are distributed normally multivariate.

12. The significance of the parameters testing, it can be tested partially and simultaneously with the following hypotheses and equation:
   a. Significance test of partial parameters. This test is measured using a t-test with the following hypothesis (Slatina et al., 2023).

**Table 1.** Hypothesis Testing Significance of Partial Parameters

| Endogenous Parameters | Exogenous Parameters |
|---|---|
| $H_1 : \phi \neq 0$ (significant) | $H_1 : \theta \neq 0$ (significant) |
| $H_0 : \phi = 0$ (not significant) | $H_0 : \theta = 0$ (not significant) |

   If $|t_{test}| \geq t_{\frac{a}{2};(n-b)}$, then $H_0$ rejected so it can be stated that the parameters are significant.
   b. Significance test of simultaneous parameters. This test is measured using Granger causality with the hypothesis (Akal, 2015), $H_0 : \phi_p \neq 0$ versus $H_1 : \phi_1 = \phi_2 = \cdots = \phi_k = 0$. If $|F_{test}| \geq F_{a;p;n-b}$, then $H_0$ rejected so it can be stated independent variables cause the dependent variable.

13. VARX model specification and parameter estimation. According to (Sukono and Ibrahim, 2023), the general form of the VARX model is:

$$\boldsymbol{Z_t} = \boldsymbol{\beta} + \boldsymbol{\phi}_1 \boldsymbol{Z}_{t-1} + \boldsymbol{\phi}_2 \boldsymbol{Z}_{t-2} + \ldots + \boldsymbol{\phi}_p \boldsymbol{Z}_{t-p} + \boldsymbol{\theta}_1 \boldsymbol{X}_{t-1} + \ldots + \boldsymbol{\theta}_q \boldsymbol{X}_{t-q} + \boldsymbol{a}_t \tag{6}$$

   The VARX parameter estimation in this study uses the OLS method with Equation (6).
14. Determining the optimum VARX lag length with Akaike Information Criterion (AIC) with the equation (Nicholson, 2017):

$$AIC(p,q) = ln\left(\left|\overline{\Sigma(\boldsymbol{p})}\right|\right) + \frac{2k^2p + kmq}{n} \tag{7}$$

   where $\overline{\Sigma(\boldsymbol{p})} = \frac{1}{n}\sum_{t=1}^{n} \overline{\boldsymbol{u}}_t |\overline{\boldsymbol{u}_t}|'$ is the residual covariance estimation matrix of the VARX(p,q) model estimation, $\overline{u_t}$ is residuals at time $t$ for the VARX(p,q) model, where $n$ is the number of observations, and $m$ is the number of exogenous variables in the model. The lowest AIC value will be selected as the best model.

15. Perform a stability test on the VARX model as in step 10 using only the endogenous coefficients and repeat step 11 and 12 on the VARX model that has been obtained.
16. Forecasting the test data and the next period using the selected VARX model.
17. Testing the forecast accuracy of the VARX model using MAPE and nRMSE calculations. According to (Tsay, 2005), the MAPE equation is:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Z_t - F_t}{Z_t}\right| \times 100\% \tag{8}$$

where $Z_t$ is actual data values, $F_t$ is forecasting data values, and n is amount of data. Then, according to (Sutthichaimethee, 2020; Woodward, 2022), the RMSE equation is:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Z_t - F_t)^2} \tag{9}$$

to assess the accuracy of the RMSE obtained, it is necessary to normalize it to the average so that the nRMSE equation is obtained (Shcherbakov, 2013):

$$nRMSE = \frac{RMSE}{\bar{Z}_t} \times 100\% \tag{10}$$

## C. RESULT AND DISCUSSION

### 1. Data Exploration

Before beginning the time series modelling, we explored oil and gas and non-oil and gas export revenue data to identify patterns and characteristics. The time series data plots for oil and non-oil and gas export revenue are presented in Figure 1 and Figure 2. Figure 1 indicates that the oil and gas export revenue data is estimated to be non-stationary with respect to the mean, as the data does not fluctuate consistently around a single mean line ($\mu_1 = 1205.7$), but instead exhibits a clear trend (both upward and downward). Furthermore, the plot also shows indications of non-stationarity with respect to the variance, characterized by inconsistent fluctuation patterns in amplitude over time.

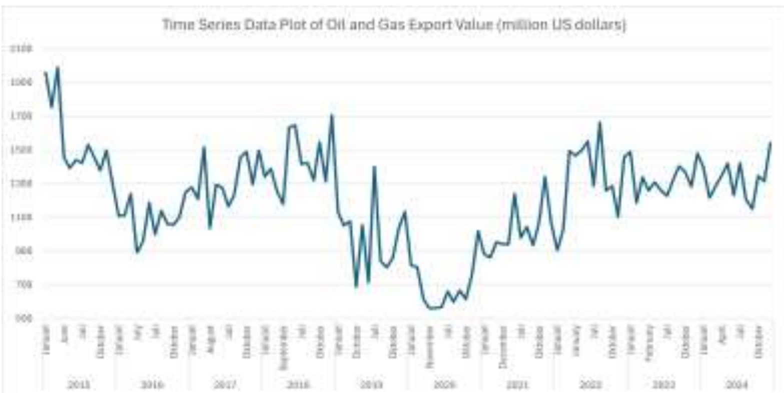


**Figure 1**. Time Series Data Plot of Oil and Gas Export Value

Figure 2 indicates non-stationarity with respect to the mean. The data does not fluctuate consistently around the mean ($\mu_2 = 15646.5$), but instead exhibits a trend that tends to increase in some periods. The plot also indicates non-stationarity to the variance.

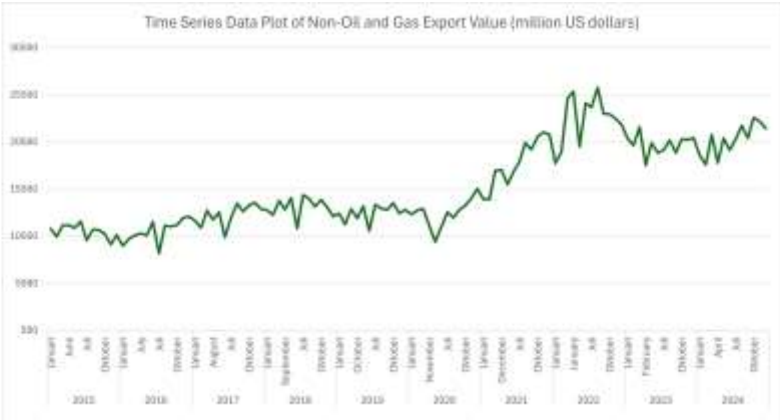


**Figure 2**. Time Series Data Plot of Non-Oil and Gas Export Value

## 2. Multicollinearity Test

To identify the presence of multicollinearity that could potentially cause a high correlation between the US Dollar exchange rate against the Rupiah ($X_{1,t}$) and the WTI oil price ($X_{2,t}$), a multicollinearity test was conducted by determining the VIF value. The coefficient of determination is $r_{x_1,x_2} = 0.18445$ with a VIF value of 1.035, which shows that there is no high multicollinearity (<10) between the exogenous variables.

## 3. Stationary Test

Based on the Box-Cox test, the optimal lambda values obtained are 1, -1, -1.5, and 0. The treatment of variables must be the same so that if each $\lambda$ value obtained is not close to 1, it can be assumed that each variable is stationary with respect to variance (Belomestny et al., 2021). Then, a stationarity test was performed on each variable using the ADF test, as shown in Table 2.

**Table 2.** ADF Test Results of Observation Data

| Variable | $ADF_{test}$ | $t_{(0.05;106)}$ | Description |
|---|---|---|---|
| $Z_{(1,t)}$ | -2.78 | -3.45 | Failed to reject $H_0$ |
| $Z_{(2,t)}$ | -2.43 | -3.45 | Failed to reject $H_0$ |
| $X_{(1,t)}$ | -4.76 | -3.45 | Reject $H_0$ |
| $X_{(2,t)}$ | -2.95 | -3.45 | Failed to reject $H_0$ |

The ADF test results on Table 2 indicate that there are variables with $|ADF_{test}| < |t_{(0.05;106)}|$ meaning these variables are not stationary with respect to the mean. Therefore, a first-order differencing process is required and the ADF test is repeated.

**Table 3.** ADF Test Results of First-Order Differencing Data

| Variable | $ADF_{test}$ | $t_{(0.05;106)}$ | Description |
|---|---|---|---|
| $Z_{(1,t)}$ | -17.18 | -3.45 | Reject $H_0$ |
| $Z_{(2,t)}$ | -9.27 | -3.45 | Reject $H_0$ |
| $X_{(1,t)}$ | -11.83 | -3.45 | Reject $H_0$ |
| $X_{(2,t)}$ | -7.96 | -3.45 | Reject $H_0$ |

The Table 3 shows that all variables have a $|ADF_{test}| > |t_{(0.05;106)}|$, thus proving that all variables are stationary with respect to the mean after first-order differencing. Therefore, the data used for modeling will be first-order differencing data.

## 4. VARX Modelling

The VARX model development process begins with determining the optimal lag (p) for the endogenous variables, as the model involves two different lags (p for endogenous and q for exogenous). The first stage involves constructing a regular VAR model and selecting the best candidate for lag p based on information criteria, such as the AIC. The selected model with lag p then undergoes a comprehensive evaluation—including stability tests and diagnostic tests to confirm its validity and stability. If all diagnostic tests are successfully passed, the process continues with determining the optimal lag (q), which is also accompanied by statistical testing, resulting in a VARX model that is overall valid and stable.

a. VAR Modelling

Before building a VAR model, it is necessary to determine the maximum lag limit to narrow the search for the optimal lag and adjust it to the amount of data. Based on Equation (4), the maximum lag limit set is 5. Therefore, the modeling and parameter estimation process will be carried out for VAR models with lags (p) ranging from 1 to 5. Model specifications are carried out using Equation (1). Next, the parameter estimation process is carried out using Equation (2) to obtain models VAR(1) to VAR(5). Furthemore, the residual values are determined to calculate each model's residual covariance matrix. This covariance matrix then becomes the basis for calculating the AIC value of each model. The AIC values obtained for each model, as shown in Table 4.

**Table 4.** AIC Value VAR(p)

| $p$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **AIC** | 25.162 | 25.136 | 25.097 | 25.168 | 25.164 |

Based on Table 4, the lowest AIC value is at lag 3, making VAR(3) the tentative best model candidate. However, further testing of the stability and fit of the model shows that although the VAR(3) model is stable, the multivariate Portmanteau test at all lags rejects $H_0$. This indicates that the residuals do not meet the white noise assumption. In addition, the correlation coefficient test also shows a non-normal distribution. Therefore, despite its superior AIC, the VAR(3) model is not optimal and needs to be re-evaluated by considering other lags.

The testing results indicate that all VAR models with different lag lengths are stable, confirming their feasibility for further analysis. However, only the model with lag 5 satisfies the white noise assumption for the residuals, whereas models with lower lags still exhibit autocorrelation. Although the correlation coefficient test shows that the residuals are not normally distributed, this limitation can be disregarded since normality is not a strict requirement in VAR modeling. Therefore, the VAR(5) model is considered the most appropriate and reliable for use in analysis and forecasting.. According to Armstrong (2007), a model's forecasting ability is its most critical attribute. Consequently, the VAR(5) model is the optimal choice and will be used for the subsequent VARX(p,q) modeling.

Before proceeding to VARX modeling, the VAR(5) model was tested for parameter significance to see the contribution of parameters in the model and the correlation between variables. Based on the partial parameter significance test, there were 7 significant parameters: $\phi_{11}^{(1)}, \phi_{11}^{(2)}, \phi_{11}^{(3)}$ , $\phi_{22}^{(1)}, \phi_{22}^{(2)}, \phi_{12}^{(2)}$, and $\phi_{21}^{(5)}$ where $\phi_{12}^{(2)}$, and $\phi_{21}^{(5)}$ indicating a correlation between the two endogenous variables: parameter $\phi_{12}^{(2)}$ which means $Z_{2,t}$ at lag-2 (the previous 2 periods) has a significant influence on $Z_{1,t}$, and $\phi_{21}^{(5)}$ which means $Z_{1,t}$ at lag-5 has a significant influence on $Z_{2,t}$. But, based on the results of the Simultaneous Parameter Significance test, it can be seen that each F count has $|F_{test}| < F_{a;p;n-b}$. The results indicate that there is neither a one-way nor two-way causal relationship between the two variables, making the predictive power of the VAR(5) model weak, so it has limitations in fully explaining the dynamics of the two endogenous variables and cannot effectively predict one based on the other. To improve predictive accuracy, the inclusion of exogenous variables is required.

b. VARX Modelling

After obtaining the best VAR(p) model, namely VAR(5), the next step is to expand the model by incorporating exogenous variables into the VAR(5) model to produce the VARX(5,q) model. Using a similar process, the VARX order is limited to the 5th order. Because all orders (from 0 to 5) do not exhibit white noise, the appropriate step is to increase the order constraint, so that in the VARX(5,q) model, the order to be observed is limited to the 6th order. Similarly, with the VAR model, the VARX(5,6) model is first specified using Equation (6). Then, the parameter estimation process is carried out using the OLS method to obtain the VARX(5,6) model for the oil and gas ($Z_{1,t}$) and non-oil and gas export values ($Z_{2,t}$) as endogenous variables, along with the rupiah exchange rate against the US dollar ($X_{1,t}$) and the WTI oil price ($X_{2,t}$) as exogenous variables:

$$
\begin{aligned}
Z_{1,t} = -18.814 &- 0.712\, Z_{1,t-1} - 0.521\, Z_{1,t-2} - 0.394\, Z_{1,t-3} - 0.191\, Z_{1,t-4} + 0.021\, Z_{1,t-5} \\
&- 0.012\, Z_{2,t-1} + 0.014\, Z_{2,t-2} + 0.004\, Z_{2,t-3} - 0.002\, Z_{2,t-4} - 0.002\, Z_{2,t-5} \\
&- 0.064\, X_{1,t-1} + 0.074\, X_{1,t-2} + 0.111 X_{1,t-3} + 0.107\, X_{1,t-4} + 0.039\, X_{1,t-5} \\
&+ 0.030\, X_{1,t-6} + 6.953\, X_{2,t-1} + 11.272\, X_{2,t-2} + 8.332\, X_{2,t-3} - 0.550\, X_{2,t-4} \\
&+ 9.909\, X_{2,t-5} - 1.084\, X_{1,t-6} + a_{1,t}
\end{aligned}
$$

$$
\begin{aligned}
Z_{2,t} = {} & 172.425 - 1.549\, Z_{1,t-1} - 0.733\, Z_{1,t-2} - 0.786\, Z_{1,t-3} - 0.859\, Z_{1,t-4} + 1.136\, Z_{1,t-5} \\
& - 0.661\, Z_{2,t-1} - 0.550\, Z_{2,t-2} - 0.396\, Z_{2,t-3} - 0.068\, Z_{2,t-4} - 0.080\, Z_{2,t-5} \\
& + 0.311\, X_{1,t-1} - 0.853\, X_{1,t-2} + 0.061 X_{1,t-3} - 0.164\, X_{1,t-4} - 0.022\, X_{1,t-5} \\
& + 0.421\, X_{1,t-6} + 102.138\, X_{2,t-1} + 27.410\, X_{2,t-2} + 92.453\, X_{2,t-3} \\
& + 11.571\, X_{2,t-4} + 74.306\, X_{2,t-5} + 55.779\, X_{1,t-6} + a_{2,t}
\end{aligned}
$$

The specification and modeling process is carried out similarly at lags 0 to 5. After obtaining these models, the residual values are determined to calculate the residual covariance matrix of each model. This covariance matrix then becomes the basis for calculating the AIC value of each model. Thus, the AIC value obtained for each VARX model with Equation (7), as shown in Table 5.

**Table 5.** AIC Value VARX(5,q)

| $p$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $AIC$ | 25.164 | 24.994 | 24.961 | 24.959 | 24.990 | 24.987 | 24.984 |

Based on Table 5, the lowest AIC value is at lag $q = 3$, making VARX(5,3) the tentative best model candidate. However, further testing of the stability and fit of the model shows that although the VARX(5,3) model is stable and normal distribution, the multivariate Portmanteau test at all lags rejects $H_0$. This indicates that the residuals do not meet the white noise assumption. Therefore, despite its superior AIC, the VARX(5,3) model is not optimal and needs to be re-evaluated by considering other lags. The complete results of the stability and fit tests for all lags are presented in Table 6.

**Table 6.** Stability and Model Suitability Test Results VARX(5,q)

| $q$ | Stability Test | Portmenteau Test | Correlation Coefficient Test |
|---|---|---|---|
| 0 | Stable | No White Noise | Normally Distributed |
| 1 | Stable | No White Noise | Normally Distributed |
| 2 | Stable | No White Noise | Normally Distributed |
| 3 | Stable | No White Noise | Normally Distributed |
| 4 | Stable | No White Noise | Normally Distributed |
| 5 | Stable | No White Noise | Normally Distributed |
| 6 | Stable | White Noise | Normally Distributed |

Table 6 shows that VARX(5,6) meets the stability test and model fit assumptions. Thus, the VARX(5,6) model is the best model and can be used for forecasting.

## 5. Stability Test VARX Model

This test is used to ensure that the VARX(5,6) model does not explode. If a model explodes, the prediction results will experience extreme fluctuations (increasing or decreasing indefinitely). The stability test in this study uses the eigenvalue modulus, which begins by substituting the endogenous lag coefficient value of VARX(5,6) into the matrix in Equation (3) as follows:

$$C=\begin{bmatrix} -0.7 & -0.01 & 0.5 & 0.01 & -0.4 & 0.004 & -0.2 & -0.002 & -0.02 & -0.002 \\ -1.6 & -0.66 & -0.7 & -0.55 & -0.8 & -0.34 & -0.9 & -0.07 & 1.14 & -0.08 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}_{10\times 10}$$

Furthermore, calculate the determinant $|C-\lambda I|=0$ to obtain the characteristic equation. The values of $\lambda$ and its modulus are obtained, which are presented in Table 7.

**Table 7.** The Stability Test Results on the VARX(5,6)

| No | $\lambda$ | $\lvert\lambda\rvert$ |
|---|---|---|
| 1 | $0.237 - 0.716i$ | 0.754 |
| 2 | $0.237 + 0.716i$ | 0.754 |
| 3 | $-0.087 - 0.731i$ | 0.736 |
| 4 | $-0.087 + 0.731i$ | 0.736 |
| 5 | $-0.623 - 0.301i$ | 0.692 |
| 6 | $-0.623 + 0.301i$ | 0.692 |
| 7 | $-0.6440$ | 0.644 |
| 8 | $0.136 - 0.350i$ | 0.375 |
| 9 | $0.136 + 0.350i$ | 0.375 |
| 10 | $-0.054$ | 0.054 |

Based on Table 7, it can be seen that all $|\lambda|$ have values $<1$, thus the VARX(5,6) model is proven to be stable.

## 6. Diagnostic Test VARX Model

a. White Noise Test

The purpose of testing the residual white noise assumption is to determine whether there is a correlation between the residual vectors from the VARX(5,6) model that has been formed. If there is a correlation, the model is considered inadequate and needs to be improved. Based on Equation (4), the results of the portmenteau test on the residuals of the VARX(5,6) model, as shown in Table 8.

**Table 8.** Multivariate Portmenteau Tests Results on VARX(5,6)

| h | $Q_h$ | $df$ | $X^2$ Critical | Description |
|---|---|---|---|---|
| 6 | 5.665 | 4 | 9.49 | $H_0$ is accepted (white noise) |
| 7 | 10.366 | 8 | 15.51 | $H_0$ is accepted (white noise) |
| 8 | 11.951 | 12 | 21.03 | $H_0$ is accepted (white noise) |
| 9 | 20.662 | 16 | 26.30 | $H_0$ is accepted (white noise) |
| 10 | 28.388 | 20 | 31.41 | $H_0$ is accepted (white noise) |
| 11 | 33.208 | 24 | 36.41 | $H_0$ is accepted (white noise) |

Based on Table 8, $H_0$ is accepted for each lag because the $Q_h \geq X^2_{4(h-5);0,05}$. It is proven that the residuals VARX(5,6) models comply the white noise assumption.

b. Normally Distribution Test
The purpose of this test is to see whether the residuals of the VARX(5,6) model meet the normality assumption. The Q-Q plot correlation coefficient test is used with Equation (5). Based on this equation, the correlation value between the chi-square quantile and the Mahalanobis distance is $r_Q = 0.9892$ which is greater than $r_{Q_{(0.05;101)}} = 0.9876$, which means $H_0$ is not rejected. Therefore, it can be proven that the residuals are multivariately normally distributed.

c. Partial Parameter Significance Test
Based The purpose of this test is to ensure that the parameter coefficients in the VARX(5,6) model obtained contribute to the model. This test uses the t-test for endogenous and exogenous parameters. The results of the partial significance test show that there are 15 significant parameters: parameters $\phi_{11}^{(1)}, \phi_{11}^{(2)}, \phi_{11}^{(3)}$, which means that variable $Z_{1,t}$ is influenced by its past values, where lags 1, 2, and 3 represent (1, 2, and 3 previous periods). Similarly, parameters $\phi_{22}^{(1)}, \phi_{22}^{(2)}, \phi_{22}^{(3)}$ mean that $Z_{2,t}$ is influenced by its past values with lags 1, 2, and 3. Furthermore, exogenous variables also contribute, such as parameters $\theta_{12}^{(1)}, \theta_{12}^{(2)}, \theta_{12}^{(3)}, \theta_{12}^{(5)}$, which means that variable $X_{2,t}$ influences $Z_{1,t}$ at lag-1, 2, and 5. In addition, the parameters $\theta_{22}^{(1)}, \theta_{22}^{(3)}, \theta_{22}^{(5)}, \theta_{22}^{(6)}$ indicate that the variable $X_{2,t}$ influences $Z_{2,t}$ at lag- 1, 3, 5, and 6. Furthermore, the parameter $\theta_{21}^{(2)}$ indicates that the variable $X_{1,t}$ influences$Z_{2,t}$ at lag-2.

d. Simultaneous Parameter Significance Test
This test is used to determine the relationship between variables, such as the relationship between endogenous variables or between endogenous variables and exogenous variables, both separately and simultanously. This test uses the Granger causality test and the results are presented in Table 9.

**Table 9.** Results Significance Test Parameter Simultaneous VARX(5,6)

| Variable | | $F_{Stat}$ | df | $F_{0.05;5;102-b}$ | Description |
|---|---|---|---|---|---|
| **Dependent** | **Independent** | | | | |
| $Z_{1,t}$ | $Z_{2,t}$ | 0.677 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{1,t}$ | 1.847 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{2,t}$ | 4.410 | 5 | 2.323 | Reject $H_0$ |
| | $Z_{2,t}, X_{1,t}, X_{2,t}$ | 2.489 | 15 | 1,769 | Reject $H_0$ |
| $Z_{2,t}$ | $Z_{1,t}$ | 1.519 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{1,t}$ | 1.335 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{2,t}$ | 5.078 | 5 | 2.323 | Reject $H_0$ |
| | $Z_{1,t}, X_{1,t}, X_{2,t}$ | 3.450 | 15 | 1,769 | Reject $H_0$ |
| $X_{1,t}$ | $Z_{1,t}$ | 0.543 | 5 | 2.323 | Failed to reject $H_0$ |
| | $Z_{2,t}$ | 0.571 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{2,t}$ | 0.648 | 5 | 2.323 | Failed to reject $H_0$ |
| | $Z_{1,t}, Z_{2,t}, X_{2,t}$ | 0.577 | 15 | 1,769 | Failed to reject $H_0$ |
| $X_{2,t}$ | $Z_{1,t}$ | 0.539 | 5 | 2.323 | Failed to reject $H_0$ |
| | $Z_{2,t}$ | 1.861 | 5 | 2.323 | Failed to reject $H_0$ |
| | $X_{1,t}$ | 1.240 | 5 | 2.323 | Failed to reject $H_0$ |
| | $Z_{1,t}, Z_{2,t}, X_{1,t}$ | 1.296 | 15 | 1,769 | Failed to reject $H_0$ |

Based on the analysis results, a one-way individual Granger causality relationship is evident from the exogenous variable $X_{2,t}$ to the endogenous variables $Z_{1,t}$ and $Z_{2,t}$. This indicates that the WTI oil price variable ($X_{2,t}$) individually influences the movement of both export revenue variables in Indonesia ($Z_{1,t}$ and $Z_{2,t}$). Conversely, the dollar exchange rate against the rupiah ($X_{1,t}$) individually does not show a one-way causal relationship with the two endogenous variables ($Z_{1,t}$ and $Z_{2,t}$) (although in the partial test there is one parameter that is significant for $Z_{2,t}$ at lag-1, this variable can be considered insignificant). Therefore, exchange rate movements are not sufficient to explain the movement of both export revenues. However, the results of the collective causality test indicate that the variables $Z_{1,t}, X_{1,t}$, and $X_{2,t}$ collectively influence $Z_{2,t}$and the variables $Z_{2,t}, X_{1,t}$, and $X_{2,t}$ collectively influence $Z_{1,t}$ . This confirms that the combined effect of these variables, including the exchange rate $(X_{1,t})$, significantly predicts the movement of the endogenous variable. Furthermore, the results of both (individual and collective) indicate that the two exogenous variables $X_{1,t}$ and $X_{2,t}$ are not affected by the endogenous variable, thus reinforcing the exogenous nature of the model: they are not affected by the endogenous variable but instead influence the endogenous variable, both individually and collectively.

## 7. Evaluation of VARX Model Results

Before evaluating the model with MAPE and nRMSE, the predicted values of each export were aggregated to obtain Indonesia's total export revenue. The MAPE and nRMSE were then calculated on the total export forecast, considering the rupiah–dollar exchange rate and WTI oil prices. The results of the model evaluation using MAPE with Equation (8) and nRMSE with Equation (10) are presented in Table 10.

**Table 10.** Evaluation of VARX(5,6) Model Results on Test Data

| Evaluation | Total Export Revenue in Indonesia |
|---|---|
| MAPE | 6.895 % |
| RMSE | 7.817 % |

Based on Tables 10, the MAPE and nRMSE values for Indonesia's total export revenue are 6.895% and 7.817%, respectively. This indicates that the forecasting results for Indonesia's total export revenue are considered very good (<10%), allowing for forecasting for the next period.

## 8. Forecasting

The results of the forecast of total export data in Indonesia using the VARX(5,6) model with the influence of exogenous variables of the rupiah exchange rate against the US dollar and the price of WTI oil from January 2025 – July 2025 can be seen in Table 11, as follows:

**Table 11.** Total Export Revenue Data Prediction Results with VARX(5,6) Model

| No | Month | Export Value | | Total Export Revenue |
|---|---|---|---|---|
| | | Export Oil and Gas | Export Non Oil and Gas | |
| 1 | January | 1,254.56 | 21,398.18 | 22,652.74 |
| 2 | February | 1,310.97 | 21,109.07 | 22,420.04 |
| 3 | March | 1,432,17 | 21,587.75 | 23,019.92 |
| 4 | April | 1,318.59 | 21,940.01 | 23,321.60 |
| 5 | May | 1,316.69 | 21,204.06 | 22,520.75 |
| 6 | June | 1,374,76 | 20,602.77 | 21,977.53 |
| 7 | July | 1,302.67 | 22,015.73 | 23,318.41 |

Furthermore, the results obtained in Table 11 were compared with the actual values and re-measured using MAPE and nRMSE. The MAPE was 5.566% and the nRMSE was 6.40%, indicating that the VARX(5,6) model provided excellent forecasting results in forecasting total export revenue in Indonesia from January 2025 to July 2025.

## D. CONCLUSION AND SUGGESTIONS

This study advances export forecasting research by proposing a VARX-based framework that captures the joint effects of domestic export dynamics and external macroeconomic shocks. The results highlight the dominant role of global oil prices in transmitting external volatility to Indonesia's export revenue, while underscoring the importance of interaction effects between oil prices and exchange rates rather than isolated macroeconomic indicators. These findings position the VARX(5,6) model as a methodological refinement over conventional VAR approaches through the explicit integration of exogenous global drivers. From a policy perspective, the proposed framework offers a data-driven early warning tool to support export stabilization strategies that are responsive to global oil price fluctuations. Future research may extend this approach by incorporating broader external indicators, adopting long-horizon multivariate models to capture equilibrium relationships, and refining model parsimony to enhance interpretability without sacrificing predictive accuracy.

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