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Exploring Non-Mathematics Students' Reasoning in Solving Function Continuity Problems in Calculus Courses

Sri Suryanti[1]*, Clinton Chidibere Anyanmwu[2]
[1]Mathematics Education Department, Universitas Negeri Surabaya, Indonesia
[2]Department of Education, Institute of Science, Innovation and Culture,
Rajamangala University of Technology Krungthep, Thailand
srisuryanti@unesa.ac.id

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ABSTRACT

Understanding the concept of function continuity is one of the main conceptual challenges for non-mathematics students in learning calculus, as they tend to rely on algorithmic procedures rather than reasoning conceptually. This study aims to explore and compare the types of mathematical reasoning used by non-mathematics students in solving function continuity problems in basic calculus courses, using Lithner's reasoning framework. Using qualitative descriptive, this study compares two first-year calculus classes from two non-mathematics study programs using Lithner's framework. The research instruments comprised three written assignments on function continuity, developed according to the categories of Imitative Reasoning (IR) and Creative Reasoning (CR), in addition to task-based interviews (think-aloud) conducted to investigate students' cognitive processes. Data were analyzed by categorizing mathematical reasoning into Memorized Reasoning (MR), Algorithmic Reasoning (AR), Local Creative Reasoning (LCR), and Global Creative Reasoning (GCR), accompanied by an inter-rater reliability assessment. The results indicate differences in reasoning patterns between engineering and general education students, especially regarding their propensity to employ imitative reasoning (IR) or creative reasoning (CR) when confronted with continuity-of-function problems. These results offer a significant critique of the utilization of Lithner's framework in the analysis of calculus tasks especially the continuity of functions and propose minor adjustments to enhance the categorization of reasoning, making it more suitable for non-mathematics students.

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A. INTRODUCTION

This study investigates the reasoning strategies used by first-year non-mathematics students when solving functional continuity problems in elementary calculus courses. Specifically, this study examines differences in the reasoning strategies used by students from engineering and general education programs when faced with tasks related to the concept of functional continuity. In the context of higher education, both in Indonesia and elsewhere, calculus courses for mathematics students and non-mathematics students are generally differentiated by the level of material complexity and the degree of mathematical rigor used (Matic, 2015; Pepin et al., 2021; Biza et al., 2022). However, these differences do not necessarily indicate that opportunities for developing mathematical reasoning are also different; this question is the focus of our study.

The analysis was conducted by examining student assignments in the form of structured exercises, quizzes, and midterm exams, using a reasoning framework developed by Lithner (2008). Lithner (2008) says that reasoning is "the line of thought used by someone to make statements and reach conclusions in completing assignments." This concept encompasses not only formal proofs but also both high and low quality mathematical arguments. The framework divides problems into two main groups: imitative reasoning (IR) and creative reasoning (CR). IR consists on imitative methodologies, whereas CR entails mathematically substantiated argumentation. In this study, the word "tasks" includes all of the work that students have to do, such as homework, quizzes, and questions on the final exam (Norqvist, 2018; Renninger et al., 2023; Gíslason, 2024).

The standard view holds that learning mathematics plays a crucial role in developing logical and analytical thinking skills (Wang et al., 2025). The dominant viewpoint maintains that the study of mathematics is crucial for developing logical and analytical reasoning skills (Wang et al., 2025). Concerns continue to be expressed regarding the potential that certain students may complete mathematics courses by reproducing symbolic procedures without fully understanding their underlying conceptual meaning (Rittle-Johnson & Siegler, 2021). Classeroom activities substantially impact students' cognitive growth. Previous studies (Wijaya et al., 2015; Lithner, 2017; Russo & Hopkins, 2017; Clarke & Roche, 2018; Suryanti et al., 2022; Suryanti et al., 2024) indicate that the characteristics of tasks can influence the student learning outcomes. Bergqvist (2007) and Jonsson et al. (2014) observed that students often relied on rote memorization of procedures when confronted with tasks requiring minimal cognitive engagement. Consequently, students exhibited a diminished capacity to solve novel problems or apply mathematical knowledge across diverse contexts. Lithner (2017) demonstrated that prioritizing CR duties can improve mathematical proficiency. So, assignments can be seen as chances to learn math (Clarke & Roche, 2018; Hwang & Ham, 2021).

An interesting question to ask is: what types of assignments are typically given in introductory calculus courses to non-mathematics students, and how do these assignments provide opportunities for creative and imitative reasoning? El-Sabagh (2021) stressed that the type and structure of tests have a big effect on how pupils learn. They came up with a way to sort questions on tests and assignments and found that majority of them were procedural and repeated. Bergwall & Hemmi (2017) reported similar findings in their analysis of calculus exams at four Swedish universities: approximately 70% of the questions could be solved using imitative reasoning alone, and most exams could be passed without creative reasoning.

In the United States, Tallman et al. (2016) developed the Exam Characterization Framework to analyze 150 university-level calculus exams and found that the majority of questions were low in cognitive demand, rarely involved real-life contexts, and rarely required explanations or demonstrations of conceptual understanding. In contrast, Mac an Bhaird et al. (2017) found variation in the cognitive orientation of tasks between different instructors despite using the same textbook, and that exams tended to have a higher proportion of rich tasks than homework. Several factors may explain the prevalence of procedural questions on calculus final exams. Swanson & Collins (2018) observe that certain instructors believe an excessive number of CR problems may elevate student failure rates, whereas Peng et al. (2024) highlight the perception that higher-order reasoning is solely pertinent to high-achieving students. This view has implications for assessment designs that discourage creative reasoning, particularly for non-mathematics students who typically have a more limited conceptual background.

In this study, we consider various analytical frameworks used in previous studies to classify assignments in college-level calculus courses. However, our focus is on the arguments and reasoning processes used by non-mathematics students when solving problems involving the continuity of functions. The Lithner reasoning framework is most appropriate because it allows analysis of both procedural and conceptual reasoning without requiring formal proofs. Since all calculus students should be able to make mathematical arguments (Cardetti & LeMay, 2019), using this framework makes it possible to compare groups of students with diverse academic backgrounds in a meaningful way. Based on this framework, this study aims to answer the following research questions (RQ): (1) *RQ1. What types of reasoning do non-mathematics students use when solving problems involving continuity of functions?*; (2) *RQ2. How do these reasoning patterns reflect their understanding of the concept of continuity of functions?*; and (3) *RQ3. What difficulties or misconceptions arise in their reasoning processes?*

B. THEORETICAL FRAMEWORK

Lithner (2008) distinguishes two main types of mathematical reasoning: Imitative Reasoning (IR) and Creative Reasoning (CR). Imitative reasoning describes a form of thinking in which students solve a problem by copying or recalling previously learned steps without developing new conceptual understanding. Creative thinking, on the other hand, means coming up with new, plausible mathematical arguments based on a comprehension of the problem's mathematical features. Within this approach, IR is subdivided into two subcategories: Memorized Reasoning (MR) and Algorithmic Reasoning (AR). MR occurs when the solution strategy relies entirely on recalling a known answer, without reflecting on its mathematical meaning. The hallmark of MR is that students choose a strategy based on remembering the complete answer, and the solution step consists solely of rewriting it. In the context of introductory calculus courses, this form of reasoning is often seen when students are asked to rewrite the formal definition of function continuity or state a theorem that has been taught, such as the statement that "the function $f(x)$ is continuous at the point x_0 if and only if $\lim_{x \to x_0} f(x) = f(x_0)$. At this stage, students do not reason the conceptual meaning of the condition, but only remember it as a formula.

Meanwhile, Algorithmic Reasoning (AR) occurs when students choose a solution strategy by recalling a specific procedure or algorithm they have learned. In this type of reasoning, students follow the solution steps mechanically without understanding the mathematical justification. Lithner (2008) emphasized that implementing strategies in AR is routine and trivial for the reasoner, so the only obstacles that may arise are technical errors or carelessness. In the context of function continuity, algorithmic reasoning is evident when students directly apply limit substitution procedures or algebraic manipulations, or use the ε–δ definition, without understanding the conceptual reasoning behind why these conditions indicate continuity. In other words, students may obtain procedurally correct results but fail to explain why a function is categorized as continuous or discontinuous at a particular point.

Unlike IR, Creative Reasoning (CR) is a form of constructive reasoning in which students construct their own sequence of problem-solving steps, grounded in a logical understanding of the problem's mathematical nature. Lithner (2008) defines CR as reasoning that has three main characteristics: novelty, plausibility, and mathematical foundation. Novelty means students create or reconstruct a sequence of reasoning that is new to them; feasibility means students can put forward reasonable arguments to support the steps and conclusions taken; while mathematical foundation means that the arguments put forward are rooted in the intrinsic properties of the mathematical objects involved in the reasoning. Bergqvist (2007) then developed two subcategories of CR: Local Creative Reasoning (LCR) and Global Creative Reasoning (GCR). LCR occurs when a problem can actually be solved with a known algorithm, but students need to make non-trivial local modifications to reach a solution. Conversely, GCR occurs when no algorithm is available, so students must develop an entirely new reasoning strategy to solve the problem.

In the context of this study, LCR occurs when students adapt standard limit calculation procedures or algebraic manipulations to handle functions with unusual properties, such as those that are not defined at a particular point but still have a finite limit. GCR, on the other hand, happens when students come up with new conceptual arguments to show how a discontinuous function can be made continuous by changing the value of the function at a certain point, or when they use narrative reasoning to explain results they got without using standard methods. So, the main difference between IR and CR isn't how hard the assignment is, but how students think about the connections between concepts, procedures, and the math arguments that support them.

C. METHODS

1. Research Context and Subjects

This research was conducted in two non-mathematics study programs at two universities with different calculus learning characteristics: an engineering program (Indonesia) and a general education program (Thailand). The cross-national context was deliberately chosen not to compare national education systems but to to represent two spectrums of calculus learning approaches commonly found in higher education: one with a more procedural and computational orientation (an engineering program) and the other with a more conceptual and applied orientation (a general education program). Selecting these two programs enables a comparative analysis of the different types of reasoning used by non-mathematics students when solving problems involving the continuity of functions.

Both study programs offer a similar Basic Mathematics course, including introductory calculus, as part of their first-year curriculum. Engineering students generally have a stronger mathematical background. Students in general education programs, on the other hand, come from a wider range of backgrounds and usually haven't had much formal math teaching. This disparity shows that the two groups of students are not all at the same degree of preparedness for math, which could affect how they think about calculus concepts.

The study participants were 36 students from engineering programs and 32 from general education programs. All participants were first-year students taking a Basic Mathematics course in the odd semester of the current academic year. The engineering class taught students how to utilize calculus in engineering, whereas the general education class taught students how to understand functions and how to use them in real life. Even while both classes utilized the same books and lesson plans, the ways they taught and tested were very different, especially when it came to how hard the assignments were and how deep the ideas were examined.

2. Research Design

This study employed a qualitative-descriptive approach with a task-based analysis design. This approach was chosen because it allowed researchers to deeply explore students' reasoning processes based on their responses to calculus tasks specifically designed to uncover the types of reasoning used. The primary objective of this study was not to measure students' success rates, but rather to identify how they construct, modify, or imitate solution strategies in the context of function continuity.

The primary instrument in this study was a set of written tasks comprising three problem items on function continuity, varying in cognitive level and expected reasoning type. These tasks were designed based on Lithner (2008) framework, taking into account the diversity between Imitative Reasoning (IR) and Creative Reasoning (CR). For example, one problem asked students to determine whether a given function was continuous at a given point using only its formal definition (indicative MR/AR). In contrast, another asked them to conceptually explain how a discontinuous function could be made continuous by redefining its values (indicative LCR/GCR).

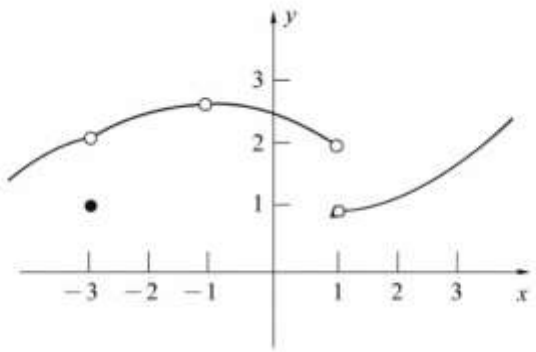
Example Task 1.

Given function $f(x) = \frac{x^2-1}{x-1}$, defined for all $x \neq 1$

Determine whether the function f(x) is continuous on $x = 1$

Example Task 2.

Look at the following function graph.



a. Determine whether the function f(x) is continuous on $x = 1$, and explain your reasons.
b. Determine the intervals where the function f(x) is continuous
c. Mention the points where the function(f (x)) is not continuous and explain your reasons

Example Task 3.
A function f(x) is defined as follows:

$$G(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2, \\ k, & x = 2. \end{cases} \quad (1)$$

Determine the value of k so that the function is continuous. In addition to written assignments, data were collected through task-based interviews with 8 participants from each study program. The interviews utilized a think-aloud approach, prompting students to articulate their cognitive processes while addressing problems. This method gave us a better idea of how students think.

3. Data Collection and Analysis

The data collection method took four weeks and included doing assignments, interviewing people, and checking the results. We gathered all of the students' answer papers and used Lithner (2008) reasoning classification framework to look at them. We put each student's answer into one of four reasoning categories: Memorized Reasoning (MR), Algorithmic Reasoning (AR), Local Creative Reasoning (LCR), and Global Creative Reasoning (GCR). Two researchers independently performed the coding to guarantee the veracity of the analysis. The inter-rater agreement was 0.87, which means that the interpretations were very consistent. Discrepancies in the classification results were then discussed until consensus was reached. Interview data were used to enrich the interpretation of the written assignment analysis results, particularly in identifying verbal evidence indicating a shift from imitative reasoning to creative reasoning

D. RESULT AND DISCUSSION

1. RQ1. *What types of reasoning do non-mathematics students use when solving problems involving continuity of functions?*

The tasks used in this study (see example Tasks 1–3) were classified using Lithner's (2008) reasoning framework. In the two non-mathematics study programs studied—Engineering (Indonesia) and General Education (Thailand), each course included three types of assessments: practice questions, submitted questions, and exam questions. This methodology mirrors the framework of prior research (Bergqvist, 2007; Jonsson et al., 2014), which employed diverse problem types to evaluate the probability of Imitative Reasoning (IR) and Creative Reasoning (CR). Table 1 and Table 2 below present the classification results of all student assignments based on reasoning categories: Memorized Reasoning (MR), Algorithmic Reasoning (AR), Local Creative Reasoning (LCR), and Global Creative Reasoning (GCR).

Table 1. Number of tasks in each reasoning category for Engineering Mathematics

Required Reasoning Type	Practice Questions	Submitted Questions	Exam Questions	Total
IR	88 (59%)	40 (27%)	12 (8%)	140 (94%)
MR	0	0	0	0
AR	88 (59%)	40 (27%)	12 (8%)	140 (94%)
CR	9 (6%)	4 (3%)	2 (1%)	15 (6%)
LCR	7	3	1	11
GCR	2	1	1	4
Total	97	44	14	155

Table 2. Number of tasks in each reasoning category for General Education Mathematics

Required Reasoning Type	Practice Questions	Submitted Questions	Exam Questions	Total
IR	72 (55%)	28 (21%)	10 (8%)	110 (84%)
MR	0	0	0	0
AR	72 (55%)	28 (21%)	10 (8%)	110 (84%)
CR	15 (11%)	5 (4%)	1 (1%)	21 (16%)
LCR	10	4	1	15
GCR	5	1	0	6
Total	87	33	11	131

The classification results show that most non-mathematics students, from both engineering and general education programs, predominantly use imitation reasoning (IR) to solve function-continuity problems. The highest proportion comes from the Algorithmic Reasoning (AR) category, indicating students' tendency to rely on mechanistic procedures such as limit substitution, simplification of algebraic expressions, and the direct application of the definition of continuity without conceptual argumentation. In contrast, the number of assignments categorized as Creative Reasoning (CR) is relatively small, especially for summative assignments such as final exams. However, there is an interesting pattern: practice assignments provide more opportunities for Local Creative Reasoning (LCR) to emerge—for example, when students have to interpret function graphs or explain types of discontinuities using their own arguments.

The analysis showed that AR was more common among engineering students, whereas LCR was a little more common among general education students. These findings demonstrate that task design affect the reasoning styles employed by students. Tasks that focus on procedures and results, like those in engineering, tend to encourage IR. On the other hand, assignments that promote conceptual reflection and verbal explanation, such as those in general education settings, offer more opportunities for CR. The high level of AR shows that students' learning styles are very well matched with the tasks and tests they have to do. In service classes (engineering & general education), assignments often emphasize procedural accuracy, computational efficiency, and concise numerical answers thus, students practice memorizing algorithms and worked examples rather than constructing new conceptual arguments. Analysis of large-scale calculus exams in the US revealed that low cognitive demands rarely characterize exam items that require explanation/justification (Tallman et al., 2016), thus systematically "encouraging" IR/AR over CR; this pattern is consistent with our data. On the coursework side, Ellis et al., (2015) study found that homework did provide "practice space" for understanding.

However, the responsibility for constructing knowledge was mainly shifted to students, and the items remained procedural in nature, which, again, fosters AR when there is no explicit task design intervention.

Both of our program contexts are also important. Engineering students are accustomed to a culture of assessment based on correctness and procedural efficiency (e.g., limit substitution, algebraic manipulation), so when they encounter continuity of functions, they optimize familiar algorithmic paths; in contrast, in general education, there is slightly more discursive space, so LCR occurs when interpreting graphs/arguments but remains minor due to the time constraints and formatting of summative assessments. Recent findings by Mkhatshwa, (2025) show that even when students demonstrate quantitative reasoning (e.g., using diagrams), the covariational reasoning aspect is weak, so the transition from procedure to conceptual argument is often "disconnected" a finding that corroborates our low CR for continuity topics that require coordinated limit-value functions. Finally, task design is a very important factor. Recent research shows that non-routine or unusual tasks make CR more likely, while most calculus textbook exercises invite IR (El Turkey et al., 2024). A newly proposed task design framework for fostering creativity shows how instructors can modify prompts, representations, and constraints to require students to make conceptual decisions rather than execute algorithms. This explains why we see a slight increase in LCR on practice questions due to their more open-ended nature but still fall behind formative/summative pressures.

2. **RQ2. *How do these reasoning patterns reflect their understanding of the concept of continuity of functions?***

Analysis of student assignment results and think-aloud interview data revealed three main reasoning patterns reflecting varying levels of understanding of the concept of functional continuity. These three patterns generally align with the classifications of AR, LCR, and GCR reasoning, as described in the theoretical framework (Lithner, 2008).

Pattern 1: Procedural–Algorithmic (AR)

The majority of engineering students (83%) and some general education students (61%) demonstrated a procedural–algorithmic reasoning pattern, in which the thinking process focuses on applying mechanistic steps to determine the continuity of a function, without considering its mathematical meaning.

For example, in Task 1

$$f(x) = \frac{x^2 - 1}{x - 1} \tag{2}$$

Students routinely perform factorization and substitution without examining the context of the formal definition of continuity. One engineering student stated in an interview:

> *"I know how to do it: just factor it and then cancel the denominator. If the results are the same, it means it's continuous."* (Engineering Student, S2)

Although the procedure used yields the correct answer, the argument behind it is mechanistic they judge continuity based solely on the result of the limit calculation, not on the equality of the three principal components (left-hand limit, right-hand limit, and function value).
This pattern suggests that students' understanding tends to be limited to conceptual imagery (Tall & Vinner, 1981), where they associate continuity with "a continuous graph" or "a function that can be drawn without lifting a pencil," without being able to relate it to the formal definition. This finding aligns with Mkhatshwa (2025) findings, which report that calculus students often exhibit weak covariational reasoningthey recognize visual changes but fail to conceptually reason about the relationship between limit values and function values.
Studies by Tallman et al. (2016) and Ellis et al. (2015) also found that overemphasizing routine practice leads students to "memorize their way through" without developing a structural understanding. In the context of this study, this is reflected in engineering students' tendency to respond quickly with symbols but to have difficulty explaining the meaning of continuity verbally.

Pattern 2: Locally Adaptive (LCR)
Some students, especially those in general education, were able to change the tactics they had learnt when they saw unexpected function graphs (Task 2). In this instance, students not only utilized processes but also made modifications based on visual observation and logical analysis. For example, in a function graph showing an open point at x=1 and a closed point at x=3, one student explained:

> *"At point one, the left and right limits are the same, but because the point is open, the values are different, so it shouldn't be continuous. But it can be made continuous by redefining it."* (General Education Student, S7).

This response demonstrates Local Creative Reasoning (LCR)—the student connected the visual aspect (open/closed points) with the formal concept of continuity and proposed a conceptual refinement (a redefinition of the function) even though they had not yet used the formal symbol. This pattern shows how students go from having a concept image to having a concept definition when they learn the mathematical rules that make a function continuous. This discovery aligns with the findings of Gehrtz et al. (2024), which indicated that active participation in the discussion of students' written work enhanced their capacity to substantiate their mathematical arguments. Moreover, these findings corroborate El Turkey et al. (2024) assertion that non-routine activities might elicit more sophisticated and credible reasoning, particularly when students encounter scenarios that compel them to link graphical representations with formal definitions.

Pattern 3: Global Conceptual (GCR)
The third pattern, which appeared least frequently (in only 8% of participants, primarily two general education students), was characterized by the ability to construct an entirely new conceptual argument. Students in this group not only recognized discontinuity but could also explain how the continuity condition could be generally satisfied by reconstructing the relationships between the function's parts.

In Task 3, which asked students to determine the value of k so that the function

$$G(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases} \quad (3)$$

becomes continuous at $x = 2$, One student said,

> *"For the function to be continuous, the value at $x = 2$ must be the same on both sides." So, I first discover the limit, which is 4. This means that $k = 4$ so that the value of the function is equal to the limit.* (Engineering Student, S3)

This answer shows Global Creative Reasoning (GCR) since the students built a full mathematical argument based on the idea of continuity instead of just memorizing formulas. They showed that they were possible and based on arithmetic, which is what Lithner (2008) wanted. The findings align with Spencer-Tyree et al. (2024), who demonstrated that exploration-driven learning activities, can enhance students' conceptual comprehension and argumentative skills in calculus, particularly when required to articulate the mathematical reasoning underlying numerical outcomes. Students outside the field of mathematics demonstrate a cognitive progression from algorithmic to conceptual reasoning approaches, as exemplified by the differences observed between engineering and general education students. Engineering students generally operate within the imitative reasoning (IR/AR) framework, prioritizing speed and procedural effectiveness.

3. **RQ3. *What difficulties or misconceptions arise in their reasoning processes?***

Analysis of student work and think-aloud interviews revealed that difficulties in understanding the concept of functional continuity are not solely due to a lack of procedural skills, but instead to a fragmentation between symbolic, graphical, and conceptual representations. Based on thematic groupings, three main categories of difficulties were identified: (1) misconceptions about the formal definition of continuity, (2) difficulty relating limits to function values, and (3) representational barriers in interpreting function graphs. Most engineering and general education students associate continuity with "continuous graphs" without addressing formal mathematical requirements. In interviews, the following statements frequently emerged:

> *"A function is continuous if its graph is continuous, unbroken. If there are gaps, it is not continuous."* (Engineering Student, S5)
> *"Basically, if it can be drawn without lifting a pencil, it is continuous."* (General Education Student, S3)

Statements like these demonstrate a strong intuitive image Tall & Vinner (1981) but are not connected to an actual concept definition. Students understand continuity visually, not through the relationship between the three main components of the formal definition: the existence of a limit, the equivalence of a function value to the limit, and the connectedness of the domain. This finding is consistent with studies conducted by Radmehr & Drake (2017), which indicates that students frequently misinterpret their intuitive grasp of core calculus concepts, including limits and continuity. The study confirms that without directed exploration of multiple representations, students often develop pseudo-conceptual understanding understanding that appears procedurally correct, but is not based on the logical structure of the definition. Another prominent difficulty was students' failure to connect the limit value with the function value at a specific point. In Task 3, a lot of students got the limit right but didn't connect it to the requirement of continuation. A student in engineering said,

"*The limit is 4, but I don't know how it relates to k, I believed we only needed to change thing"s* (Engineering Student, S6)

Meanwhile, another general education student said:

"I got the limit to 4, but the function value at that point isn't necessarily 4. So it should be defined so that it's the same." (General Education Student, S8)

This comparison demonstrates a difference in the degree of conceptual integration between the two groups. Engineering students tended to stop at procedural results, while general education students began to connect the functional relationship between the limit and the function value. This finding confirms Mkhatshwa (2025) research, which reported weak covariational reasoning among calculus students, they could calculate but did not yet understand the meaning of continuous change in function values. Similarly, Ellis et al. (2015) emphasized that this kind of difficulty often arises because students fail to see the limit as a process concept (something that happens as x approaches a particular value), rather than an object concept (a single value that must be calculated). Furthermore, representational barriers and graph misinterpretations. When shown a function graph, like in Task 2, most students only thought about how it looked and not what the open and closed points meant. They made a mistake when they said the function was continuous just because the lines were "connected," without looking at the function's result at a specific location.

"I think it's continuous because the lines are connected, the line with dots is simply an example" (Engineering Students, S4)

"I know there's a hole at point one, but the function still works, so I think it can still be called continuous." (General Education Student, S9)

Analysis of these responses indicates that students struggled to coordinate visual and symbolic representations. They didn't understand that an open point signifies that the function's value isn't defined at that point. This supports what Jonsson et al. (2014) found: calculus homework that exclusively focuses on procedural forms without any visual investigation doesn't help students think creatively. Yu (2024) research also showed that weaknesses in covariational reasoning are closely related to students' inability to interpret the relationship between graph shape and changes in function values a condition identical to what we observed in this study.

These three types of difficulties demonstrate a general pattern: non-mathematics students experience conceptual fragmentation between symbols, meanings, and graphical representations. This results in them being able to perform only imitative-level reasoning despite having considerable learning experience. Other research Radmehr & Drake (2017) reveal a comparable phenomenon, students' epistemological framing of calculus is predominantly pragmatic and outcome-oriented, rather than focused on deep comprehension. Consequently, the challenges found in this study are not merely human deficiencies, but rather a consequence of a mainly procedural instructional ecology. Jonsson et al. (2014) contend that the sole effective method to tackle this fragmentation is to formulate tasks that necessitate argumentation grounded in concepts and diverse representations, rather than mere symbol manipulation. Thus, there is a need for pedagogical interventions that integrate the exploration of graphs, symbols, and applicative contexts so that non-mathematics students can develop creative reasoning rooted in accurate mathematical understanding.

E. CONCLUSION AND SUGGESTIONS

This study, which analyzes three types of calculus tasks and includes comprehensive interviews, concludes that non-mathematics students predominantly employ imitative reasoning, particularly algorithmic reasoning (AR), when solving functional continuity problems. In contrast, creative reasoning (CR) is observed only to a limited degree in assignments requiring graph interpretation or the reconstruction of conceptual arguments. This reasoning pattern suggests that students' understanding is predominantly based on representations of "unbroken graphs," rather than being fully aligned with the formal definition of continuity, which involves the equivalence of limits and function values. Students who demonstrate Local Creative Reasoning can adapt to new situations and begin to link visual aspects with formal concepts, while only a few achieve Global Creative Reasoning by constructing complete mathematical arguments. The difficulties encountered, particularly definitional misconceptions, the failure to connect limits to function values, and representational errors. Therefore, calculus instruction for non-mathematics students should be designed to encourage the study of diverse representations and reflective opportunities that foster creative reasoning and deepen conceptual comprehension. Theoretically, these findings reinforce and expand Lithner's reasoning framework by demonstrating that, within the context of calculus instruction for non-mathematics students, creative reasoning is more frequently expressed as Local Creative Reasoning influenced by task design and representations, rather than through formal proof.

REFERENCES

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *The Journal of Mathematical Behavior*, *26*(4), 348–370. https://doi.org/https://10.1016/j.jmathb.2007.11.001

Bergwall, A., & Hemmi, K. (2017). The state of proof in Finnish and Swedish mathematics textbooks—Capturing differences in approaches to upper-secondary integral calculus. *Mathematical Thinking and Learning*, *19*(1), 1–18. https://doi.org/https://doi.org/10.1080/10986065.2017.1258615

Biza, I., González-Martín, A. S., & Pinto, A. (2022). 'Scaffolding'or 'filtering': a review of studies on the diverse roles of calculus courses for students, professionals and teachers. *International Journal of Research in Undergraduate Mathematics Education*, *8*(2), 389–418. https://doi.org/https://10.1007/s40753-022-00180-1

Cardetti, F., & LeMay, S. (2019). Argumentation: Building students' capacity for reasoning essential to learning mathematics and sciences. *Primus*, *29*(8), 775–798. https://doi.org/https://doi.org/10.1080/10511970.2018.1482581

Clarke, D., & Roche, A. (2018). Using contextualized tasks to engage students in meaningful and worthwhile mathematics learning. *The Journal of Mathematical Behavior*, *51*, 95–108. https://doi.org/https://doi.org/10.1016/j.jmathb.2017.11.006

El Turkey, H., Karakok, G., Cilli-Turner, E., Satyam, V. R., Savić, M., & Tang, G. (2024). A Framework to Design Creativity-Fostering Mathematical Tasks. *International Journal of Science and Mathematics Education*, *22*(8), 1761–1782. https://doi.org/10.1007/s10763-024-10449-3

Ellis, J., Hanson, K., Nuñez, G., & Rasmussen, C. (2015). Beyond plug and chug: An analysis of calculus I homework. *International Journal of Research in Undergraduate Mathematics Education*, *1*(2), 268–287. https://doi.org/https://doi.org/10.1007/s40753-015-0007-x

El-Sabagh, H. A. (2021). Adaptive e-learning environment based on learning styles and its impact on development students' engagement. *International Journal of Educational Technology in Higher Education*, *18*(1), 53. https://doi.org/https://doi.org/10.1186/s41239-021-00289-4

Gehrtz, J., Hagman, J. E., & Barron, V. (2024). Engagement with student written work as an instantiation of and proxy for how college calculus instructors engage with student thinking. *The Journal of Mathematical Behavior*, *76*, 101187. https://doi.org/10.1016/j.jmathb.2024.101187

Gíslason, I. (2024). Interactions and tensions between mathematical discourses and schoolwork discourses when solving dynamic geometry tasks: what is internally persuasive for students? *Research in Mathematics Education*, *26*(3), 494–520. https://doi.org/https://10.1080/14794802.2023.2177882

Hwang, J., & Ham, Y. (2021). Relationship between mathematical literacy and opportunity to learn with different types of mathematical tasks. *Journal on Mathematics Education*, *12*(2), 199–222. https://doi.org/https://doi.org/10.22342/jme.12.2.13625.199-222

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior*, *36*, 20–32. https://doi.org/https://doi.org/10.1016/j.jmathb.2014.08.003

Jukic Matic, L. (2015). Non-Mathematics Students' Reasoning in Calculus Tasks. *International Journal of Research in Education and Science*, *1*(1), 51–63. https://doi.org/https://doi.org/10.21890/ijres.11418

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/https://doi.org/10.1007/s10649-007-9104-2

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *Zdm*, *49*(6), 937–949. https://doi.org/https://doi.org/10.1007/s11858-017-0867-3

Mac an Bhaird, C., Nolan, B. C., O'Shea, A., & Pfeiffer, K. (2017). A study of creative reasoning opportunities in assessments in undergraduate calculus courses. *Research in Mathematics Education*, *19*(2), 147–162. https://doi.org/https://doi.org/10.1080/14794802.2017.1318084

Mkhatshwa, T. P. (2025). An examination of calculus students' quantitative and covariational reasoning in the context of differentials. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *44*(4), 287. https://doi.org/https://doi.org/10.1093/teamat/hraf003

Norqvist, M. (2018). The effect of explanations on mathematical reasoning tasks. *International Journal of Mathematical Education in Science and Technology*, *49*(1), 15–30. https://doi.org/https://doi.org/10.1080/0020739X.2017.1340679

Peng, J., Sun, M., Yuan, B., Lim, C. P., van Merriënboer, J. J. G., & Wang, M. (2024). Visible thinking to support online project-based learning: Narrowing the achievement gap between high- and low-achieving students. *Education and Information Technologies*, *29*(2), 2329–2363. https://doi.org/10.1007/s10639-023-11896-1

Pepin, B., Biehler, R., & Gueudet, G. (2021). Mathematics in engineering education: A review of the recent literature with a view towards innovative practices. *International Journal of Research in Undergraduate Mathematics Education*, *7*(2), 163–188. https://doi.org/https://doi.org/10.1007/s40753-021-00139-8

Radmehr, F., & Drake, M. (2017). Exploring students' mathematical performance, metacognitive experiences and skills in relation to fundamental theorem of calculus. *International Journal of Mathematical Education in Science and Technology*, *48*(7), 1043–1071. https://doi.org/10.1080/0020739X.2017.1305129

Renninger, K. A., Gantt, A. L., & Lipman, D. A. (2023). Comprehension of argumentation in mathematical text: what is the role of interest? *ZDM – Mathematics Education*, *55*(2), 371–384. https://doi.org/10.1007/s11858-022-01445-4

Rittle-Johnson, B., & Siegler, R. S. (2021). The relation between conceptual and procedural knowledge in learning mathematics: A review. In *The Development of Mathematical Skills* (pp. 75–110). Psychology Press. https://doi.org/10.4324/9781315784755-6

Russo, J., & Hopkins, S. (2017). Student reflections on learning with challenging tasks: 'I think the worksheets were just for practice, and the challenges were for maths.' *Mathematics Education Research Journal*, *29*(3), 283–311. https://doi.org/10.1007/s13394-017-0197-3

Spencer-Tyree, B., Bowen, B. D., & Olaguro, M. (2024). The Impact of Computational Labs on Conceptual and Contextual Understanding in a Business Calculus Course. *International Journal of Research in Undergraduate Mathematics Education*. https://doi.org/10.1007/s40753-024-00255-1

Suryanti, S., Nusantara, T., Parta, I. N., & Irawati, S. (2022). Problem-based task in teacher training program: Mathematics teachers' beliefs and practices. *Journal on Mathematics Education*, *13*(2), 257–274. https://doi.org/10.22342/jme.v13i2.pp257-274

Suryanti, S., Nusantara, T., Subanji, S., & Wijaya, A. P. (2024). *How to engage students in online mathematics learning through problem-based tasks: Students' perspective*. 020034. https://doi.org/10.1063/5.0225603

Swanson, H., & Collins, A. (2018). How failure is productive in the creative process: Refining student explanations through theory-building discussion. *Thinking Skills and Creativity*, *30*, 54–63. https://doi.org/10.1016/j.tsc.2018.03.005

Tallman, M. A., Carlson, M. P., Bressoud, D. M., & Pearson, M. (2016). A Characterization of Calculus I Final Exams in U.S. Colleges and Universities. *International Journal of Research in Undergraduate Mathematics Education*, *2*(1), 105–133. https://doi.org/10.1007/s40753-015-0023-9

Wang, M., Mohd Matore, M. E. E., & Rosli, R. (2025). A systematic literature review on analytical thinking development in mathematics education: trends across time and countries. *Frontiers in Psychology*, *16*. https://doi.org/10.3389/fpsyg.2025.1523836

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, *89*(1), 41–65. https://doi.org/10.1007/s10649-015-9595-1

Yu, F. (2024). Extending the covariation framework: Connecting covariational reasoning to students' interpretation of rate of change. *The Journal of Mathematical Behavior*, *73*, 101122. https://doi.org/10.1016/j.jmathb.2023.101122