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Exchange Rate Prediction of BRICS Countries against US Dollar Based on Multiresponse Fourier series Estimator

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ABSTRACT

The dominance of the US dollar (USD) as the global reserve currency has begun to face structural challenges since the 2007-2008 financial crisis, which triggered the strengthening of the BRICS alliance. Although this alliance now controls 35% of the world's GDP and is actively pursuing de-dollarization, analysis of the volatility of their collective currencies is often limited to univariate parametric models that fail to capture inter-country dependencies and complex periodic fluctuation patterns. This study aims to fill this gap by applying a nonparametric multiresponse Fourier series regression to simultaneously model the interdependence of the five major BRICS currencies against the USD. Using weekly secondary data from June 2009 to February 2025 (817 observations) from investing.com, this study positions time as the predictor and the exchange rates of the five BRICS currencies as the response. The analysis results show that the best estimation model is obtained through a sine function without a trend component with an optimal oscillation parameter k=1, based on a minimum Generalized Cross Validation (GCV) value of 0.000702363. The prediction results from the training data produce a MAPE value of 4.7521%, which classifies the analysis as highly accurate. These findings strategically support the validation of the de-dollarization movement, providing a predictive instrument for developing countries to reduce their dependence on the USD, as well as strengthening the bargaining position of Eastern economies in a more multipolar international financial order.

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A. INTRODUCTION

The global economy has been dominated by Western powers, particularly the group of economically advanced countries known as the G7, since the end of the second world (Cochrane & Zaidan, 2024). But financial crisis that hit the United States (USA) in 2007-2008 slightly affected the supremacy and prestige of the US Dollar (USD) as the world's central currency. This triggered a greater opportunity for countries in the eastern region to rise up to challenge the dominance of western countries to change the global economic governance system (Liu & Papa, 2022). With the same historical background and goals, even significant economic differences on global issues, several eastern countries such as Brazil, Russia, India, China, and South Africe came together in 2009 to form an economic and geopolitical alliance known as BRICS (Han & Papa, 2022).

The explosion of multinational trade led to extraordinary economic benefits for the USA such as receiving low borrowing costs due to high global demand for its currency (Meisenzahl et al., 2021). The power of dollar reflected in the US Dollar Index (USDX) continues to rise, aggressively raising the USA interest rate from 0.25% to 5.50% throughout 2022-2023 (Zhang & Yan, 2024). However, other data shows that 35% of the world's GDP has been controlled by BRICS countries, which is greater than the GDP of G7 countries such as the USA which is only around 30% (Sadovnichiy et al., 2024). Also, 17% of the world's trade is currently accounted for by BRICS countries, which means that some of the chain effects of world trade have depended on the movement of BRICS countries (Zhang et al., 2019).

The BRICS group's determination to create an economic system that no more depends on the USD monopoly, gave rise to the concept of dedollarization, which is an initiative to replace the use of USD with local currencies in trade and other (Abbas et al., 2025). BRICS countries' efforts to devalue their currencies will result in a decrease in the dollar exchange rate against local currencies (Popov, 2024). In addition, several recent conflicts have also strengthened the opinion that the power of USD is not an absolute power. Therefore, it is necessary to conduct research to predict the value of the USD against the currencies of the BRICS countries. This research will be very useful given the need for a precise measuring tool to validate the extent to which the movement of de-dollarization actually reduces the hegemony of the USD (Lee & Sims, 2024). Without accurate predictions using a multivariate approach, it is difficult for BRICS countries to determine the right momentum in collectively implementing local currency policies. It will also reveal how long it will take for local currencies to potentially replace the USD as the global currency (Huang, 2024).

Previous studies on exchange rate prediction have been conducted by Saparudin & Maulidina (2019), who predicted the exchange rate of the US dollar (USD) to the Indonesian rupiah (IDR), showing that there would be a decline in the exchange rate of the US dollar (USD). Rosita & Moonlight (2024) in their research also stated that there was a significant decline in the rupiah (IDR) against the dollar (USD) in early 2021 using three direct comparison methods, namely Single Exponential Smoothing (SES), Holt's Method, and Holr-Winters Method. Based on information obtained from previous studies, researchers tend to focus on univariate analysis (one variable) using parametric or smoothing methods such as Holt-Winters.

However, these methods have limitations in capturing the complex dynamics of exchange rate data, which have irregular periodic fluctuations and interdependence between currencies within an economic bloc (Salisu et al., 2021). Therefore, this study offers a novel approach through the use of the Multiresponse Fourier Series Estimator in a non-parametric statistical framework. Unlike previous studies, the multiresponse approach allows for simultaneous analysis of the five BRICS currencies, so that the correlation between currency fluctuations can be captured in a single estimation model (Sukran et al., 2025).

The advantage of Fourier series lies in its flexibility in modeling repetitive and seasonal data patterns without being bound to specific distribution assumptions, which are often violated by volatile financial data. Fourier series estimators enable in-depth analysis of seasonal patterns and periodic cycles in time series data, which is useful for detecting long-term and recurring trends (Mardianto et al., 2021). This approach is suitable for exchange rate data that fluctuates repeatedly over a certain period, which is often influenced by inflation, monetary policy, etc.

The results of this study are crucial, given that the phenomenon of de-dollarization is no longer merely political discourse, but rather a structural shift in the global economy. By accurately predicting the exchange rates of BRICS currencies against the USD, this study provides an empirical basis for measuring the effectiveness of local currencies in challenging the supremacy of the US dollar. For Indonesia, which has now expressed its interest in joining this alliance, these predictions will be a strategic instrument in mitigating exchange rate risks and formulating monetary policies that are adaptive to the new economic order. Thus, this can also help accelerate the success of dedollarization efforts to eliminate the US dollar's monopoly in trade and international cooperation. This is also in line with supporting the achievement of SDGs 10 and 17 on global partnerships to reduce global economic disparities, especially for Indonesia, which has just joined the BRICS alliance.

B. METHODS

This study employs a quantitative time series modeling approach using secondary weekly exchange rate data of BRICS countries (Brazil, Russia, India, China, and South Africa) obtained from the Investing.com website, covering the period from June 21, 2009 to February 9, 2025. The exchange rate series constitute the research subjects in this study, and the research instruments consist of the secondary data source and statistical computation tools used for data processing and analysis. The research variables include time as the predictor variable and the weekly exchange rates of BRICS countries as the response variables. Data analysis techniques applied in this study include descriptive statistical analysis, Bartlett's test, Fourier series estimator modeling, model selection based on Generalized Cross-Validation (GCV), and accuracy evaluation using Mean Absolute Percentage Error (MAPE). A detailed description of the variables is provided in Table 1.

Table 1. Characteristics of Research Variables

Variables	Description
Predictor	Time (t)
Response	USD/BRL Rate (y_1)
	USD/RUB Rate (y_2)
	USD/INR Rate (y_3)
	USD/CNY Rate (y_4)
	USD/ZAR Rate (y_5)

1. Multiresponse Fourier Series Estimator

The Fourier series estimator is a nonparametric approach used in multiresponse regression, where there are multiple correlated response variables associated with one or more predictor variables (Rencher, 2002). This estimator is constructed using a combination of sine and cosine functions, making it particularly effective for approximating periodic functions (Adrianingsih et al., 2021). It is commonly applied when the data exhibit an unknown pattern with potential seasonal trends. In multiresponse modeling, parameter estimation using the Fourier series estimator is best handled through the Weighted Least Squares (WLS) method, which incorporates weights typically based on error variance (Mardianto et al., 2019). Consider a dataset consisting of paired observations $(y_{i1},\ y_{i2},\dots,y_{iq},t_i)$, where t_i represents the time at the i_{th} observation. The general form of the nonparametric regression equation for time series data is expressed in Equation (1).

$$y_{ij} = m_j(t_i) + \varepsilon_{ij} \quad , \quad \varepsilon_{ij} \sim IIDN(0, \sigma_j^2) \tag{1}$$

The variance in Equation (2) has a value of σ_j^2 and is heterogeneous, which means that there are differences in variance between one response variable and another response variable. Equation (1) is written as follows:

$$\begin{gathered} y_{i1} = m_1(t_i) + \varepsilon_{i1} \\ y_{i2} = m_2(t_i) + \varepsilon_{i2} \\ \vdots \\ y_{iq} = m_q(t_i) + \varepsilon_{iq} \end{gathered} \tag{2}$$

with $i = 1,2,\dots,n$ representing the number of observations and $j = 1,2,\dots,q$ denoting the number of response variables. The function $m_j(t_i)$ in Equation (1) can be approximated using a Fourier series. As a result, Equation (3) is derived as follows.

$$\begin{gathered} m_1(t_i) = \frac{a_{01}}{2} + \gamma_1 t_{i1} + \sum_{\lambda=1}^{K} (a_{\lambda 1} \cos \lambda t_{i1} + \beta_{\lambda 1} \sin \lambda t_i) \\ m_2(t_i) = \frac{a_{02}}{2} + \gamma_2 t_{i2} + \sum_{\lambda=1}^{K} (a_{\lambda 2} \cos \lambda t_{i2} + \beta_{\lambda 2} \sin \lambda t_i) \\ \vdots \\ m_q(t_i) = \frac{a_{0q}}{2} + \gamma_q t_{iq} + \sum_{\lambda=1}^{K} (a_{\lambda q} \cos \lambda t_{iq} + \beta_{\lambda q} \sin \lambda t_i) \end{gathered} \tag{3}$$

By substituting Equation (3) into (2), Equation (4) is obtained as follows.

$$\begin{gathered} y_{i1} = \frac{a_{01}}{2} + \gamma_1 t_{i1} + \sum_{\lambda=1}^{K} (a_{\lambda 1} \cos \lambda t_{i1} + \beta_{\lambda 1} \sin \lambda t_i) + \varepsilon_{i1} \\ y_{i2} = \frac{a_{02}}{2} + \gamma_2 t_{i2} + \sum_{\lambda=1}^{K} (a_{\lambda 2} \cos \lambda t_{i2} + \beta_{\lambda 2} \sin \lambda t_i) + \varepsilon_{i2} \\ \vdots \\ y_{iq} = \frac{a_{0q}}{2} + \gamma_q t_{iq} + \sum_{\lambda=1}^{K} (a_{\lambda q} \cos \lambda t_{iq} + \beta_{\lambda q} \sin \lambda t_i) + \varepsilon_{iq} \end{gathered} \tag{4}$$

Furthermore, by applying Equation (4) for $i = 1,2,\dots,n$ the resulting vector equation is obtained as follows

$$\begin{gathered} y_1 = T_1 \beta_1 + \varepsilon_1 \\ y_2 = T_2 \beta_2 + \varepsilon_2 \\ \vdots \\ y_q = T_q \beta_q + \varepsilon_q \end{gathered} \tag{5}$$

Thus, based on Equation (5), the multiresponse Fourier series nonparametric regression model can be expressed in matrix form, as shown in Equation (6)

$$y = T_\lambda \beta_\lambda + \varepsilon \tag{6}$$

with

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{pmatrix}; T = \begin{pmatrix} T_1 & 0 & \dots & 0 \\ 0 & T_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & T_q \end{pmatrix}; \beta_\lambda = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_q \end{pmatrix}; \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_q \end{pmatrix}$$

One of the methods used to estimate parameters in the case of multiresponse modeling is the Weighted Least Square (WLS) method because it is classified as a simple method, and the residuals do not depend on certain distribution assumptions. The WLS estimation method is relatively simple because it involves weights based on the error variance, so it is not bound by the distribution of the error. Basically $\boldsymbol{W}$ as a weight matrix acts to accommodate both the heteroscedasticity and correlation between each response, which are typical characteristics of exchange rate data (Nurcahayani et al., 2021). Given $\boldsymbol{I}$ is an identity matrix. With $\boldsymbol{W}$ is the weighting form of the variance-covariance matrix of the error which has elements σ_{ij} described in Equation (7) below.

$$\boldsymbol{W} = \boldsymbol{\Sigma} \otimes \boldsymbol{I} = \begin{pmatrix} \sigma_1^2\mathbf{I} & \sigma_{12}\mathbf{I} & \dots & \sigma_{1q}\mathbf{I} \\ \sigma_{21}\mathbf{I} & \sigma_2^2\mathbf{I} & \dots & \sigma_{2q}\mathbf{I} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{q1}\mathbf{I} & \sigma_{q2}\mathbf{I} & \dots & \sigma_q^2\mathbf{I} \end{pmatrix} \otimes \boldsymbol{I}_{\boldsymbol{nq}\times\boldsymbol{nq}} \tag{7}$$

In this case, the error variances are made equal, that is $\sigma_{11} = \sigma_{22} = \ldots = \sigma_{qq} = \sigma^2\boldsymbol{I}$. To estimate $\boldsymbol{\beta}_\lambda$ in Equation (6) using the WLS method, which is to minimize the goodness of fit as follows.

$$\min_{\beta \in R^{q(2k+2)}} [S(\boldsymbol{\beta})] \tag{8}$$

Equation (8) can be interpreted, leading to the formulation of Equation (9)

$$\begin{aligned} S(\beta) = \varepsilon^T W \varepsilon &= (y - TB_\lambda)^T W (y - TB_\lambda) \\ &= y^T W y - 2B_\lambda^T T^T W y + B_\lambda^T T^T W T B_\lambda \end{aligned} \tag{9}$$

necessary conditions for the S function in Equation (9) to attain a minimum are as follows

$$\frac{\partial S}{\partial \beta_\lambda} = 0 - 2T^T W y + 2T^T W T \beta_\lambda = 0 \tag{10}$$

From Equation (10), the estimator $\widehat{\boldsymbol{\beta}}_\lambda$ is obtained as in Equation (11) as follows

$$\hat{\beta}_\lambda = (T^T W T)^{-1} T^T W y \tag{11}$$

The estimation results obtained using the WLS method are then substituted into each $\hat{y}_{ij}$ in Equation (4), As a result, the multiresponse nonparametric time series regression model employing the Fourier series estimator approach is formulated as shown in Equation (12) below.

$$\hat{y}_{ij} = \frac{\hat{a}_{0j}}{2} + \hat{\gamma}_j t_{i1} + \sum_{\lambda=1}^{K} (\hat{a}_{\lambda j} \cos \lambda t_i + \hat{\beta}_{\lambda j} \sin \lambda t_i) \tag{12}$$

2. Selection of Optimal Oscillation Parameter

Selecting the oscillation parameter or λ value is a critical factor to consider. The Generalized Cross Validation (GCV) method is commonly used to determine the optimal λ value (Mariati et al., 2021). A higher λ leads to a rougher function estimation, while a lower λ produces a smoother estimation. Therefore, determining the optimal λ is essential for obtaining the best estimator. Based on this, for each λ the estimator $m(t)$ can be formulated as shown below.

$$\hat{y} = H_\lambda \tag{13}$$

with

$$H_\lambda = T(T^T W T)^{-1} T^T W$$

Based on the H_λ matrix, the optimal value of λ is determined by identifying the value that minimizes the GCV criterion as shown below:

$$GCV(\lambda) = \frac{MSE(\lambda)}{((nq)^{-1}\, trace[I - H_\lambda])^2} \tag{14}$$

with

$$MSE(\lambda) = (nq)^{-1} y'(I - H_\lambda) y$$

3. Best Model Selection

Selecting the optimal forecasting model is crucial to ensuring a good fit between actual and predicted data. A well-chosen model enhances accuracy and reliability, leading to more precise forecasts and better decision-making. Explains that the measure of forecasting accuracy is considered a criterion for rejecting or choosing a particular forecasting method, thus helping to determine a better option. The best model is determined using the Mean Absolute Percentage Error (MAPE) value, which measures the percentage error of predictions against actual observations. MAPE is formulated as follows (Myttenaere et al., 2016).

$$MAPE = \frac{1}{nq} \sum_{j=1}^{q} \sum_{i=1}^{n} \left| \frac{y_{ij} - \hat{y}_{ij}}{y_{ij}} \right| \times 100\% \tag{15}$$

with
y_{ij} : Actual value of the j_{th} response variable at time t_i
$\hat{y}_{ij}$: Predicted value of the j_{th} response variable at time t_i
n : Number of observations
q : Number of response variable

The accuracy of the prediction results based on the MAPE value is categorized in Table 2.

Table 2. Prediction Accuracy based on MAPE Value

MAPE	Description
$MAPE \leq 10\%$	Highly accurate prediction
$10\% < MAPE \leq 20\%$	Accurate prediction
$20\% < MAPE \leq 50\%$	Feasible prediction
$MAPE > 50\%$	Bad prediction

4. Analysis Procedure

The research data were analyzed using the Fourier series estimator method. The procedures carried out in this study to perform the analysis are as follows.

a. Collected weekly exchange rate data for Brazil, Russia, India, China, and South Africa from investing.com, covering the period from June 21, 2009 to February 9, 2025, resulting in 817 observations.
b. Divided the dataset into training and testing sets using a time-based split, with the first 80% of observations (654 weeks) as training data and the remaining 20% (163 weeks) as testing data.
c. Analyzed descriptive statistics of weekly exchange rate data over the specified data period.
d. Conducted Bartlett's test to test whether the five exchange rate variables are correlated and can be analyzed simultaneously using multiresponse modeling.
e. Performed model selection using the training data: (1) Estimated candidate models using sine, cosine, and cosine and sine functions, with and without a trend component; (2) Identified the optimal smoothing parameter λ based on the minimum GCV value; and (3) Selected the best model based on the minimum MAPE value computed from the training data
f. Evaluated model accuracy: (1) Used the selected model to predict the testing data; (2) Calculated the MAPE of testing data predictions to quantify prediction accuracy; and (3) Classified model performance according to Table 2 criteria.

C. RESULT AND DISCUSSION

1. Descriptive Statistics

The fluctuating pattern of the exchange rate data for the USD against the five BRICS currencies during 2009 to 2025 with 817 points measured over a weekly period is presented in Figure 1.

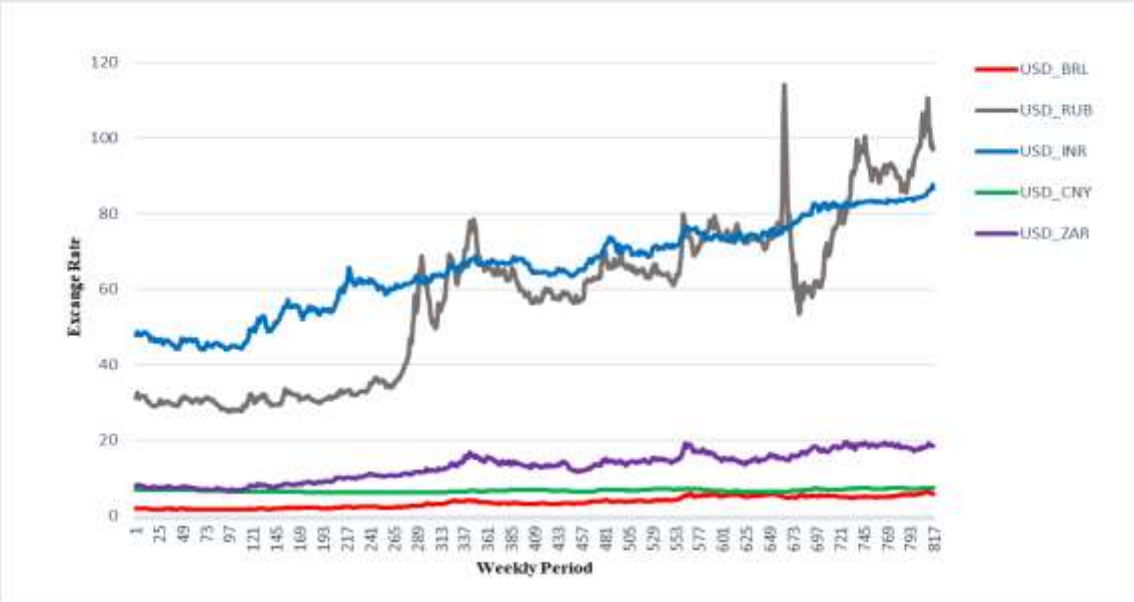

Figure 1. Time Series Plot of Research Variables

Based on Figure 1, the USD exchange rate against the five BRICS countries currencies exhibits varying volatility chracteristics. A very striking fluctuating pattern is shown by the USD/RUB exchange rate with an increasing trend in the data, although there was a significant decrease around the early period of 2022. This is in line with the dynamics of Russia's economic stability due to economic sanctions imposed by the United States (Crozet & Hinz, 2021). A different pattern is shown by the USD/CNY exchange rate with fluctuations that trend lower, indicating the stability of the Chinese economy as the second largest GDP owner after the United States. Additionally, the descriptive statistical analysis results for the five research variables is presented in Table 3.

Table 3. Descriptive Statistics

Variables	Mean	StDev	Minimum	Maximum
USD/BRL	3,5355	1,3929	1,5490	6,1964
USD/RUB	57,339	21,376	27,407	114,253
USD/INR	65,743	11,935	43,925	87,627
USD/CNY	6,6394	0,3434	6,0488	7,3430
USD/ZAR	12,944	3,763	6,572	19,645

Based on Table 3, USD/RUB and USD/INR exchange rates exhibit the highest volatility with standard deviations of 21.376 and 11.935, respectively. USD/INR fluctuated considerably, ranging from 43.925 to 87.627, with an average of 65.743, indicating substantial exchange rate movements for India throughout the study period. USD/BRL and USD/ZAR show moderate volatility levels with standard deviations of 1.3929 and 3.763, respectively. USD/BRL varied between 1.5490 and 6.1964 with a mean of 3.5355, while USD/ZAR ranged from 6.572 to 19.645 with an average of 12.944. These moderate standard deviations indicate that Brazil and South Africa experienced notable exchange rate fluctuations, though not to the extreme degree observed in Russia. In contrast, USD/CNY displays the most stable pattern with the smallest standard deviation of 0.3434 and an average of 6.6394. The varying volatility patterns across the five currencies reflect the heterogeneous economic conditions and exchange rate dynamics within the BRICS countries.

2. Bartlett's Test

In this study, before conducting further research, Bartlett's test will be carried out first to test the correlation or homogeneity of variance between data with the following hypothesis (Hair et al., 2014)
H_0: There is no correlation between data
H_1: There is a correlation between the data

Bartlett's test statistics are as follows.

$$\chi^2 = -\left[(n-1) - \frac{(2p+5)}{6}\right] ln|\boldsymbol{R}| \tag{16}$$

where n represents the number of observations, p is the number of variables, and $|\boldsymbol{R}|$ is the determinant of the Pearson correlation matrix. The test criterion used is to reject H_0 if the p-value < 0,05 (Watkins, 2018). The results are shown in Table 4.

Table 4. Bartlett Test Results

Chi-Square Statistic Value	Degree of Freedom	P-Value
6379,514	10	0,000

Based on Table 4, p-value = 0,000 < α = 0.05. This result leads to the rejection of H_0, confirming a correlation between the exchange rate data of BRICS countries. As a result, a multiresponse modelling approach can be applied to the weekly exchange rate data of all BRICS countries using the multiresponse Fourier series estimator.

3. Modeling the USD Rate against BRICS

The time series plot in Figure 1 shows a fluctuating pattern, similar to trigonometric functions such as sine and cosine. Given this pattern, identifying the best model to estimate the relationship between the predictor and response variables requires selecting the optimal smoothing parameter λ. The optimal λ is determined by minimizing the GCV value (Mariati et al., 2021), with the results summarized in Table 5.

Table 5. Comparison of Optimal λ Value and MAPE Value of Training Data

Function	Optimal λ	GCV	MAPE
Sine with trend	7	0,000555858	18,51843
Cosine with trend	8	0,000571542	18,14683
Cosine and Sine with trend	7	0,000559824	18,29007
Sine no trend	1	0,000702363	4,75210
Cosine no trend	1	0,000702469	6,251019
Cosine and Sine no trend	1	0,000702316	6,182423

According to Table 5, the function that includes the trend component achieves an optimal λ value in 7 and 8, while the estimator without the gamma function attains the same optimal λ value at 1. The analysis then proceeded with the estimation process on both training and testing data for each function. The best model was determined based on the minimum MAPE value (Myttenaere et al., 2016), which was found in the sine function without a trend component at an optimal λ of 1. This shows that the multiresponse model formed has met the criteria for model goodness. So the modeling analysis can be continued using the Fourier series estimator for the multiresponse case according to Equation (3) with j=1,2,3,4,5 and $\lambda = 1$. Based on the optimal λ value of 1, the multiresponse nonparametric regression model is derived using the Fourier series estimator approach with a sine function without a trend component. The general form of this model is presented in Equation (17) as follows.

$$\begin{aligned}
\hat{y}_{i1} &= \frac{\hat{a}_{01}}{2} + \left(\hat{\beta}_{\lambda 1} \sin\ \lambda t_{1i}\right)\\
\hat{y}_{i2} &= \frac{\hat{a}_{02}}{2} + \left(\hat{\beta}_{\lambda 2} \sin\ \lambda t_{2i}\right)\\
\hat{y}_{i3} &= \frac{\hat{a}_{03}}{2} + \left(\hat{\beta}_{\lambda 3} \sin\ \lambda t_{3i}\right)\\
\hat{y}_{i4} &= \frac{\hat{a}_{04}}{2} + \left(\hat{\beta}_{\lambda 4} \sin\ \lambda t_{4i}\right)\\
\hat{y}_{i5} &= \frac{\hat{a}_{05}}{2} + \left(\hat{\beta}_{\lambda 5} \sin\ \lambda t_{5i}\right)
\end{aligned} \tag{17}$$

4. Estimation Result of USD Rate Prediction against BRICS

After obtaining the multiresponse Fourier series model, the following step involves estimating the model using the optimal smoothing parameter λ (Nurcahayani et al., 2021). The results of the multiresponse Fourier series estimation are displayed in Table 6.

Table 6. Best Model Parameter Estimation

Parameter	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
$\frac{1}{2}\hat{a}_{0j}$	1,7771	25,3946	45,7378	6,3919	6,7138
$\hat{\beta}_{k4}$	0,1253	0,4169	-0,0173	0,0018	-0,0099

By looking at Table 6, the Fourier series estimator model for the USD exchange rate against the five BRICS countries can be formulated in Equation (18) below.

$$\begin{aligned} \hat{y}_{i1} &= 1{,}7771 + 0{,}1253\ sin(\lambda t_{1i}) \\ \hat{y}_{i2} &= 25{,}3946 + 0{,}4169\ sin(\lambda t_{2i}) \\ \hat{y}_{i3} &= 45{,}7378 - 0{,}0173\ sin(\lambda t_{3i}) \\ \hat{y}_{i4} &= 6{,}3919 + 0{,}0018\ sin(\lambda t_{4i}) \\ \hat{y}_{i5} &= 6{,}7138 - 0{,}1253\ sin(\lambda t_{5i}) \end{aligned} \tag{18}$$

The results of multiresponse prediction analysis on testing data indicate MSE = 0.9539, R-Square = 0.9809, and MAPE = 4.7521%. The model accuracy results based on the MAPE value presented in Table 2 indicate a highly accurate prediction category, as the MAPE value is below 10%. This accuracy is further illustrated by the comparison of actual and predicted values for the testing data as shown in Figure 2.

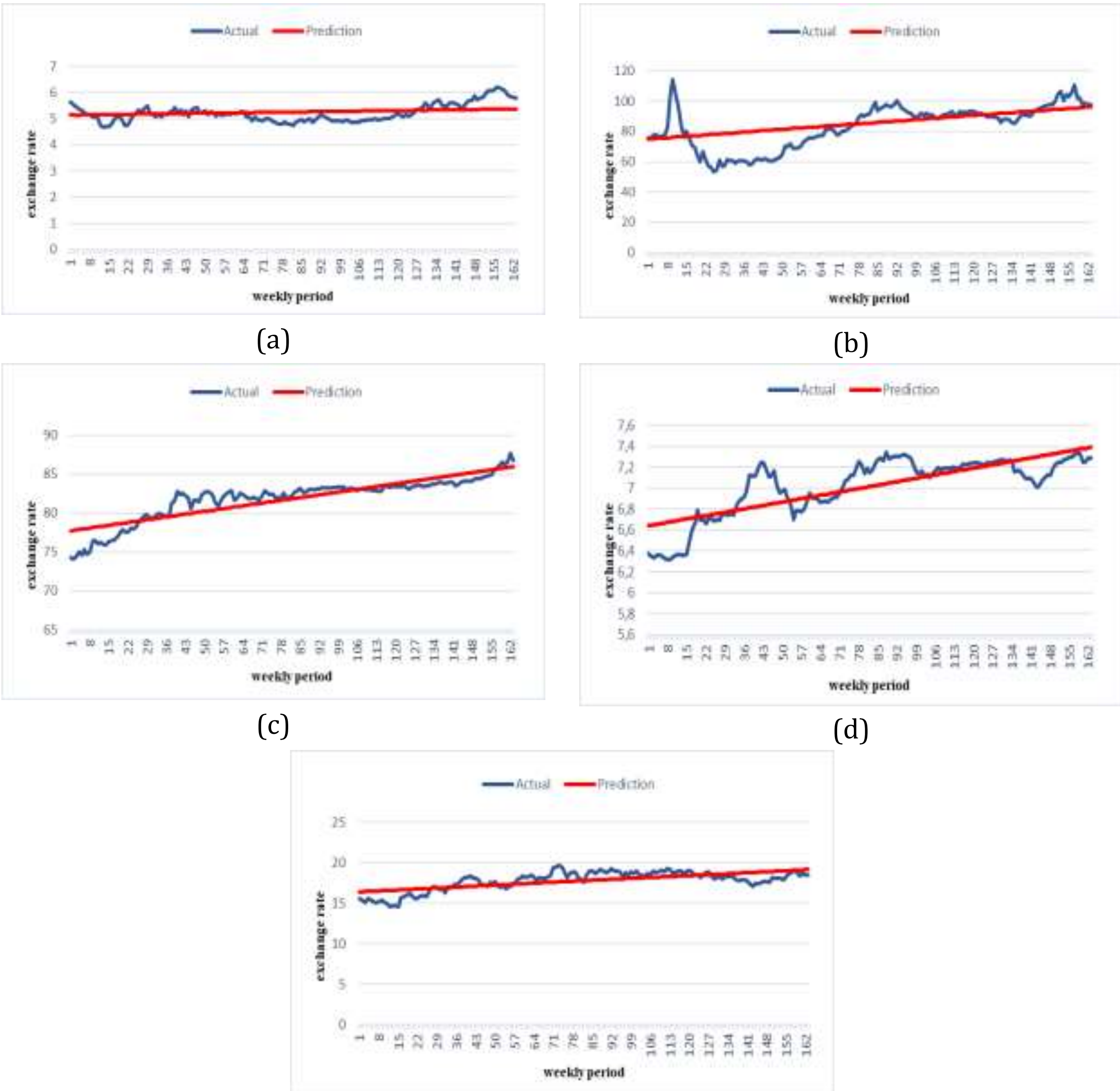

Figure 2. Plot of Prediction Results (a) USD/BRL, (b) USD/RUB, (c) USD/INR, (d) USD/CNY, (e) USD/ZAR

The prediction results of this study demonstrate strong predictive performance compared to previous studies on BRICS exchange rates. This study achieves a Mean Absolute Percentage Error (MAPE) of 4.7521%, which falls into the highly accurate category. The findings regarding BRICS currency volatility patterns are consistent with those reported in earlier research. A previous study reported that the Russian ruble, as reflected by the USD/RUB exchange rate, is the most depreciated and most volatile currency among BRICS countries, while the Chinese yuan (USD/CNY) is the most stable (Kumar et al., 2024). This is consistent with the descriptive analysis results of the present study, which show that USD/RUB exhibits the highest volatility with a standard deviation of 21.376, whereas USD/CNY shows the lowest volatility with a standard deviation of 0.3434. Similar findings are also reported in a study using an oil price–based machine learning model, which found that the Chinese yuan achieved the best prediction accuracy with the lowest MSE and RMSE among all BRICS currencies (Ahmed, 2025). This further reinforces the result of this study that USD/CNY exhibits the most stable pattern.

These consistent findings with previous studies indicate that the proposed multiresponse Fourier series model is not only statistically accurate but also substantively aligned with established economic dynamics of BRICS exchange rates. The validity of the model is supported by its evaluation on out-of-sample testing data, where a low MAPE value indicates strong predictive performance. This study employs weekly exchange rate data and relies solely on historical exchange rate movements of the five BRICS countries. Nevertheless, the generalization of the results should be interpreted with caution, as the model is constructed based on observed exchange rate patterns within the analyzed period and does not explicitly account for external economic shocks or additional explanatory variables.

D. CONCLUSION AND SUGGESTIONS

The results indicate that the five research variables are correlated and exhibit a fluctuating pattern, with the USD exchange rate against the Russian currency showing the highest standard deviation. The analysis using the Fourier series nonparametric regression approach identifies the optimal smoothing parameter (λ) as 1, based on the minimum GCV value. Consequently, the best estimation model is obtained using the sine function without a trend component, yielding a MAPE value of 4.7521%. This demonstrates that the proposed multiresponse Fourier series model is able to achieve the research objective by providing highly accurate predictions of the USD exchange rate against the five BRICS currencies.

These findings also imply that the proposed approach is capable of capturing heterogeneous exchange rate dynamics within the BRICS group, where currencies such as the Russian ruble exhibit higher volatility, while others such as the Chinese yuan display more stable patterns. In the context of dedollarization, this information is relevant as an empirical description, given that exchange rate stability is frequently discussed in the literature as one of the supporting factors for strengthening the role of local currencies in trade and monetary cooperation among BRICS countries.

Nevertheless, this study relies solely on historical exchange rate data and does not explicitly account for external economic shocks or additional explanatory factors that may affect currency movements. Therefore, the generalization of the results should be interpreted with caution. For future research, incorporating additional variables, such as exchange rates from all BRICS supporting countries, could enhance the study's impact and provide broader insights for policymakers in mitigating risks associated with the BRICS alliance's dedollarization efforts.

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