

Mathematical Modeling and Computational Simulation of Stunting Dynamics under Sanitation and Nutritional Influence

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ABSTRACT

Article History:

Received : 23-05-2026

Revised : 11-08-2026

Accepted : 12-08-2026

Online : 20-10-2026

Keywords:

Computational

Simulation;

Environmental

Sanitation;

Mathematical Modeling;

Nutritional

Intervention;

Python;

Stunting.



Stunting remains a major health issue in Central Kalimantan due to poor nutrition and sanitation. While previous mathematical models have largely focused on the direct impact of nutrition intervention, they often neglect the crucial influence of environmental sanitation, creating a gap in accurately describing stunting dynamic. To address this, this study develops a new mathematical model by integrating environmental sanitation (San) parameter into a system of ordinary differential equation (ODE). Utilizing an applied mathematical modeling approach, the model parameters were estimated using 2022 baseline data primarily sourced from the Provincial Health Office, Bapperida, and TPPS, while toddler birth and mortality rates were obtained from the Central Kalimantan Statistics Agency (BPS). This sanitation parameter mathematically contributes as a multiplier that modifies the transition rate of at risk toddlers and the overall effectiveness of nutrition intervention. After identifying the equilibrium points and proving their local stability using the Routh-Hurwitz Criteria, we conducted an analytical sensitivity analysis to find the most critical factors. We then ran numerical simulations using the Runge-Kutta 4(5) method in Python to project future trends. The simulations compared the current sanitation level with a scenario where sanitation improves by 20%. The computational results reveals that combining nutritional interventions with a 20% sanitation improvement drastically accelerates stunting reduction and potentially lowering the prevalence to 0.99% within twenty years. Therefore, nutritional programs must be supported by basic sanitation infrastructure improvements to effectively eradicate childhood stunting.



<https://doi.org/10.31764/jtam.v10i4.39901>



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A. INTRODUCTION

Stunting is a condition of growth failure in toddlers due to chronic malnutrition and repeated infections in the first 1,000 days of life (Mulyani et al., 2025; Pradnyawati et al., 2023; Salim et al., 2026). Although the prevalence of stunting in Central Kalimantan has successfully decreased to 23.5% in 2023, the initial figure was still quite high in 2022, at 26.9% (Ministry of Health of the Republic of Indonesia, 2022b). Many programmes focus on improving toddler nutrition, but the prevalence of stunting remains above the WHO threshold (20%), indicating the need to evaluate other determinants (Astuti et al., 2025; Fitri et al., 2025; Himmah et al., 2025). It is not only factors related to the child that play a crucial role, maternal health and numerous pregnancy complications have been shown to permanently impair fetal growth and

development in the womb then increase the risk of stunting as the child grows (Widyawati & Ristandiati, 2026).

In addition to calorie and nutrient intake, the risk of stunting is significantly influenced by external factors such as poor environmental hygiene and sanitation (Hasanah et al., 2020; Mulyani et al., 2025; Wijayanto et al., 2026; Wicaksono et al., 2021). Particularly in Southeast Asia, a recent literature review confirmed that environmental sanitation, drinking water sources, and family toilet facilities are fundamental risk factors that crucially trigger the high prevalence of stunting in toddlers (Azriani et al., 2024). This regional challenge is highly prevalent in Indonesia. For instance, a study conducted in the neighboring region of West Kalimantan confirms that recurrent diarrhea which is linked to poor environmental hygiene and sanitation serve a major determinants significantly increasing the risk of child stunting (Sartika et al., 2021). Specifically in the focus area of this research, a significant risk of stunting was found in communities in Central Kalimantan, particularly in the Kahayan River area, due to the lack of sanitation facilities and the habit of disposing of waste directly into the river (Shinta et al., 2020). These unhygienic environmental condition triggers repeated infectious diseases in toddlers, such as diarrhea, which ultimately inhibits the process of nutrient absorption.

Environmental constraints often undermine the outcomes of effective nutrition programs. Major research in Indonesia Sjarif et al. (2025) demonstrates that the regular provision of animal based protein, such as milk and eggs, is highly effective in preventing growth faltering and supporting the development of at-risk children. Nevertheless, achieving optimal results from this intervention still requires a supportive surrounding environmental health context. Nutritional intervention provided to toddlers at risk of stunting will not be optimal without improvements in environmental sanitation (Wegmüller et al., 2026). No matter how well defined specific nutritional interventions are implemented, stunting prevalence will be slow to decline if sanitation levels in the area remain poor. Poor sanitation infrastructure and difficult access to clean water are the main triggers of nutritional problems (Cooten et al., 2019; Sahiledengle et al., 2022). Although the government has implemented various specific and sensitive nutritional interventions, the results achieved have not been optimal due to an over focus on health and nutrition aspects alone (Saputri et al., 2020). Therefore, an integrated approach is needed that takes into account hygiene and sanitation factors so that nutritional intervention programs can work effectively and stunting prevalence can be reduced more quickly (Cumming & Cairncross, 2016).

In international literatures, the determinants of childhood stunting have traditionally been evaluated using static statistical methods, such as structural equation path analysis (Locks et al., 2025; Sharma et al., 2025). A limitation of this approach is its inability to capture changes in conditions or the rate at which children transition between groups over time. Meanwhile, standard epidemiological disease transmission models, such as SEIR or SIR, are epidemiologically inappropriate because stunting is not, fundamentally, an infectious disease. Furthermore, although recent studies have examined stunting through the lenses of environmental factors and multi-stakeholder collaboration (Astuti et al., 2025), they have not employed mathematical formulas to project the long-term progression of cases.

Therefore, previous research provides novelty by modifying a deterministic compartmental model based on a more appropriate system of ordinary differential equations (ODEs). The resulting stunting dynamics model focuses purely on the effectiveness of nutritional interventions (Himmah et al., 2025). As a development of this research, the model in this study was modified to accommodate environmental factors by adding environmental sanitation level (*San*) parameters whose values range from 0 (very bad) to 1 (very good) to the stunting dynamics transition rate. The inclusion of this parameter is in line with empirical evidence showing that poor sanitation facilities are strongly associated with a high risk of stunting in children (Li et al., 2020). Conversely, access to safe drinking water and good sanitation has been proven to reduce the risk of stunting (Batool et al., 2023; Patlan- Hernandez et al., 2021).

The sanitation parameter directly affects the compartmental transition rates. Deteriorating sanitation ($San \rightarrow 0$) will increase the rate of at risk toddlers (α) and stunting progression (β), while simultaneously reducing the effectiveness of nutritional interventions (ε) and the recovery rates. Through numerical simulations computationally executed using Python programming, this study aims to mathematically investigate and prove the extent to which improvements in environmental sanitation can accelerate the reduction of stunting prevalence in Central Kalimantan.

This study fills a theoretical gap in our previous study Himmah et al. (2025), which assumed a constant and independent transition rate based solely on nutritional factors. This limitation occurred because the previous model focused purely on nutritional intake as the primary factor causing stunting, ignoring the important role of environmental sanitation. Epidemiologically, poor sanitation can lead to recurrent infectious diseases such as diarrhea, which can inhibit nutrient absorption. Therefore, no matter how effective nutritional interventions is, a toddler nutrient absorption will not be optimal due to poor sanitation condition. Consequently, modeling nutritional recovery and stunting progression as independent constants is mathematically inaccurate. To address this weakness, we modified the transition rate of the ordinary differential equation (ODE) system by integrating environmental sanitation level (*San*) as a multiplier.

B. METHODS

This research employed a literature review method and applied mathematical modeling approach. The research workflow is divided into ten sequential steps. First, the preliminary study phase was conducted to explore the dynamics of stunting and identify risk factors, specifically focusing on the influence of environmental sanitation conditions on the effectiveness of nutritional interventions in Central Kalimantan. Second, in the Data Collection stage, secondary data regarding stunting prevalence, nutritional intervention coverage, and demographic rates were gathered from regional health and statistical agencies. Specifically, stunting and intervention data were primarily source from the Provincial Health Office, The Regional Planning, Research, and the Innovation Agency (Bapperida), and the Stunting Reduction Acceleration Team (TPPS), while data on toddler birth and mortality rates were obtained from the Central Kalimantan Provincial Statistics Agency (BPS). Third, during the Sanitation Data Pre-Processing, the environmental sanitation level (*San*) parameter was

estimated using an index method calculation involving data on percentage of households with access to improved drinking water and sanitation facilities, which was then normalized using The Min-Max Normalization Method.

Fourth, the Model Formulation was carried out to develop a compartmental mathematical model in the form of an ordinary differential equation system (ODE) that represents the dynamics of stunting incidence with the modification of the addition of the sanitation level parameter (San). Fifth, Equilibrium Point Analysis was performed to determine the steady states of the system, identifying both the stunting free equilibrium point (TE_0) and the stunting equilibrium point (TE^*). Sixth, a local stability analysis was evaluated by analyzing the eigenvalues of the Jacobian matrix and applying the Routh-Hurwitz stability criterion to access the behavior of the system around this equilibrium point. Seventh, an analytical Parameter Sensitivity Analysis was executed using the normalized sensitivity index method to determine the most critical factors influencing stunting dynamics. Eighth, Numerical Simulations were conducted using the 5th order Runge-Kutta (specifically Python's SciPy solve-ivp RK45 solver) to project future stunting prevalence based on estimated baseline parameters. Ninth, a Comparative Analysis of Sanitation Impact was performed to compare stunting reduction trajectories across different scenarios, without sanitation support ($San = 0$), under current sanitation levels ($San = 0.295625$) and with a 20% sanitation improvement ($San = 0.69275$). Tenth, the Interpretation of Results and Conclusions was carried out to translate the simulation findings and mathematical properties into actionable policy recommendations for the regional Stunting Reduction Acceleration Team (TPPS) to accelerate stunting eradication. The complete stages of this research are visually illustrated in the flowchart in Figure 1.

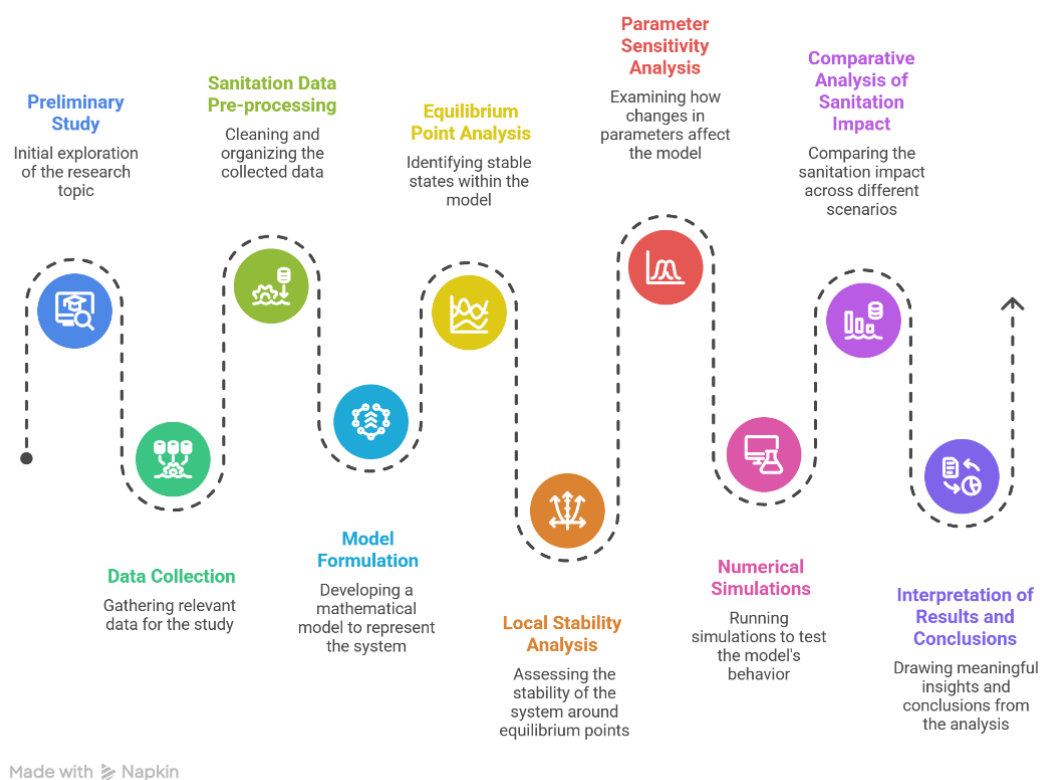


Figure 1. Flowchart of the Research Methodology (Generated by Napkin AI)

C. RESULT AND DISCUSSION

1. Mathematical Modeling of Stunting with the Influence of Sanitation and Nutritional Intervention

The proposed mathematical model is a system of ordinary differential equations (ODE). To formulate this mathematical model, we divide the population into four compartment variables and several parameters, including environmental sanitation (San) factors, as presented in Table 1.

Table 1. Description of Variables and Parameters

Symbol	Description
Variables	
R	Proportion of toddlers at risk of stunting
S	Proportion of toddlers stunted
I	Proportion of toddlers intervened
N	Proportion of toddlers normal
Parameters	
α	The rate of entry of new toddlers into the stunting risk compartment
β	The transition rate of at-risk toddlers into the stunted compartment
π	The transition rate to receive nutritional intervention from compartment R
ω	The transition rate to receive nutritional intervention from compartment S
δ	The recovery rate of successfully intervened toddlers
k	The recovery proportion of successfully intervened toddlers
ε	The effectiveness of the nutritional intervention program
θ	The coverage of the nutritional intervention program
μ	Natural toddler mortality rate
γ	The exit rate (exceeding 5 years old)
San	Environmental sanitation level index ($0 \leq San \leq 1$)

In order to simplify the complicated reality, this mathematical model is built on a number of basic epidemiological presumptions. First, the population is considered to be closed, which means that the natural death rate (μ) is constant throughout all compartments and there is no demographic mobility (migration) during the simulation period. Furthermore, we assume that toddlers receiving the intervention are evenly dispersed within the age range of 0 to 5 years, and both stunted and at-risk toddlers have equal access to the intervention program. The model also assumes stunted toddlers cannot return to normal height for their age. However, nutritional interventions are highly effective in treating malnutrition and preventing its worsening. Therefore, toddlers who successfully recover to compartment N are defined as those who have achieved normal nutritional status.

The novelty of this model is the addition of an environmental sanitation level parameter (San) which ranges from 0 (very poor) to 1 (very good). The addition of the sanitation factor (San) to the model has an impact on several model parameters. Poor sanitation ($San \rightarrow 0$) will increase the rate of entry of new toddlers into the at-risk group (α), and the transition rate from at-risk to stunting (β), while simultaneously reducing the effectiveness of nutritional interventions (ε), slowing the rate of toddler entry into nutritional intervention programs (ω), and slowing the rate of recovery (δ), as shown in Figure 2.

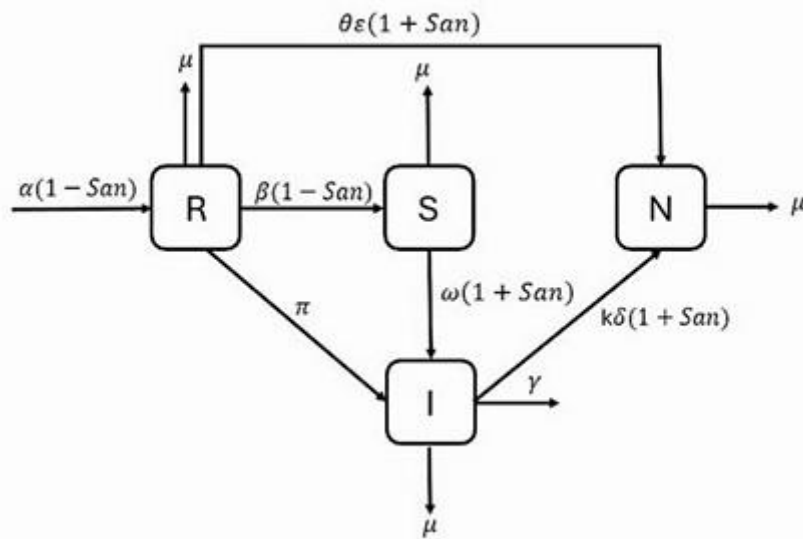


Figure 2. Compartment Diagram of Stunting Model with Sanitation and Nutritional Intervention

Based on the compartment diagram in Figure 2, the mathematical model of stunting dynamics is formulated as follows:

- a. Proportion of toddlers at risk of stunting (R)

The ordinary differential equation (ODE) that describes the change in the proportion of toddlers at risk of stunting over time is formulated as follows:

$$\frac{dR}{dt} = \alpha(1 - San) - \beta(1 - San)R - \pi R - \theta\epsilon(1 + San)R - \mu R \quad (1)$$

Unlike other epidemiological models, the dynamics in this compartment are influenced by environmental sanitation factors (San). Equation (1) shows that the dynamics of changes in the proportion of toddlers at risk of stunting (R) over time are driven by the interaction between the addition of new cases and the transition of toddlers to other compartments. This proportion increases with the entry of the number of toddlers at risk of stunting at a rate of $\alpha(1 - San)$.

Mathematically, the term $(1 - San)$ acts as an attenuation factor for the infection and deterioration rates. If sanitation is extremely poor ($San \rightarrow 0$), the value of this multiplier becomes $(1 - 0) = 1$. This implies that toddlers in unhygienic environments are at the highest risk of contracting infectious diseases, causing the rate of new cases (α) and stunting progression (β) to operate at their maximum baseline capacity. Conversely, if sanitation is improved to the ideal level ($San \rightarrow 1$), the multiplier approaches $(1 - 1) = 0$. Epidemiologically, this ideal sanitation reduces risks caused by the environment, thereby significantly reducing the rate of toddlers entering the at risk compartment or deteriorating into the stunting group.

Therefore, the population in this R compartment will decrease when toddlers at risk experience worsening conditions and enter the stunting toddler group (S) at a rate of $\beta(1 - San)R$ where sanitation factors can accelerate the rate of deterioration. In addition, this decrease in proportion also occurs due to the presence of at risk toddlers

who were successfully prevented from stunting due to nutritional intervention and entered the N compartment at a rate of $\theta\varepsilon(1 + San)R$, sanitation factors again play an important role in changing the nutritional status of toddlers to normal. Another factor is the number of toddlers who began receiving nutritional interventions and moved to compartment I at a rate of πR , as well as the proportion of toddlers in this compartment who left the system due to natural death at a rate of μR .

- b. Proportion of stunted toddlers (S),

The differential equation representing the dynamic changes in the proportion of stunted toddlers is formulated as follows:

$$\frac{dS}{dt} = \beta(1 - San)R - \omega(1 + San)S - \mu S \quad (2)$$

Based on Equation (2), the increase in population in this compartment occurred purely due to the worsening condition of toddlers at risk (R) of becoming stunted at rate $\beta(1 - San)R$. On the other hand, the proportion of stunted toddlers will decrease as stunted toddlers enter the nutrition intervention program (I) at a rate of $\omega(1 + San)S$. In this term, the factor $1 + San$ indicates that adequate environmental sanitation improvements will accelerate the process of attracting toddlers to receive nutrition intervention. In addition, the rate of decline in the proportion of stunted toddlers (S) was also caused by the natural death of stunted toddlers at a rate of μS .

- c. Proportion of toddlers receiving nutritional intervention (I)

The differential equation representing the dynamic changes in the proportion of toddlers receiving nutritional intervention is formulated as follows:

$$\frac{dI}{dt} = \pi R + \omega(1 + San)S - k\delta(1 + San)I - \mu I - \gamma I \quad (3)$$

Compartment I represents the dynamics of toddlers receiving nutritional intervention. The increase in the proportion of toddlers in this compartment comes from the inclusion of some toddlers at risk of stunting (R) at a rate of πR , as well as the withdrawal of stunted toddlers (S) to receive nutritional intervention at a rate of $\omega(1 + San)S$.

Some toddlers who received nutritional intervention were able to recover their nutritional status to normal. The recovery of some toddlers with a proportion of k and a recovery rate of $\delta(1 + San)I$ reduced the proportion of toddlers in group I , where good sanitation accelerated this recovery process. Other factors that reduced the proportion of compartment I were natural death of toddlers with a rate of μI , as well as the exit of toddlers from compartment I because their age had passed the maximum age limit of the intervention period (age 5 years), with a rate of γI .

- d. Proportion of toddlers with normal nutritional status (N)

The mathematical equation that represents the dynamic changes in the proportion of toddlers with normal nutritional status is formulated as follows:

$$\frac{dN}{dt} = k\delta(1 + San)I + \theta\varepsilon(1 + San)R - \mu N \quad (4)$$

The population in compartment (N) is formed from the success of prevention and recovery intervention programs. The proportion of (N) increases as the number of toddlers from the intervention group (I) who successfully recover and achieve normal nutritional status at a rate of $k\delta(1 + San)$. The proportion of toddlers in this group also increases because toddlers at risk of stunting (R) whose deterioration is successfully prevented through early intervention so that their nutritional status improves, at a rate of $\theta\epsilon(1 + San)R$. In both mechanism, environmental sanitation conditions (San) act as a multiplier that accelerates the rate of recovery and prevention.

Specifically, the term $(1 + San)$ acts to boost the effectiveness of the intervention. If sanitation is extremely poor ($San \rightarrow 0$), the effectiveness of the nutritional program is limited to its minimum level, which is $\theta\epsilon(1 + 0) = \theta\epsilon$. However, if sanitation is ideal ($San \rightarrow 1$), the effectiveness the nutritional intervention will double to $\theta\epsilon(1 + 1) = 2\theta\epsilon$. This proves that improving sanitation helps toddlers absorb nutrients better by preventing infectious diseases. As a result, this highly increases the success of the nutrition intervention in returning toddlers to normal nutritional status (N). Finally, the decrease in the proportion in this group is only caused by the natural mortality factor of toddlers at a rate of μN .

The mathematical model of stunting dynamics involving complete nutritional and sanitation intervention based on Equations (1), (2), (3), and (4) is:

$$\begin{aligned}\frac{dR}{dt} &= \alpha(1 - San) - \beta(1 - San)R - \pi R - \theta\epsilon(1 + San)R - \mu R, \\ \frac{dS}{dt} &= \beta(1 - San)R - \omega(1 + San)S - \mu S, \\ \frac{dI}{dt} &= \pi R + \omega(1 + San)S - k\delta(1 + San)I - \mu I - \gamma I, \\ \frac{dN}{dt} &= k\delta(1 + San)I + \theta\epsilon(1 + San)R - \mu N.\end{aligned}\tag{5}$$

with the initial conditions $R(0) > 0, S(0) > 0, I(0) > 0$, and $N(0) > 0$.

2. The Equilibrium Point

After obtaining the ordinary differential equation system (ODEs) as a model of stunting dynamics (Model 5), the next step is to investigate the longterm behaviour of the system solution. This behaviour can be analysed through an equilibrium point, which represents a condition where the system solution does not change over time, meaning the proportion of toddlers in each compartment no longer changes. This steady state condition is obtained by setting each rate of change to zero, namely $\frac{dR}{dt} = 0$, $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, and $\frac{dN}{dt} = 0$. By solving this equation, the model produces two equilibrium points: a stunting free equilibrium point and a stunting equilibrium point.

a. Stunting-Free Equilibrium Point (TE_0)

The stunting free equilibrium occurs when there are no stunted toddlers in the population, defined as $TE_0 = (R_0, S_0, I_0, N_0) = (0, 0, 0, 0)$. This condition occurs only if environmental sanitation is very good ($San = 1$). This suggests that stunting cannot be completely eradicated in the absence of optimal sanitation.

b. Stunting Equilibrium Point (TE^*)

The stunting equilibrium point represents a condition where stunted toddlers exist in the population ($S \neq 0$). This equilibrium point, $TE^* = R^*, S^*, I^*, N^*$, are obtained by solving the system when it is equated to zero. To simplify the mathematical expressions, we define the following substitution constants representing the total outflow rates from each compartments:

$$K_1 = \beta(1 - San) + \pi + \mu + \theta\varepsilon(1 + San),$$

$$K_2 = \omega(1 + San) + \mu,$$

$$K_3 = k\delta(1 + San) + \mu + \gamma.$$

By utilizing these constants, the exacr coordinates of the stunting equilibrium point can be simplified as follows:

$$\begin{aligned} TE^* &= R^*, S^*, I^*, N^* \\ &= \left(\frac{\alpha(1 - San)}{K_1}, \frac{\beta(1 - San)^2 R^*}{K_2}, \frac{\pi R^* + \omega(1 + San) S^*}{K_3}, \frac{k\delta(1 + San) I^* + \theta\varepsilon(1 + San) R^*}{\mu} \right) \end{aligned}$$

3. Equilibrium Point Stability Analysis

The stability analysis of the equilibrium point is determined by evaluating the eigenvalues derived from the Jacobian matrix of the system. The Jacobian Matrix (J_f) is constructed by taking the partial derivatives of each ordinary differential equation to each variable (R, S, I, N). For this model, the general Jacobian matrix is formulated as follows:

$$J_f = \begin{bmatrix} -\beta(1 - San) + \pi + \theta\varepsilon(1 + San) & 0 & 0 & 0 \\ \beta(1 - San) & -(\omega(1 + San) + \mu) & 0 & 0 \\ \pi & \omega(1 + San) & -(k\delta(1 + San) + \mu + \gamma) & 0 \\ \theta\varepsilon(1 + San) & 0 & k\delta(1 + San) & -\mu \end{bmatrix}$$

To determine the stability of the system, the characteristic polynomial of the Jacobian Matrix defined as $P(\lambda) = \det(\lambda I - J_f) = 0$ is evaluated to obtain the eigenvalue (λ). Based on the matrix structure, the eigenvalue of system (5) is obtained: $\lambda_1 = -(\beta(1 - San) + \pi + \mu + \theta\varepsilon(1 + San))$, $\lambda_2 = -(\omega(1 + San) + \mu)$, $\lambda_3 = -(k\delta(1 + San) + \mu + \gamma)$, $\lambda_4 = -\mu$.

It is known that all the defined parameters represent biological and epidemiological rates, which means their values are always positive. Futhermore, the environmental sanitation index (San) operates within the interval $0 \leq San \leq 1$. Consequently, the terms $(1 - San)$ and $(1 + San)$ will always yield positive values. Given these conditions, it is mathematically proven that all eigenvalues are strictly negative ($\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0, \lambda_4 < 0$). Since all real parts of the

eigenvalues are negative, then based on the Routh-Hurwitz Criterion, the mathematical model of stunting dynamics with the influence of sanitation and nutritional interventions is locally asymptotically stable. To visualize this mathematical property, a longterm numerical simulation was conducted. As illustrated in Figure 3, the proportion of toddlers in each compartment slowly but consistently converges to a constant state, supporting the analytical findings of the system's local asymptotic stability.

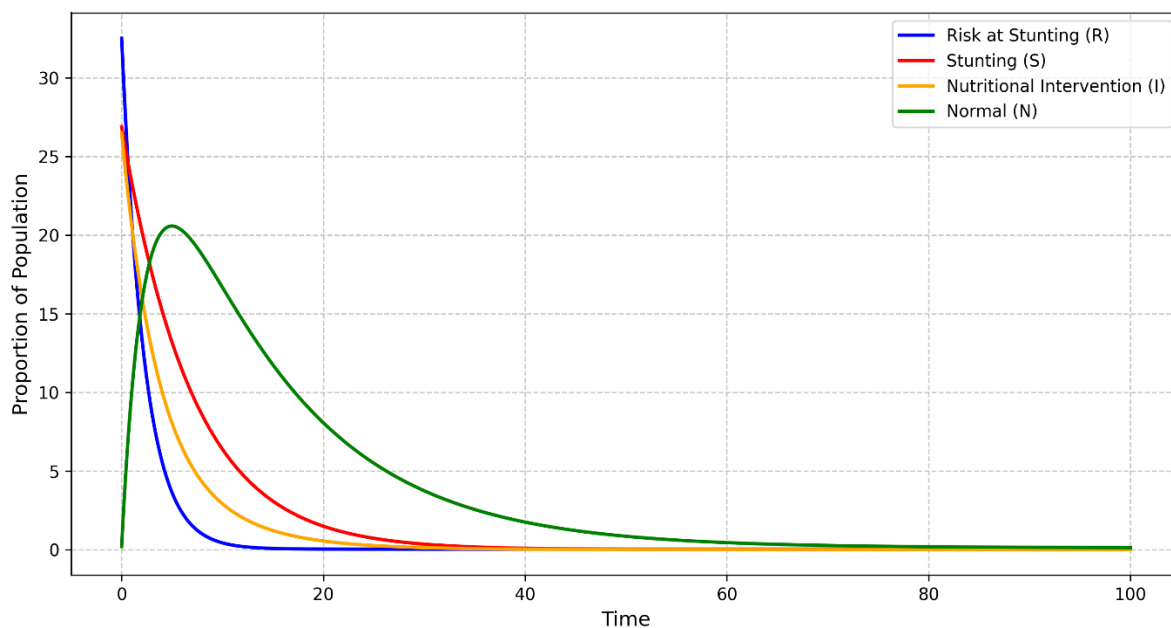


Figure 3. Simulation of Locally Asymptotically Stable Equilibrium Point for The Stunting Model with the Influence of Nutritional Intervention and Sanitation

Epidemiologically, this steady-state condition indicates that if there are small disturbances or fluctuations in the population dynamics, the system will return to its equilibrium over time.

4. Numerical Simulation

Python programming was used to run numerical simulations in order to project the stunting dynamics in Central Kalimantan and assess the reasonableness of the model's interpretation. To illustrate the computational approach, the numerical solution procedure developed using the Python programming. The computational process makes use of the `solve_ivp` function from the SciPy library and applies the Runge-Kutta method of order 5(4) or RK45. The final result of this procedure is a time series of population dynamics which is then visualized in the form of a line chart. The detailed computational algorithm is presented as a pseudocode in Algorithm 1.

Algorithm for Stunting Dynamics Simulation

This algorithm describes the computational procedure for simulating stunting dynamics using a modified Ordinary Differential Equation (ODE) model. Specifically designed for implementation in Python.

Algorithm 1: Computational Procedure for Stunting Dynamics Simulation

Input:

Initial population condition (R_0, S_0, I_0, N_0) , simulation time range (T_{end}) , parameters $(\alpha, \beta, \pi, \omega, \mu, \delta, k, \varepsilon, \theta, \gamma)$; raw sanitation access data (San_{raw}) .

Output:

Time series of population dynamics $R(t), S(t), I(t), N(t)$.

Procedure DataPreProcessing (San_{raw}) :

$San_{norm} \leftarrow -(San_{raw} - San_{min}) / (San_{max} - San_{min})$ // Normalisation process

return San_{norm}

Procedure ODE_System (t, Y, San_{norm}) :

$R, S, I, N \leftarrow Y$,

$\frac{dR}{dt} \leftarrow \alpha(1 - San) - \beta(1 - San)R - \pi R - \theta\varepsilon(1 + San)R - \mu R$,

$\frac{dS}{dt} \leftarrow \beta(1 - San)R - \omega(1 + San)S - \mu S$,

$\frac{dI}{dt} \leftarrow \pi R + \omega(1 + San)S - k\delta(1 + San)I - \mu I - \gamma I$, (5)

$\frac{dN}{dt} \leftarrow k\delta(1 + San)I + \theta\varepsilon(1 + San)R - \mu N$.

return $\left[\frac{dR}{dt}, \frac{dS}{dt}, \frac{dI}{dt}, \frac{dN}{dt} \right]$

Initialisation:

$San_{op} \leftarrow \text{DataPreProcessing} San_{raw}$

$Y_0 \leftarrow [R_0, S_0, I_0, N_0]$

Execution (Python SciPy solve_ivp/ RK45):

$Solution \leftarrow \text{solve_ivp}(fun = \text{ODE_System}, t_{span} = [0, T_{end}],$

$y_0 = Y_0, method = 'RK45', args = San_{op}))$

Solution using Runge-Kutta 5(4)

Plotting:

Generate Line Chart $(Solution.t, Solution.y)$

The simulation utilized the established initial population conditions and parameter values, which were estimated based on actual data primarily from the Provincial Health Office, Bapperida, and TPPS, with demographic rates (birth and mortality) sourced from the Central Kalimantan Statistics Agency (BPS). Specifically for the newly introduced environmental sanitation parameter (San), the exact value was calculated prior to the simulation using the index method. This was done by multiplying the percentage of households with access to proper sanitation facilities (73.6%), access to proper drinking water (77.72%), and overall proper sanitation access (76.31%) (Ministry of Health of the Republic of Indonesia, 2022a). The multiplication resulted in an index of 43.65%. The results obtained are then normalized to range between 0 and 1 using the Min-Max Normalisation. By applying linear changes to the original data, the Min Max normalisation approach creates a balance of value comparisons

between the data before and after the operation (Henderi et al., 2021). Mathematically, the normalisation equation used in this study is formulated as follows:

$$San_{norm} = (San - San_{min}) / (San_{max} - San_{min}) \quad (6)$$

Where San_{norm} represent the normalized sanitation value, San is the actual sanitation value obtained from the index method calculation (43.65%), San_{min} is the assumed minimum bound (20%), and San_{max} is the assumed maximum bound (100%). By substituting these values into the equation, the calculation yields:

$$San_{norm} = (43.65 - 20) / (100 - 20) = 0.295625$$

This normalized result is then defined as the operational environmental sanitation parameter ($San = 0.295625$) used for the numerical simulation. The complete numerical values used for this simulation, including the calculated sanitation parameter, are presented in Table 2.

Table 2. Initial Conditions and Parameter Values

Symbol	Value	Source/Type
Initial Condition		
$R(0)$	32.52	$\frac{\text{Total at risk toddlers in 2022}}{\text{Total toddler population}} \times 100\%$
$S(0)$	26.900	$\frac{\text{Total stunted toddlers in 2022}}{\text{Total toddler population}} \times 100\%$
$I(0)$	26.4965	$\frac{\text{Total toddlers receiving intervention in 2022}}{\text{Total toddler population}} \times 100\%$
$N(0)$	0.19	$\frac{\text{Total normally recovered toddlers in 2022}}{\text{Total toddler population}} \times 100\%$
Parameters		
α	0.0168	$\frac{\text{Number of new births}}{\text{Total toddler population}}$
β	0.00692	$\frac{\text{Number of new stunted case}}{\text{Total toddlers at risk}}$
π	0.0081	$\frac{\text{Number of at risk toddlers entering intervention}}{\text{Total toddlers at risk}}$
ω	0.05	assumption (due to the unavailability of empirical data regarding the exact proportion of stunted toddlers enetering the intervention program)
δ	0.1912	$\frac{\text{Number of recovered toddlers}}{\text{Total toddlers receiving intervention}}$
k	0.20	assumption (due to the unavailability of empirical data on targeted intervention capacity)
ε	0.30	Expert interview estimation (indicating 30% baseline effectiveness for nutritional interventions without sanitation support)
θ	0.891	$\frac{\text{Number of at risk toddlers receiving intervention}}{\text{Total toddlers at risk}}$
μ	0.081	$\frac{\text{infant deaths} + \text{toddlers deaths}}{1000}$ (converted from the recorded epidemiological
γ	0.20	$\frac{1}{\text{Maximum age limit (5 years)}}$ (assuming a uniform age distribution, 1/5 of the cohort exits the intervention program annually)
San	0.295625	$(San - San_{min}) / (San_{max} - San_{min})$

The numerical simulation results of model (5) over a 30 year period with the initial condition and parameter values in Table 2 are presented in Figure 4.

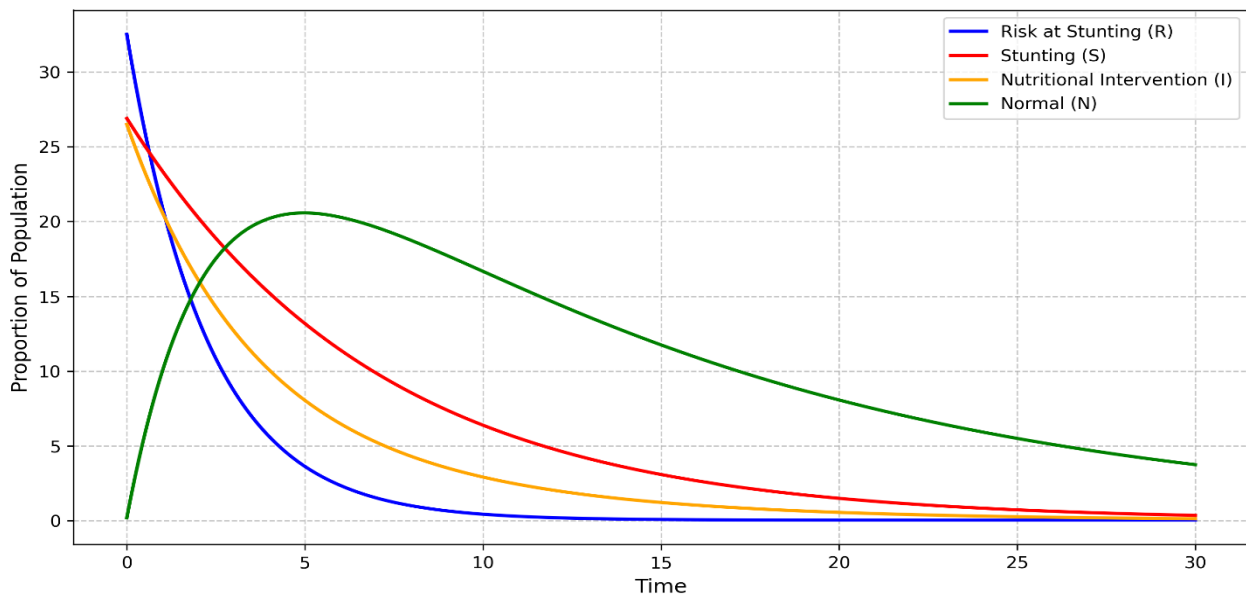


Figure 4. Simulation of Mathematical Model for Stunting with the Influence Nutritional Intervention and Sanitation ($San = 0.295625$)

Based on the current sanitation level ($San = 0.295625$), the simulation demonstrate an initial decrease in stunting prevalence from 26.9% in 2022 to 23.34% in the first year (2023) . This downward trend is projected to continue, with the prevalence dropping to 20.22% in 2024 and 17.49% in 2025. Long term projection indicate that the prevalence will further decline to 6.47% by the tenth year (2032) and reach 1.49% by the twentieth year (2042). Ultimately, by the thirtieth year (2052), the prevalence is projected to flatten out at 0.35%.

To demonstrate the reliability of this mathematical model quantitatively, the authors validated it against actual data using the Absolute Percentage Error (APE) method. The simulated stunting prevalence in the first year (2023) was compared with actual data recorded by the Ministry of Health for the same year (23.5%). The error was calculated mathematically as follows:

$$\begin{aligned}
 APE &= \frac{|Actual\ Data - Simulation\ Data|}{Actual\ Data} \times 100\% \\
 &= \frac{|23.5\% - 23.34\%|}{23.5\%} \times 100\% = 0.68\%.
 \end{aligned}
 \tag{7}$$

The result show that the margin of error (0.68%) is much smaller than acceptable 5% limit, confirming a highly accurate goodness-of-fit. Therefore, the proposed model is reliable and valid for projecting long-term stunting dynamics.

5. Analysis of Sensitivity

A local sensitivity was carried out analytically to determine the most important factors affecting stunting dynamics under the influence of environmental sanitation. The relative changes in the endemic equilibrium of stunting (S^*) are measured by this mathematical study in response to changes in a specific parameter by utilizing partial derivatives. The normalized sensitivity index of parameter p , $C_p^{S^*}$, for the proportion of stunted toddlers in a state of equilibrium, S^* , is defined as (Himmah et al., 2025):

$$C_p^{S^*} = \frac{\partial S^*}{\partial p} \times \frac{p}{S^*} \quad (8)$$

Based on the established mathematical model integrating the sanitation factor, the analytical equilibrium point for stunted toddlers (S^*) is given by:

$$S^* = \frac{\alpha\beta(1 - San)^2}{[\beta(1 - San) + \pi + \theta\varepsilon(1 + San)][\omega(1 + San) + \mu]}$$

In demonstrating this analytical procedure, we specifically highlight the parameters α and ω . This is because in our previous study (Himmah et al., 2025), these two parameters were the most highly sensitive. Parameter α serves as the main threat factor (the sources of new cases), while ω represents the most impactful transition rate for intervention solutions.

By applying the partial derivative principles, the normalized sensitivity index of parameter α to S^* is analytically derived as:

$$C_\alpha^{S^*} = \frac{\partial S^*}{\partial \alpha} \times \frac{\alpha}{S^*} = 1$$

A rise in the number of new toddlers at risk of stunting will directly and proportionately raise the long term equilibrium prevalence of stunting, as demonstrated mathematically by this positive index (+1). Furthermore, the analytical sensitivity index for parameter ω is derived as:

$$C_\omega^{S^*} = \frac{\partial S^*}{\partial \omega} \times \frac{\omega}{S^*} = \frac{-\omega(1 + San)}{\omega(1 + San) + \mu}$$

Since all parameters are positive, the sensitivity index for the parameter ω is negative. Mathematically, this indicates that increasing ω will strictly reduce stunting prevalence under equilibrium state. Crucially, the sanitation parameter (San) multiplies multiple rates in the system, resulting in a complex negative derivative $\left(\frac{\partial S^*}{\partial San}\right)$. Because it strongly controls the infection (β) and recovery (ω) rates, numerical scenario comparisons are necessary to translate these analytical results into practical policy recommendations.

6. Comparative Analysis of Sanitation Impact

To provide a more comprehensive understanding of how environmental sanitation (San) parameters specifically influence stunting dynamics, a comparative analysis of different scenarios was conducted. The scenarios compared were the stunting prevalence projections generated by the baseline nutrition intervention model (without sanitation) established in our previous study (Himmah et al., 2025) with a newly proposed model incorporating current sanitation conditions ($San = 0.295625$) and a hypothetical optimal scenario with improved sanitation conditions.

Next, Model (5) was re-simulated under the hypothetical scenario by increasing the environmental sanitation level. This was systematically done by increasing the current access percentages by 20%. Consequently, the proportion of households with access to proper sanitation facilities (healthy latrines) increased to 88.32%, access to proper drinking water increased to 93.26%, and overall proper sanitation access increased to 91.57%. By applying the index method, this improvement yielded a raw sanitation level of 75.42%. Min-Max Normalization was then performed, resulting in an operational sanitation value of $San = 0.69275$. A 20% hypothetical improvement was selected for this optimal scenario as it represents a realistic yet significant infrastructure development target, elevating the current sanitation access to approach universal coverage. Furthermore, this scenario mathematically demonstrates the level of sensitive intervention required to accelerate the stunting decline towards the government's regional target of 15.38%. The comparison of the projected stunting prevalence over a 30-year simulation period based on these three different scenario is summarized in Table 3.

Table 3. Comparison of Stunting Prevalence Based on Different Scenario

Scenario Model	Sanitation Level (San)	Stunting Prevalence (%)					
		Year 1 (%)	Year 2 (%)	Year 3 (%)	Year 10 (%)	Year 20 (%)	Year 30 (%)
Nutritional Intervention Only	0	23.74	20.92	18.42	7.59	2.03	0.55
Intervention Current Sanitation ⁺	0.295625	23.34	20.22	17.49	6.47	1.49	0.35
Intervention Improved Sanitation ⁺	0.69275	22.84	19.38	16.44	5.17	0.99	0.19

From an epidemiological perspective, the analytical and numerical results demonstrate that relying solely on nutritional interventions is insufficient to eradicate stunting. Without optimal sanitation improvements, even the most robust nutritional intervention programs will always leave residual stunting cases in the population. Biologically, the sensitivity analysis reveals that the influx of new at-risk toddlers (α) is the primary threat, while environmental sanitation acts as a booster for both prevention and recovery. The simulation shows that when infrastructure is improved by 20% the decline in stunting prevalence becomes faster and aligns with the government's regional target of emphasizing that combining nutritional interventions with improved WASH (Water, Sanitation, and Hygiene) practises is essential to accelerate the reduction of stunting (Batoool et al., 2023; Sahiledengle et al., 2022).

The proposed mathematical model, while quite robust, must be acknowledged to have several limitations. First, the model assumes a closed population system, thereby excluding the impact of dynamic migration or other demographic factors. Second, due to the unavailability of data regarding the targeted capacity of nutritional intervention programs, the recovery proportion (k) and the withdrawal rate (ω) were estimated using epidemiological assumptions without field calculations. Finally, the proposed model focuses exclusively on sanitation and direct nutritional interventions, and has not yet accommodated other interventions that could affect stunting dynamics, such as maternal health and toddler immunization programs. These limitations serve as a foundation for future model development.

D. CONCLUSION AND SUGGESTIONS

This study has successfully developed a novel deterministic compartmental model (ODE) by integrating the environmental sanitation factor (San) into stunting dynamics alongside nutritional interventions, filling the critical gap left by earlier epidemiological models that primarily focused on nutritional aspects. The mathematical contribution of this research lies in the analytical proof that a stunting-free equilibrium (TE_0) can only be achieved under optimal sanitation conditions ($San = 1$). Furthermore, the Routh-Hurwitz analysis indicates that stunting will persist and be difficult to eradicate if environmental improvements are neglected. The analytical sensitivity analysis also mathematically proves that the influx rate of at risk toddlers (α) is the primary factor driving the high number of stunting cases. Meanwhile, good sanitation plays a crucial role as a booster that accelerates recovery and suppresses infection.

These findings are strongly validated by the numerical simulations. Nutritional intervention programs cannot function as a stand-alone solution; combining them with a 20% improvement in sanitation infrastructure is proven to drastically accelerate the reduction of stunting cases. To achieve the regional target of 15.38%, the Central Kalimantan Provincial Government and the Stunting Reduction Acceleration Team (TPPS) must fundamentally synergize specific nutritional interventions with robust basic sanitation infrastructure. For future research, we suggest developing this mathematical model by applying stochastic or spatial model approaches to capture regional and demographic variations. Furthermore, the integration of time series data and extensive field validation are also required to enhance the accuracy and reliability of the model.

ACKNOWLEDGEMENT

This research is fully supported and funded by the Directorate of Research, Technology, and Community Service (DRTPM), Ministry of Education, Culture, Research, and Technology of The Republic of Indonesia, under the Beginner Lecturer Research Grant (Hibah Penelitian Dosen Pemula- PDP) scheme for the fiscal year 2024. The authors also express their deepest gratitude to STMIK Palangka Raya for providing the necessary facilities and institutional support throughout the execution of this research.

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