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On Relations between Some Types of (α, β)-Intuitionistic Fuzzy Ideals of Ternary Semigroups

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ABSTRACT

In this article, the notion of (α, β)-intuitionistic fuzzy ideals (briefly, (α, β)-IF ideals) of ternary semigroups is described using ”belong to” relation (ϵ) and “quasi-coincidence with” relation (q) connecting two objects, i.e., an intuitionistic fuzzy point (IFP, for short) and an intuitionistic fuzzy set (briefly, IFS). Throughout this paper, $\alpha \in \{\epsilon, q, \epsilon \vee q\}$ and $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$. The main purposes of this research are to construct the definition of (α, β)-intuitionistic fuzzy ideals of ternary semigroups and to investigate the relations between some types of these ideals. To achieve these goals, we use literature review method to study previous researches regarding (α, β)-fuzzy ideals of ternary semigroups and (α, β)-IF ideals of semigroups. As a result, we find the conditions for an IFS and an ideal of a ternary semigroup to be classified as an $(\alpha, \epsilon \vee q)$-IF ideal of ternary semigroup. Relations between some types of (α, β)-IF ideals of a ternary semigroup are also discussed here.

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A. INTRODUCTION

A ternary semigroup is defined as a non-empty set X with a ternary operation $\#$, such that two conditions are hold: firstly, X closed to $\#$. Secondly, $\#$ satisfied associative law (Nakkhasen, 2020). By using the set $G = \{i, 0, -i\}$ as an example, it had been proven that any semigroup can be expressed into an equivalent ternary semigroup, but not necessarily otherwise, for G is a ternary semigroup, but it is not a semigroup under the multiplicative operation of complex numbers (Akram, 2012; Lalithamani et al., 2020). Many studies have been conducted on ternary semigroups. Kar and Maity, (2011) worked on some properties regarding ideals of ternary semigroups. Since then, various structures of ideals of ternary semigroups had also been studied, e.g. fuzzy ideals, fuzzy A-ideals and fuzzy bi-ideals of ternary semigroups (Kar & Sarkar, 2012; Suebsung et al., 2019).

The notion of fuzzy set was first studied by Zadeh in 1965. In his research (Zadeh, 1965), a fuzzy subset is conceptualized as a function from non-empty set X to unit closed interval, i.e., $f: X \to [0,1]$. Many structures have been developed using the concept of fuzzy subset, e.g, fuzzy points, fuzzy ternary subsemigroups and fuzzy ideals of ternary semigroups, as it is mentioned in (Davvaz et al., 2014). Furthermore, the relations between fuzzy points and fuzzy sets had also been subjects of interest for many studies, e.g. the concept of (α, β)-fuzzy ideals in various algebraic structures (Akram, 2012; Khan et al., 2018). Additionally, Davvas, et al. worked on (α, β)-fuzzy ideals of ternary semigroups in their paper (Davvaz et al., 2014). Then, Muhiuddin and Al-Roqi, presented the notion of Classification of (α,β)-Fuzzy Ideals in BCK/BCI-Algebra (Muhiuddin & Al-Roqi, 2016).

The concept of intuitionistic fuzzy sets (IFS, for short) is first introduced by Atanassov as a generalization of fuzzy concept (Atanassov, 1986). IFS has been applied to many real life problems that deal with vagueness and imprecise measurement (Davvaz & Hassani Sadrabadi, 2016; Ren et al., 2021; Xiao, 2019; Xue et al., 2021). An Atanassov IFS can be described as a pair of functions valued in [0,1], which are called the membership function and the non-membership function (Aslam et al., 2012). This concept has also been implemented into various structures and researches in algebra (Hamouda, 2017; Kang, 2016). Furthermore, Akram, examined Intuitionistic Fuzzy Points and Ideals of Ternary Semigroup in his research (Akram, 2012). Later, Abdullah and Hussein worked on (α, β)-intuitionistic fuzzy bi-ideals of semigroups by implementing the concept of "quasi-coincident with" relation (q) and "belong to" relation (ϵ) (Abdullah & Hussain, 2017).

In this research, the definition of (α, β)-intuitionistic fuzzy ideals of ternary semigroups is proposed. The definition is constructed based on the results of two previous researches regarding some ideals in semigroups, i.e. the definition of (α, β)-IF Ideals of semigroups (Abdullah & Hussain, 2017) and the definition of (α, β)-fuzzy ideals of ternary semigroups (Davvaz et al., 2014). Some properties regarding this new structure is also proposed. Throughout the paper, q denotes "quasi-coincident with" relation and ϵ denotes "belong to" relation, where $\alpha \in (\epsilon, q, \epsilon \vee q)$ and $\beta \in (\epsilon, q, \epsilon \vee q, \epsilon \wedge q)$. Furthermore, relations between some types of (α, β)-intuitionistic fuzzy ideals of ternary semigroups are also established, based on the concept in (Muhiuddin & Al-Roqi, 2016).

B. METHODS

This research is conducted through a literature review. The definition of (α, β)-IF ideals of ternary semigroups is constructed by combining the definition of (α, β)-fuzzy ideals of ternary semigroups (Davvaz et al., 2014) and the definition of (α, β)-IF ideals of semigroups (Abdullah & Hussain, 2017). After the definition of (α, β)-IF ideals of ternary semigroups is established, the next step is to investigate whether some of the properties in (α, β)-IF ideals of semigroups are also hold in (α, β)-IF ideals of ternary semigroups. Furthermore, We also investigate the relations among some types of (α, β)-IF ideals of a ternary semigroup, based on the values of α and β.

Now, we would like to recall some concepts which are useful for the next sections. Throughout this article, q denotes "quasi-coincident with" relation, ϵ denotes "belong to" relation, $\alpha \in \{\epsilon, q, \epsilon \vee q\}$ and $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$. Also, please note that S is a semigroup under binary operation and T is a ternary semigroup, unless otherwise stated. Finally, for the sake of simplicity, (α, β)-IF is a short term for (α, β)-Intuitionistic Fuzzy.

Definition 1. (Rao, 2018) A semigroup S is an algebraic structure $(S, \#)$ consisting of a non-empty set S together with an associative binary operation #.

Definition 2. (Davvaz et al., 2014; Reddy & Shobhalatha, 2018) A non-empty set T whose elements are closed under the ternary operation [ ] of multiplication is said to be a ternary semigroup, if [ ] satisfies the associative law defined as follows:

$$\big[[abc]de\big] = [a[bcd]e] = \big[ab[cde]\big], \text{for all } a, b, c, d, e \in X. \quad (1)$$

For the sake of simplicity, we shall write $[abc]$ as defined in (1) as abc. For non-empty subsets of A, B and C of T, let $ABC := \{abc \mid a \in A, b \in B \text{ and } c \in C\}$.

A non-empty subset X of a ternary semigroup T is said to be a ternary subsemigroup of T, if $XXX \subseteq X$. A non-empty subset Y of a ternary semigroup T is called a right (resp. lateral, left) ideal of T, if $YTT \subseteq Y$ (resp. $TYT \subseteq Y, TTY \subseteq Y$). Furthermore, a non-empty subset Y of a ternary semigroup T is said to be an ideal of T, if Y is a right, lateral and left ideal of T.

Next, we are going to recall some concepts about fuzzy set.

A fuzzy subset f of a universe X is a function from X into the unit closed interval [0,1], i.e. $f: X \to [0.1]$ (Zadeh, 1965).

Definition 3. (Davvaz et al., 2014) A fuzzy subset f of a non-empty set X of the form

$$f : X \to [0,1], y \mapsto f_x(y) = \begin{cases} t \in (0,1] & if\ y = x; \\ 0 & otherwise \end{cases}$$

Is called a fuzzy point with support x and value t and is denoted by $[x; t]$. Consider a fuzzy point $[x; t]$, a fuzzy subset f and $\alpha \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$. We define $[x, t]\alpha f$ as follows:

a. $[x; t]\epsilon f$ (resp. $[x; t]qf$) means that $f(x) \geq t$ (resp. $f(x) + t > 1$) and in this case we say that $[x; t]$ belongs to (resp. quasi-coincident with) fuzzy subset f.
b. $[x; t]\epsilon \wedge qf$ (resp. $[x, t]\epsilon \vee qf$) means that $[x; t]\epsilon f$ and $[x; t]qf$ (resp. $[x; t]\epsilon f$ or $[x; t]qf$).
c. $[x; t]\bar{\alpha} f$ means that $[x; t]\alpha f$ does not hold.

Additionally, if f is a fuzzy subset of X, characterized by $f(x) \leq 0.5$ for all $x \in X$, then the set $\{[x; t] | [x; t]\epsilon \wedge qf\}$ is empty.

Definition 4. (Davvaz et al., 2014) A fuzzy subset f of T is said to be a $(\epsilon, \epsilon \vee q)$-fuzzy ternary subsemigroup of T, if for all $a, b, c \in T$ and $t, r, s \in (0,1]$,

$$[a; t]\epsilon f, [b; r]\epsilon f, [c; s]\epsilon f \to [abc; \min\{t, r, s\}]\ \epsilon \vee qf. \quad (2)$$

Definition 5. (Davvaz et al., 2014) A fuzzy subset f of T is called $(\epsilon, \epsilon \vee q)$-fuzzy left (resp. lateral, right) ideal of T, if for all $x, y, z \in T$ and $t \in (0,1]$,

$$[z; t]\epsilon f \text{ (resp. } [y; t]\epsilon f, [x; t]\epsilon f) \to [xyz; t]\ \epsilon \vee qf. \quad (3)$$

A fuzzy subset f is said to be a $(\epsilon, \epsilon \vee q)$-fuzzy ideal of T, if it is a $(\epsilon, \epsilon \vee q)$-fuzzy lateral ideal, a $(\epsilon, \epsilon \vee q)$-fuzzy right ideal and a $(\epsilon, \epsilon \vee q)$-fuzzy left ideal of T.

Now, we review some intuitionistic fuzzy logic concepts.

Definition 6. (Abdullah & Hussain, 2017) Let X be a non-empty set. An Intuitionistic fuzzy set (briefly, IFS) A is object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) | \ x \in X\},$$

Where the function $\mu_A : X \to [0,1]$ denotes the degree of membership (namely $\mu_A(x)$) and the function $\gamma_A : X \to [0,1]$ denotes the degree of non-membership (namely $\gamma_A(x)$), of each element $x \in X$ to the set A, respectively, and $\mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

Please note that, throughout the paper, we use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) | \ x \in X\}$.

Definition 7. (Abdullah et al., 2011; Akram, 2012) Let c be a point in a non-empty set X. If $a \in (0,1]$ and $b \in [0,1)$ are two real number such that $0 \leq a + b \leq 1$, then an intuitionistic fuzzy point (IFP, for short) in X, written as $c_{(a,b)}$ is defined to be intuitionistic fuzzy subset of X, given by

$$c_{(a,b)}(x) = \begin{cases} (a,b) & if \ \ x = c; \\ (0,1) & otherwise \end{cases}$$

Where a is the degree of membership of $c_{(a,b)}$ and b is the degree non-membership of $c_{(a,b)}$ and $c \in X$ is the support of $c_{(a,b)}$.

Let $c_{(a,b)}$ be an IFP in X, $A = (\mu_A, \gamma_A)$ be an IFS in X and $\alpha \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$. Then, $c_{(a,b)}$ is said to belong to A, written $c_{(a,b)}\epsilon A$, if $\mu_A(c) \geq a$ and $\gamma_A(c) \leq b$. We say that $c_{(a,b)}$ is quasi-coincident with A, written $c_{(a,b)}qA$, if $\mu_A(c) + a > 1$ and $\gamma_A(c) + b < 1$. Furthermore, $c_{(a,b)}\epsilon \wedge qA$ (resp. $c_{(a,b)}\epsilon \vee qA$) means that $c_{(a,b)}\epsilon A$ and $c_{(a,b)}qA$ (resp. $c_{(a,b)}\epsilon A$ or $c_{(a,b)}qA$). Furthermore, $c_{(a,b)}\bar{\alpha}A$ means that $c_{(a,b)}\alpha A$ does not hold.

Definition 8. (Abdullah & Hussain, 2017) An IFS $A = (\mu_A, \gamma_A)$ in a semigroup S is called an $(\epsilon, \epsilon \vee q)$-intuitionistic fuzzy subsemigroup of a semigroup S if for all $x, y \in S$ and for all $t_1, t_2 \in (0,1]$ and $s_1, s_2 \in [0,1)$, the following condition holds:

$$x_{(t_1,s_1)}\epsilon A \text{ and } y_{(t_2,s_2)}\epsilon A \rightarrow (xy)_{(\min\{t_1,t_2\}, max\{s_1,s_2\})}\epsilon \vee qA. \tag{4}$$

Definition 9. (Abdullah & Hussain, 2017) An IFS $A = (\mu_A, \gamma_A)$ in a semigroup S is called an $(\epsilon, \epsilon \vee q)$-intuitionistic fuzzy right (resp. left) ideal of semigroup S if $\forall x, y \in S$ and $\forall t \in (0,1)$ and $\forall s \in [0,1)$, the following are hold.

$$x_{(t,s)}\epsilon A \ \rightarrow (xy)_{(t,s)}\epsilon \vee qA \ (resp. \ y_{(t,s)}\epsilon A \rightarrow (xy)_{(t,s)}\epsilon \vee qA) \tag{5}$$

An IFS $A = (\mu_A, \gamma_A)$ in a semigroup S is said to be an $(\epsilon, \epsilon \vee q)$-intuitionistic fuzzy ideal of semigroup S, if $A = (\mu_A, \gamma_A)$ is an $(\epsilon, \epsilon \vee q)$-IF right ideal and $(\epsilon, \epsilon \vee q)$-intuitionistic fuzzy left ideal of semigroup S.

C. RESULT AND DISCUSSION

This section consists of two main parts. First, we are going to define (α, β)-IF ideals of a ternary semigroup T. Then, we will analyze some of its properties. For the second part, we are going to investigate the relations between some types of (α, β)-intuitionistic fuzzy ideals of a ternary semigroup T and then come up with diagrams that will describe their relationships.

1. $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-Intuitionistic Fuzzy Ideals of Ternary Semigroups

Here, we define (α, β)-IF ternary subsemigroups, (α, β)-IF ideals of a ternary semigroups and analyze some of their properties.

Definition 10. An IFS $A = (\mu_A, \gamma_A)$ in a ternary semigroup T is called an (α, β)-intuitionistic fuzzy ternary subsemigroup of a ternary semigroup T, where $\alpha \in \{\epsilon, q, \epsilon \vee q\}$ and $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$, if for all $x, y, z \in T$ and for all $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$, the following condition holds:

$$x_{(a_1,b_1)}\alpha A, y_{(a_2,b_2)}\alpha A, z_{(a_3,b_3)}\alpha A \ \rightarrow xyz_{(\min\{a_1,a_2,a_3\}, max\{b_1,b_2,b_3\})}\beta A \tag{6}$$

Definition 11. An IFS $A = (\mu_A, \gamma_A)$ in a ternary semigroup T is called an (α, β)-intuitionistic fuzzy right (resp. lateral, left) ideal of a ternary semigroup T, where $\alpha \in \{\epsilon, q, \epsilon \vee q\}$ and $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$, if for all $x, y, z \in T$ and for all $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$, the following conditions hold:

$$x_{(a_1,b_1)}\alpha A \rightarrow xyz_{(a_3,b_3)}\beta A \; (resp. \, y_{(a_2,b_2)}\alpha A \rightarrow xyz_{(a_2,b_2)}\beta A,\; z_{(a_3,b_3)}\alpha A \rightarrow xyz_{(a_1,b_1)}\beta A)$$
$$x_{(a_1,b_1)}\alpha A, y_{(a_2,b_2)}\alpha A, z_{(a_3,b_3)}\alpha A \rightarrow xyz_{(\min\{a_1,a_2,a_3\},max\{b_1,b_2,b_3\})}\beta A \tag{7}$$

An IFS $A = (\mu_A, \gamma_A)$ in a ternary semigroup T is called an (α, β)-intuitionistic fuzzy ideal of a ternary semigroup T, if $A = (\mu_A, \gamma_A)$ is an (α, β)-intuitionistic fuzzy right ideal (α, β)-intuitionistic fuzzy lateral ideal and (α, β)-intuitionistic fuzzy left ideal of a ternary semigroup T.

Alternatively, **Definition 11,** can also be expressed into the following definition.

Definition 12. An IFS $A = (\mu_A, \gamma_A)$ in a ternary semigroup T is called an (α, β)-intuitionistic fuzzy ideal of a ternary semigroup T, where $\alpha \in \{\epsilon, q, \epsilon \vee q\}$ and $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$, if for all $x, y, z \in T$ and for all $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$, the following condition holds:

$$x_{(a_1,b_1)}\alpha A, y_{(a_2,b_2)}\alpha A, z_{(a_3,b_3)}\alpha A \rightarrow xyz_{(\max\{a_1,a_2,a_3\},min\{b_1,b_2,b_3\})}\beta A. \tag{8}$$

Based on statements in (6) and (7), it is clear that any (α, β)-IF left (resp. lateral, right) ideal of a ternary semigroup T is (α, β)-intuitionistic fuzzy ternary subsemigroup of T but the converse is not true.

Theorem 1. *Let D be a ternary subsemigroup of a ternary semigroup T and $P = (\mu_P, \gamma_P)$ be an IFS such that the following conditions hold:*

a. $(\forall x \in T \backslash D)$ $(\mu_P(x) = 0$ *and* $\gamma_P(x) = 1)$,

b. $(\forall x \in D)$ $(\mu_P(x) \geq 0.5$ *and* $\gamma_P(x) \leq 0.5)$.

Then, $P = (\mu_P, \gamma_P)$ is an $(\alpha, \epsilon \vee q)$-intuitionistic fuzzy ternary subsemigroup of T

Proof. This proof will be separated into three main cases, based on the values of α. First, for $\alpha = q$. Let $j, k, l \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$, be such that the following conditions hold:

$$j_{(a_1,b_1)}qP, k_{(a_2,b_2)}qP, l_{(a_3,b_3)}qP \tag{9}$$

By applying **Definition 7** to statements (9), we have

$$\begin{gathered}\mu_P(j) + a_1 > 1,\; \gamma_P(j) + b_1 < 1\,, \\ \mu_P(k) + a_2 > 1,\; \gamma_P(k) + b_2 < 1 \text{ and} \\ \mu_P(l) + a_3 > 1, \gamma_P(l) + b_3 < 1.\end{gathered} \tag{10}$$

So, $j, k, l \in P$. Then, $jkl \in P$. Thus, if $a_m \leq 0.5, b_m \geq 0.5$, for $m = 1,2,3$, then for inequalities in (10), the following statements hold:

$$\begin{gathered}\mu_P(jkl) \geq \min\{a_1, a_2, a_3\} \text{ and} \\ \gamma_P(jkl) \leq \max\{b_1, b_2, b_3\}.\end{gathered} \tag{11}$$

Therefore, based on **Definition 7** and inequalities in (11), we can infer that:

$$jkl_{(\min\{a_1,a_2,a_3\},max\{b_1,b_2,b_3\})}\epsilon P.$$

Furthermore, If $a_m > 0.5, b_m < 0.5$, for $m = 1,2,3$, we have:

$$\begin{gathered} \mu_P(jkl) + \min\{a_1, a_2, a_3\} > 1 \text{ and} \\ \gamma_P(jkl) + \max\{b_1, b_2, b_3\} < 1 \end{gathered} \tag{12}$$

Similiarly, based on **Definition 7** and inequalities in (12), then $jkl_{(\min\{a_1,a_2,a_3\},max\{b_1,b_2,b_3\})}qP$. Thus, $jkl_{(\min\{a_1,a_2,a_3\},max\{b_1,b_2,b_3\})}\epsilon \vee qP$. Hence, $P = (\mu_P, \gamma_P)$ is an $(q, \epsilon \vee q)$-intuitionistic fuzzy ternary subsemigroup of T.

Similarly, we can prove for the case of $\alpha = \epsilon$ and $\alpha = \epsilon \vee q$. This completes the proof.

Theorem 2. *Let R be an ideal of a ternary semigroup T and $I = (\mu_I, \gamma_I)$ is an IFS, such that*

a. $(\forall x \in T\backslash R)$ $(\mu_I(x) = 0$ *and* $\gamma_I(x) = 1)$

b. $(\forall x \in R)$ $(\mu_I(x) \geq 0.5$ *and* $\gamma_I(x) \leq 0.5)$

Then, $I = (\mu_I, \gamma_I)$ is an $(\alpha, \epsilon \vee q)$-intuitionistic fuzzy ideal of T.

Proof. Based on the values of α, this proof will be separated into three cases. First, for $\alpha = \epsilon$. Let $j, k, l \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$, be such that

$$x_{(a_1,b_1)}\epsilon I, y_{(a_2,b_2)}\epsilon I, z_{(a_3,b_3)}\epsilon I. \tag{13}$$

Thus, based on **Definition 7**, the inequalities below hold.

$$\begin{gathered} \mu_P(x) \geq a_1,\ \gamma_P(x) \leq b_1, \\ \mu_P(y) \geq a_2,\ \gamma_P(y) \leq b_2 \text{ and} \\ \mu_P(z) \geq a_3, \gamma_P(z) \leq b_3. \end{gathered} \tag{14}$$

By statements in (13), $x, y, z \in R$. Therefore $xyz \in R$. Now, If $a_m \leq 0.5$, $b_m \geq 0.5$, for $m = 1,2,3$, then

$$\begin{gathered} \mu_P(xyz) \geq \max\{a_1, a_2, a_3\} \text{ and} \\ \gamma_P(xyz) \leq \min\{b_1, b_2, b_3\}. \end{gathered} \tag{15}$$

Thus,

$$xyz_{(\max\{a_1,a_2,a_3\},min\{b_1,b_2,b_3\})}\epsilon I.$$

Alternatively, if $a_m > 0.5, b_m < 0.5$, for $m = 1,2,3$, then for inequalities in (14), we have

$$\begin{gathered} \mu_P(xyz) + \max\{a_1, a_2, a_3\} > 1 \text{ and} \\ \gamma_P(xyz) + \min\{b_1, b_2, b_3\} < 1. \end{gathered} \tag{16}$$

Hence by **Definition 7**, $xyz_{(\max\{a_1,a_2,a_3\},min\{b_1,b_2,b_3\})}qI$. Thus, $xyz_{(\min\{a_1,a_2,a_3\},max\{b_1,b_2,b_3\})}\epsilon \vee qI$. Therefore, $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-intuitionistic fuzzy ideal of T.

We can also prove for the case of $\alpha = q$ and $\alpha = \epsilon \vee q$ by going through similar process as above. This completes the proof.

2. Relations between Some Types of (α, β)-Intuitionistic Fuzzy Ideals of A Ternary Semigroup

Here, the relations between some types of (α, β)-IF ideals of a ternary semigroup T are established. Based on the values of α and β, there are twelve different types of (α, β)-intuitionistic fuzzy ideals of T, that is, (α, β) is any one of (ϵ, ϵ), (ϵ, q), $(\epsilon, \epsilon \vee q)$, $(\epsilon, \epsilon \wedge q)$, (q, ϵ), (q, q), $(q, \epsilon \wedge q)$, $(q, \epsilon \vee q)$, $(\epsilon \vee q, \epsilon)$. $(\epsilon \vee q, q)$, $(\epsilon \vee q, q)$, $(\epsilon \vee q, \epsilon \wedge q)$.

First, we study the relation between $(q, \epsilon \vee q)$-IF ideal and $(\epsilon, \epsilon \vee q)$-IF ideal of ternary semigroup.

Theorem 3. $((q, \epsilon \vee q) \rightarrow (\epsilon, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \vee q)$-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T.*

Proof. Let $I = (\mu_I, \gamma_I)$ be an $(q, \epsilon \vee q)$-IF left ideal of a ternary semigroup T and $x, y, z \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$.

First, we prove that $z_{(a_3,b_3)} \epsilon I \rightarrow xyz_{(a_3,b_3)} \epsilon \vee qI$ holds. Let $z \in T, a_3 \in (0,1]$ and $b_3 \in [0,1)$ be such that $z_{(a_3,b_3)} \epsilon I$. Suppose $xyz_{(a_3,b_3)} \overline{\epsilon \vee q} I$. Then $xyz_{(a_3,b_3)} \bar{\epsilon} I$ and $xyz_{(a_3,b_3)} \bar{q} I$. Thus, $\mu_I(xyz) < a_3$, $\gamma_I(xyz) > b_3$ and $\mu_I(xyz) + a_3 \leq 1$, $\gamma_I(xyz) + b_3 \geq 1$. Hence, $\mu_I(xyz) < 0.5$, $\gamma_I(xyz) > 0.5$, and so $\mu_I(xyz) < \min\{a_3, 0.5\}$ and $\gamma_I(xyz) > max\{b_3, 0.5\}$. Then, $1 - \mu_I(xyz) > 1 - \min\{a_3, 0.5\}$ and $1 - \gamma_I(xyz) \leq 1 - \max\{b_3, 0.5\}$. Therefore, $1 - \mu_I(xyz) \geq \max\{1 - \mu_I(z), 0.5\}$ and $1 - \gamma_I(xyz) \leq \min\{1 - \gamma_I(z), 0.5\}$. Hence, there exist $c \in (0,1]$ and $d \in [0,1)$, such that

$$\begin{gathered} 1 - \mu_I(xyz) \geq c > \max\{1 - \mu_I(z), 0.5\} \text{ and} \\ 1 - \gamma_I(xyz) \leq d < \min\{1 - \gamma(z), 0.5\}. \end{gathered} \tag{17}$$

Then, by inequalities in (17), we have $\mu_I(z) + c > 1$ and $\gamma_I(z) + d < 1$, that is $z_{(c,d)} qI$. Since $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \vee q)$-IF left ideal of T and $z_{(c,d)} qI$, it follow that $xyz_{(c,d)} \epsilon \vee qI$. But, now we have

$$\begin{gathered} \mu_I(xyz) + c \leq 1,\ \gamma_I(xyz) + d \geq 1 \text{ and} \\ \mu_I(xyz) \leq 1 - c < 1 - 0.5 \leq c. \end{gathered} \tag{18}$$

Thus, $xyz_{(c,d)} \bar{\epsilon} I$ and $xyz_{(c,d)} \bar{q} I$ by **Definition 7**. Hence, $xyz_{(c,d)} \overline{\epsilon \vee q} I$, a contradiction. Therefore, $xyz_{(a_3,b_3)} \epsilon \vee qI$ and thus $z_{(a_3,b_3)} \epsilon I \rightarrow xyz_{(a_3,b_3)} \epsilon \vee qI$ holds.

Next, we prove that $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I \rightarrow xyz_{(\min\{a_1,a_2,a_3\}, max\{b_1,b_2,b_3\})} \epsilon \vee qI$ holds. Let $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$ and let $g = \min\{a_1, a_2, a_3\}$ and $h = max\{b_1, b_2, b_3\}$, such that $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I$. Suppose, $xyz_{(g,h)} \overline{\epsilon \vee q} I$. Then $xyz_{(g,h)} \bar{\epsilon} I$ and $xyz_{(g,h)} \bar{q} I$. Thus,

$$\begin{gathered} \mu_I(xyz) \leq g, \gamma_I(xyz) \geq h \text{ and} \\ \mu_I(xyz) + g \leq 1, \gamma_I(xyz) + h \geq 1 \end{gathered} \tag{19}$$

So, $\mu_I(xyz) < \min\{0.5, g\}$ and$\gamma_I(xyz) > \max\{0.5, h\}$. Then,

$$\begin{gathered} 1 - \mu_I(xyz) > 1 - \min\{\min\{a_1, a_2, a_3\}, 0.5\} \text{ and} \\ 1 - \gamma_I(xyz) < 1 - \max\{\max\{b_1, b_2, b_3\}, 0.5\}. \end{gathered} \tag{20}$$

Thus,

$$\begin{gathered} 1 - \mu_I(xyz) \geq \max\{1 - \mu_I(x), 1 - \mu_I(y), 1 - \mu_I(z), 0.5\} \text{ and} \\ 1 - \gamma_I(xyz) \leq \min\{1 - \gamma_I(x), 1 - \gamma_I(y), 1 - \gamma_I(z), 0.5\}. \end{gathered} \tag{21}$$

Hence, there exist $r \in (0,1]$ and $s \in [0,1)$, such that

$$\begin{gathered} 1 - \mu_I(xyz) \geq r > \max\{1 - \mu_I(x), 1 - \mu_I(y), 1 - \mu_I(z), 0.5\} \text{ and} \\ 1 - \gamma_I(xyz) \leq s < \min\{1 - \gamma_I(x), 1 - \gamma_I(y), 1 - \gamma_I(x), 0.5\}. \end{gathered} \tag{22}$$

Now, let $\mu_I(u) = \min\{\mu_I(x), \mu_I(y), \mu_I(z)\}$ and $\gamma_I(u) = max\{\gamma_I(x), \gamma_I(y), \gamma_I(z)\}$. Then, by inequalities in (22), we have $\mu_I(u) + r > 1$ and $\gamma_I(u) + s < 1$, that is $u_{(r,s)} qI$. Since, $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \vee q)$-IF left ideal of T and $u_{(r,s)} qI$. So, $xyz_{(r,s)} \in \vee qI$. But in (18), we have $\mu_I(xyz) + c \leq 1$, $\gamma_I(xyz) + d \geq 1$ and $\mu_I(xyz) \leq 1 - r < 1 - 0.5 \leq r$,

$\gamma_I(xyz) \geq 1 - s \geq 1 - 0.5 \geq s$. Thus, $u_{(r,s)} \overline{\in} I$ and $u_{(r,s)}\bar{q}I$. Hence, $u_{(r,s)} \overline{\in \vee q} I$, which contradicts our hypothesis. Therefore, $xyz_{(g,h)} \epsilon \vee qI$ and thus, $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I \rightarrow xyz_{(g,h)} \epsilon \vee qI$ holds. Hence, $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF left ideal of T.

Similarly, we can prove for the case of $(q, \epsilon \vee q)$-IF lateral ideal and $(q, \epsilon \vee q)$-IF right ideal of T. This completes the proof.

Now, we investigate the relations between some types of (α, β)-intuitionistic fuzzy ideals of a ternary semigroup.

Theorem 4. $((\epsilon, \epsilon \wedge q) \rightarrow (\epsilon, \epsilon), (\epsilon, q), (\epsilon, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \wedge q)$-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an (ϵ, ϵ)-IF ideal, (ϵ, q)-IF ideal and $(\epsilon, \epsilon \vee q)$-IF ideal of T.*

Proof. Let $I = (\mu_I, \gamma_I)$ be an $(\epsilon, \epsilon \wedge q)$ -IF left ideal of a ternary semigroup T. Let $e = \max\{a_1, a_2, a_3\}$, $f = min\{b_1, b_2, b_3\}$, $x, y, z \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$ be such that$x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I$. Then $xyz_{(e,f)} \epsilon \wedge qI$, that is $xyz_{(e,f)} \epsilon I$ and $xyz_{(e,f)} qI$. Hence, $I = (\mu_I, \gamma_I)$ is an (ϵ, ϵ)-IF ideal, (ϵ, q)-IF ideal and $(\epsilon, \epsilon \vee q)$-IF ideal of T.
By using the similar process as in the theorem above, we can also prove the theorems below.

Theorem 5. $((q, \epsilon \wedge q) \rightarrow (q, \epsilon), (q, q), (q, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \wedge q)$-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an (q, ϵ)-IF ideal, (q, q)-IF ideal and $(q, \epsilon \vee q)$-IF ideal of T.*

Theorem 6. $((\epsilon \vee q, \epsilon \wedge q) \rightarrow (\epsilon \vee q, \epsilon), (\epsilon \vee q, q), (\epsilon \vee q, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an $(\epsilon \vee q, \epsilon \wedge q)$-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(\epsilon \vee q, \epsilon)$-IF ideal, $(\epsilon \vee q, q)$-IF ideal and $(\epsilon \vee q, \epsilon \vee q)$-IF ideal of T.*

Here, we study the relation between (ϵ, ϵ)-IF ideal and $(\epsilon, \epsilon \vee q)$-IF ideal of a ternary semigroup.

Theorem 7. $((\epsilon, \epsilon) \rightarrow (\epsilon, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an (ϵ, ϵ)-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T.*

Proof. Let $I = (\mu_I, \gamma_I)$ be an (ϵ, ϵ) -IF left ideal of a ternary semigroup T. Let $e = \max\{a_1, a_2, a_3\}$, $f = min\{b_1, b_2, b_3\}$, $x, y, z \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$ be such that $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I$. Then $xyz_{(e,f)} \epsilon I$. Therefore, $xyz_{(e,f)} \epsilon \vee qI$. Hence, $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T.

We can also prove the theorems below by using similar process as **Theorem 7**.

Theorem 8. $((\epsilon, q) \rightarrow (\epsilon, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an (ϵ, q)-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T.*

Theorem 9. $((q, \epsilon) \rightarrow (q, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an (q, ϵ)-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \vee q)$-IF ideal of T.*

Theorem 10. $((q, q) \rightarrow (q, \epsilon \vee q))$ *If $I = (\mu_I, \gamma_I)$ is an (q, q)-IF ideal of a ternary semigroup T, then $I = (\mu_I, \gamma_I)$ is an $(q, \epsilon \vee q)$-IF ideal of T.*

Theorem 11. $((\epsilon \vee q, \epsilon) \rightarrow (\epsilon \vee q, \epsilon \vee q))$ *If* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon \vee q, \epsilon)$*-IF ideal of a ternary semigroup* T*, then* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon \vee q, \epsilon \vee q)$*-IF ideal of* T*.*

Theorem 12. $((\epsilon \vee q, q) \rightarrow (\epsilon \vee q, \epsilon \vee q))$ *If* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon \vee q, q)$*-IF ideal of a ternary semigroup* T*, then* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon \vee q, \epsilon \vee q)$*-IF ideal of* T*.*
Next, by using **Theorem 3.**, we can now prove the following theorem.

Theorem 13. $((\epsilon \vee q, \epsilon \vee q) \rightarrow (\epsilon, \epsilon \vee q))$ *If* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon \vee q, q)$*-IF ideal of a ternary semigroup* T*, then* $I = (\mu_I, \gamma_I)$ *is an* $(\epsilon, \epsilon \vee q)$*-IF ideal of* T*.*

Proof. Let $I = (\mu_I, \gamma_I)$ be an $(\epsilon \vee q, \epsilon \vee q)$-IF ideal of a ternary semigroup T. Let $e = \max\{a_1, a_2, a_3\}$, $f = min\{b_1, b_2, b_3\}$, $x, y, z \in T$, $a_1, a_2, a_3 \in (0,1]$ and $b_1, b_2, b_3 \in [0,1)$ be such that $x_{(a_1,b_1)} \epsilon \vee qI, y_{(a_2,b_2)} \epsilon \vee qI, z_{(a_3,b_3)} \epsilon \vee qI$. Since $I = (\mu_I, \gamma_I)$ is an $(\epsilon \vee q, \epsilon \vee q)$-IF ideal of T. So, $xyz_{(e,f)} \epsilon \vee qI$. If $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I$, then $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T. Alternatively, If $x_{(a_1,b_1)} qI, y_{(a_2,b_2)} qI, z_{(a_3,b_3)} qI$, by **Theorem 3**, then $x_{(a_1,b_1)} \epsilon I, y_{(a_2,b_2)} \epsilon I, z_{(a_3,b_3)} \epsilon I$. Therefore, $I = (\mu_I, \gamma_I)$ is an $(\epsilon, \epsilon \vee q)$-IF ideal of T.

Finally, by using some of the theorems earlier, we can describe the relations between some types of (α, β)-IF ideal of a ternary semigroup T as stated below.

Theorem 14. *If* $I = (\mu_I, \gamma_I)$ *is an* (α, β)*-IF ideal of a ternary semigroup* T*, then we have the following relations:*

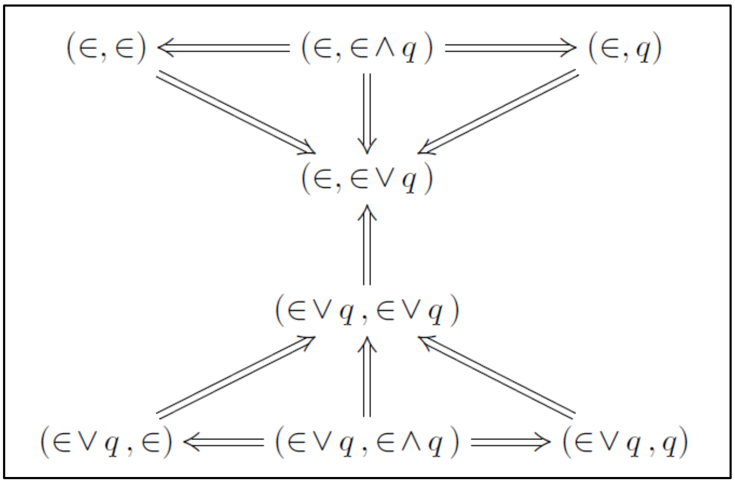

Figure 1. Relations between some types of (α, β)*-IF ideals of a ternary semigroup T – Part I*

and

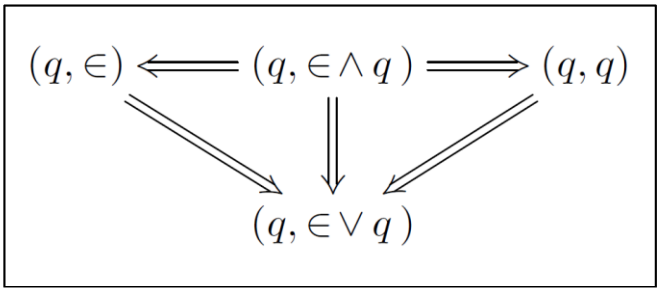

Figure 2. Relations between some types of (α, β)*-IF ideals of a ternary semigroup T – Part II*

Proof. Proof follow from **Theorem 4** through **Theorem 13**.

Now, we have concluded the result of our research. As it was stated, we have constructed a new structure, that is (α, β)-IF ideals of ternary semigroups by combining the definition of (α, β)-fuzzy ideals of ternary semigroups (Davvaz et al., 2014) and the definition of (α, β)-IF ideals of semigroups (Abdullah & Hussain, 2017). Some its properties, like **Theorem 1** and **Theorem 2** are the generalization on some properties of (α, β)-IF ideals of semigroups (Abdullah & Hussain, 2017) applied to ternary semigroups concept. On the other hand, other properties like the ones from **Theorem 4** through **Theorem 14,** are generalization of some properties of (α, β)-fuzzy ideals of ternary semigroups (Davvaz et al., 2014) introduced into intuitionistic fuzzy concept. That being said, there are also many other properties that can be inferred into this new structure, by analyzing the properties of its two foundational concepts earlier. This opens the possibilities for further research.

D. CONCLUSION AND SUGGESTIONS

The notion of (α, β)-IF ideal and (α, β)-IF ternary subsemigroup of a ternary semigroup have been established. Some of their properties have also been proven. We have provided some conditions for an ideal and an IFS of a ternary semigroup to be an $(\alpha, \epsilon \vee q)$-intuitionistic fuzzy ideal of a ternary semigroup. Based on the values of α and β, there are twelve different types of (α, β)-IF ideal of a ternary semigroup. We have describe relations between these types. It has been proven that every $(q, \epsilon \vee q)$-IF ideal of a ternary semigroup T is an $(\epsilon, \epsilon \vee q)$-IF ideal of T. For further research, the researchers suggest to investigate the relations between various types (α, β)-IF ternary subsemigroup of a ternary semigroup.

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