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Multivariate Control Chart based on Neutrosophic Hotelling T² Statistics and Its Application

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ABSTRACT

Under classical statistics Hotelling T^2 control chart is applied when the observations of quality characteristics are precise, exact, or crips data. However, in reality, under uncertain conditions, the observations are not necessarily precise, exact, or indeterminacy. As a consequence, the classical Hotelling T^2control chart is not appropriate to monitor the process for this condition. To tackle this situation, we proposed new Hotelling T^2 monitoring scheme based on a fuzzy neutrosophic concept. Neutrosophic is the generalization of fuzzy. It is used to handle uncertainty using indeterminacy. The combination of statistics based on neutrosophic Hotelling T^2 and classical Hotelling T^2 control chart will be proposed to tackle indeterminacy observations. The proposed Hotelling T^2 statistics, its call neutrosophic Hotelling T^2 (T_N^2) control chart. This chart involves the indeterminacy of observations, its call neutrosophic data and will be expressed in the indeterminacy interval. T_N^2 control charts consist T_N^2 lower chart and T_N^2 upper chart. In this paper, the neutrosophic Hotelling T^2will be applied to individual observations of glass production and will be compared by using classical Hotelling T^2 control chart. Based on T_N^2 control charts of glass production, nine points fall outside of UCL_N of lower control chart and 24 points outside from UCL_N of upper control chart. Whereas using classical Hotelling T^2 control chart, just one point outside frim UCL. From the comparison, it concluded that the neutrosophic Hotelling T^2 control chart is more suitable for the indeterminacy of observations.

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A. INTRODUCTION

A control chart is one of most the famous tools to monitor the process. Generally, there are two categories of control charts based on the types of quality characteristics: variable control chart and attribute control chart. Both of these charts, based on the number of monitored quality characteristics consist of a univariate control chart and multivariate control chart. Univariate control chart used to monitor the single quality characteristics. $\bar{X} - R$, is one of the most popular univariate control charts. Hotelling T^2 a control chart is one of the tools that is widely used to monitor the multivariate process (Montgomery, 2020). The multivariate control chart is used to monitor the process simultaneously of two or more interrelated quality characteristics. The classical Hotelling T^2 control chart suitable to monitor when all data observations are crips and precise.

In manufacture, we often meet the data observations, not price, or vague. In this case classical Hotelling T^2 control chart not suitable to use. To handle the condition control chart-based fuzzy logic proposed by Zadeh (1965). Several researchers have developed fuzzy attribute control chart; fuzzy p control chart and the extension (Pandian & Puthiyanayagam, 2013) (Sogandi et al., 2014) (Shabani & Rezayian, n.d.), fuzzy c and u control chart (Darestani et al., 2014) (Fadaei & Pooya, 2018) (Ercan-Teksen & Anagün, 2020) (Ercan-Teksen & Anagün, 2018). Meanwhile, the fuzzy variable control charts; X-Individual fuzzy control chart (Gildeh & Shafiee, 2015) (Moraditadi & Avakhdarestani, 2016a) (Alizadeh & Ghomi, 2011) (Moraditadi & Avakhdarestani, 2016b) and fuzzy EWMA and CUSUM and also the performance (Wang & Hryniewicz, 2013) (Göztok et al., 2021) (Erginel & Şentürk, 2016). And the development of fuzzy multivariate control charts: (Ghobadi et al., 2014) (Ghobadi et al., 2015) (Pastuizaca Fernández et al., 2015) (Wibawati, 2020).

The other approach to monitoring the unprecise data neutrosophic can be used. Neutrosophic is an extension of fuzzy logic (Smarandache, 2014). The logic of neutrosophic is considered indeterminacy in the measurement. However, based on literature reviews, the neutrosophic control chart is still limited. The neutrosophic control charts that have been developed are neutrosophic $\bar{X}$ (Aslam & Khan, 2019), neutrosophic S chart (Khan, Gulistan, Chammam, et al., 2020) (Khan, Gulistan, Hashim, et al., 2020), neutrosophic Exponentially Weighted Moving Average (NEWMA) X (Aslam et al., 2019). Among these charts is the univariate control chart. Meanwhile, we are often interested to monitor multivariate processes that involve the indeterminacy of observations. Recently, Aslam introduced Hotelling T^2 statistics under neutrosophic statistics (Aslam & Arif, 2020). The new statistics are the generalization of classical statistics under uncertainty conditions. This procedure is applied to chemical data. Based on comparison with classical Hotelling T^2 statistics, the proposed method is more effective. Therefore in this paper proposed new Hotelling T^2 monitoring scheme based on fuzzy neutrosophic concept and it call neutrosophic Hotelling T^2 (T_N^2) control chart. The proposed chart will be applied at spesific glass production and will be compared with the classical Hotelling T^2 control chart.

B. METHODS

1. Neutrosophic Hotelling T_N^2 Statistics

If $x_{jkN} \epsilon [x_{jkL}, x_{jkU}]$ be a random variable of neutrosophic that represents neutrosophic observation for kth variable and jth observations. This interval expresses the indeterminacy, x_{jkL} show the smallest value and x_{jkU} is the largest value. Based on this form x_{jkL} is a part of determinate and pa oart of indeterminate is $x_{jkU} I_N$, where $I_N \epsilon [I_{N]L}, I_U]$ and it can be stated as If $x_{jkN} = x_{jkL} + x_{jkU} I_N$. If $x_N \epsilon [n_L, n_U]$ are observations of neutrosophic from neutrosophic variable $p_N \epsilon [p_L, p_U]$.

$$X_N \epsilon \left[\begin{bmatrix} x_{11L} & \cdots & x_{1kL} & \cdots & x_{1pL} \\ x_{j1L} & \cdots & x_{j1L} & \cdots & x_{jpL} \\ x_{n1L} & \cdots & x_{nkL} & \cdots & x_{npL} \end{bmatrix}, \begin{bmatrix} x_{11U} & \cdots & x_{1kU} & \cdots & x_{1pU} \\ x_{j1U} & \cdots & x_{j1U} & \cdots & x_{jpU} \\ x_{n1L} & \cdots & x_{nkU} & \cdots & x_{npU} \end{bmatrix}\right]; X_N \quad (1)$$

The form of neutrosophic $X_N \epsilon [X_L, X_U]$ can be shown as

$$X_N = X_L + X_U I_N; I_N \epsilon [I_L, I_U] \quad (2)$$

The neutrosophic sample mean is presented by

$$\bar{x}_{kN} \epsilon \left[\left[\frac{1}{n_L} \sum_{j=1}^{n_L} x_{jkL}\right], \left[\frac{1}{n_U} \sum_{j=1}^{n_U} x_{jkU}\right]\right]; \bar{x}_{kN} \epsilon [\bar{x}_{kL}, \bar{x}_{kU}], \quad (3)$$

where $\bar{x}_{kN}$ can be presented as $\bar{x}_{kN} = \bar{x}_{kL} + \bar{x}_{kU} I_N; I_N \epsilon [I_L, I_U]$.
The form of the neutrosophic sample variance is

$$s_{kN}^2 \epsilon \left[\left[\frac{1}{n_L} \sum_{j=1}^{n_L} \left(x_{jkL} - \bar{x}_{kL}\right)^2\right], \left[\frac{1}{n_U} \sum_{j=1}^{n_U} \left(x_{jkU} - \bar{x}_{kU}\right)^2\right]\right]; s_{kN}^2 \epsilon [s_{kL}^2, s_{kU}^2], \quad (4)$$

where $s_{kN}^2 = s_{kL}^2 + s_{kU}^2 I_N; I_N \epsilon [I_L, I_U]$.

The formula of neutrosophic Covariance is:

$$S_{ikN} \epsilon \left[\left[\frac{1}{n_L} \sum_{j=1}^{n_L} \left(x_{jiL} - \bar{x}_{kL} \left(\left(x_{jkL} - \bar{x}_{kL}\right)\right)\right)\right], \left[\frac{1}{n_U} \sum_{j=1}^{n_U} \left(x_{jiU} - \bar{x}_{kU} \left(\left(x_{jkU} - \bar{x}_{kU}\right)\right)\right)\right]\right]; \quad (5)$$

where :
$S_{ikN} \epsilon [S_{ikL}, S_{ikU}]$, and
$S_{ikN} = S_{ikL} + S_{ikU} I_N; I_N \epsilon [I_L, I_U]$.

The Statistics neutrosophic T^2 Hotelling is :

$$T_N^2 = \left[(\bar{X}_L - \mu_{0L})' \left(\frac{1}{n_L} S_L\right)^{-1}\right] (\bar{X}_L - \mu_{0L}), (\bar{X}_U - \mu_{0U})', \left(\frac{1}{n_U} S_U\right)^{-1} (\bar{X}_U - \mu_{0U}); T_N^2 \in [T_L^2, T_U^2], \quad (6)$$

where

$T_N^2 = T_L^2 + T_U^2 I_N; I_N \epsilon [I_L, I_U]$.

$T_N^2 \sim \frac{(n_N - 1)}{(n_N - p_N)} F_{p_N, (n_N - p_N)}; T_N^2 \epsilon [T_L^2, T_U^2]$.

These statistics can be applied for testing hypothesis.

2. Classical Hotelling T^2 Control Chart

The classical Hotelling $\boldsymbol{T^2}$ control chart widely used d to monitor mean proses simultaneously of more than one interrelated quality characteristics observations. This chart was proposed by Harold Hotelling, 1947 (Montgomery, 2020). The classical Hotelling $\boldsymbol{T^2}$ control chart can be used to monitor both of subgroup and individual obeservation. The statisics of Hotelling $\boldsymbol{T^2}$ chart for individual as follows.

$$\boldsymbol{T^2} = (\boldsymbol{x} - \bar{\boldsymbol{x}})' \boldsymbol{S}^{-1} (\boldsymbol{x} - \bar{\boldsymbol{x}}). \quad (7)$$

Let p is the number of quality characteristics, suppose we have m sample, each sample consist $n = 1$. The Control limits of statistic T^2 the control chart is (Montgomery, 2020)
Upper control limit (UCL)

$$UCL = \frac{(m-1)^2}{m} \beta_{\alpha;\beta/2(m-p-10/2} \tag{8}$$

and Lower control limit (LCL),

$$LCL = 0.$$

The process is in control if all the points fall between control limits.

C. RESULT AND DISCUSSION

1. Neutrosophic Hotelling $T^2(T_N^2)$ Control Chart

In this part, we discuss the new Hotelling T^2 monitoring scheme based on a fuzzy neutrosophic concept. Control charts and hypothesis testing have a close relationship. A point plotting within the control limits is the same as failing to reject the statistical control hypothesis, whereas a point plotting outside the control limits is the same as rejecting the statistical control hypothesis. The statistics in this new chart is obtained by combining classical Hotelling T^2 control chart and neutrosophic Hotelling T^2 statistic (T_N^2) which was proposed by Aslam (Aslam & Arif, 2020). Let $p_N \epsilon [p_L, p_U]$ is the number of quality characteristics and the data matrix $X_N \epsilon [X_L, X_U]$, suppose we have m sample with individual observation. The statistics of neutrosophic Hotelling $T^2(T_N^2)$ Control Chart is given by,

$$T_N^2 = [(\bar{X}_L - \mu_{0L})'(S_L)^{-1}](\bar{X}_L - \mu_{0L}), (\bar{X}_U - \mu_{0U})', (S_U)^{-1}(\bar{X}_U - \mu_{0U}), \tag{9}$$
$$T_N^2 \in [T_L^2, T_U^2],$$

where μ_{0L} and μ_{0U} are target.

Based on equation (9), the proposed chart consists of two charts, namely lower T_N^2 control chart and upper T_N^2 control chart. The control limits of the proposed chart are as follows:

$$UCL_N = \frac{p_N(m-1)^2}{m} \beta_{\alpha;\beta/2(m-p_N-10/2} \quad \text{and} \quad LCL_N = 0. \tag{10}$$

The process is in control if all the points fall between control limits (UCL_N and LCL_N).

2. Numerical Example

The application of neutrosophic hotelling T_N^2 Control Chart using data from Quality Control division for quality characteristics of glass production. There are two quality characteristics such as Cutter line (X_{1N}) and Edge Distorsion (X_{2N}). The target of the cutter is 115 mm and the Edge distortion is 40 mm. The form of the data can be seen in Table 1.

Based on these data we find the neutrosophic mean sample are $\bar{x}_{1N} \epsilon \big[132.76, 151.03\,\big]$ and $\bar{x}_{2N} \epsilon \big[30.61, 37.27\,\big]$, and the neutrosophic covariance sample (s_N^2) is

$$s_N^2 = \begin{pmatrix} 150.56 & 43.23 & 60.40 & 38.01 \\ 43.23 & 127.22 & 69.95 & 65.84 \\ 60.40 & 69.95 & 107.99 & 90.55 \\ 38.01 & 65.84 & 90.55 & 95.33 \end{pmatrix}$$

By using equation (9) and equation (10) we calculate neutrosophic Hotelling T^2 statistic (see Table 2) and the control limits are $LCL_N = 2.23, UCL_N = 24.63$. The result of the neutrosophic hotelling T_N^2 Control Chart as shown in Figure 1.

Table 1. The neutrosophic data of the cutter and Edge distortion

Subgroup	Cutter line		Edge distortion		Subgroup	Cutter line		Edge distortion	
	X_{1NL}	X_{1NU}	X_{2NL}	X_{2NU}		X_{1NL}	X_{1NU}	X_{2NL}	X_{2NU}
1	140	158	25	30	18	139	145	28	30
2	170	198	72	73	19	136	143	30	37
3	139	145	28	30	20	137	161	32	39
4	122	166	50	52	21	131	146	42	55
5	115	145	18	21	22	130	151	31	49
6	112	152	15	37	23	141	149	34	38
7	136	143	30	37	24	139	151	33	42
8	141	149	34	38	25	133	162	36	38
9	120	155	21	26	26	123	138	34	37
10	136	143	30	37	27	139	145	28	30
11	128	144	34	40	28	143	166	17	30
12	117	145	20	30	29	141	149	34	38
13	136	143	30	37	30	140	142	20	28
14	118	160	38	48	31	136	137	29	37
15	106	155	28	38	32	141	149	24	30
16	116	155	23	30	33	139	145	28	30
17	141	149	34	38					

Table 2. The neutrosophic T_N^2 statistic of the cutter and Edge distortion

Subgroup	T_{NL}^2	T_{NU}^2	Subgroup	T_{NL}^2	T_{NU}^2
1	17.85	31.75	18	13.22	15.63
2	33.02	56.13	19	14.87	17.73
3	14.87	17.73	20	11.11	11.33
4	0.90	24.34	21	10.86	27.21
5	5.65	26.07	22	3.55	7.51
6	6.33	19.08	23	6.43	11.95
7	11.11	11.33	24	13.22	15.63
8	13.22	15.63	25	11.99	15.04
9	6.00	32.22	26	6.28	29.22
10	11.11	11.33	27	2.12	7.89
11	4.19	10.50	28	14.87	17.73
12	5.32	17.73	29	27.83	42.48
13	11.11	11.33	30	13.22	15.63
14	0.27	20.20	31	21.74	16.64
15	1.35	21.38	32	11.66	7.28
16	3.64	28.14	33	19.61	21.59
17	17.85	31.75			

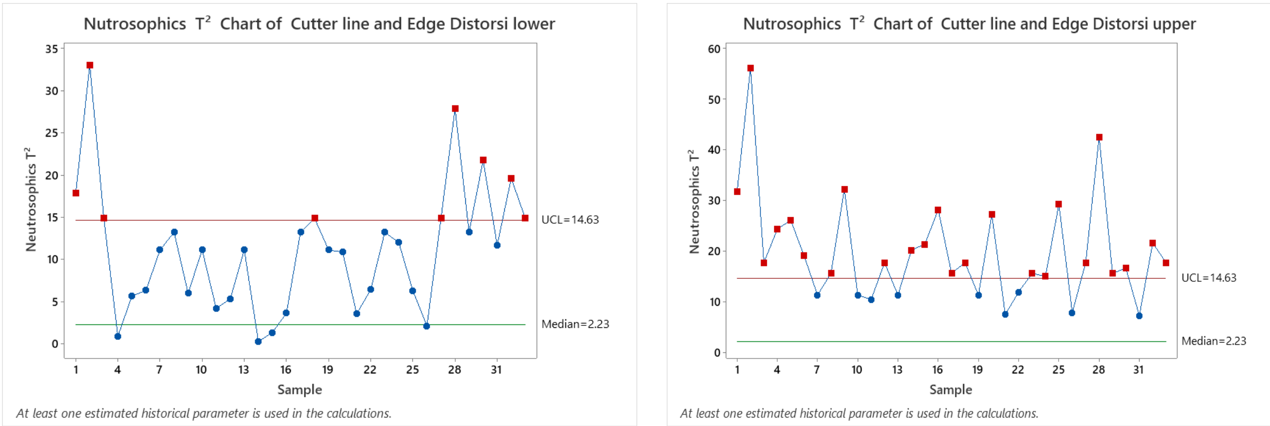

Figure 1. The neutrosophic Hotelling T_N^2 Control Chart

In Figure 1, the visualization of the neutrosophic hotelling T_N^2 control chart, indicate the process mean of glass production is out of control. Nine points fall outside of UCL_N of lower control chart and in the upper control chart closed to 73% of data outside from UCL_N. In the next step, we compare the neutrosophic hotelling T_N^2 control chart with the classical Hotelling T^2 control chart, see Figure 2. This figure shows the process mean of glass production is also out of control, but by using the classical method, just one point falls outside UCL_N. Therefore based on this case the neutrosophic hotelling T_N^2 control chart is more sensitive than with the classical hotelling T^2 control chart.

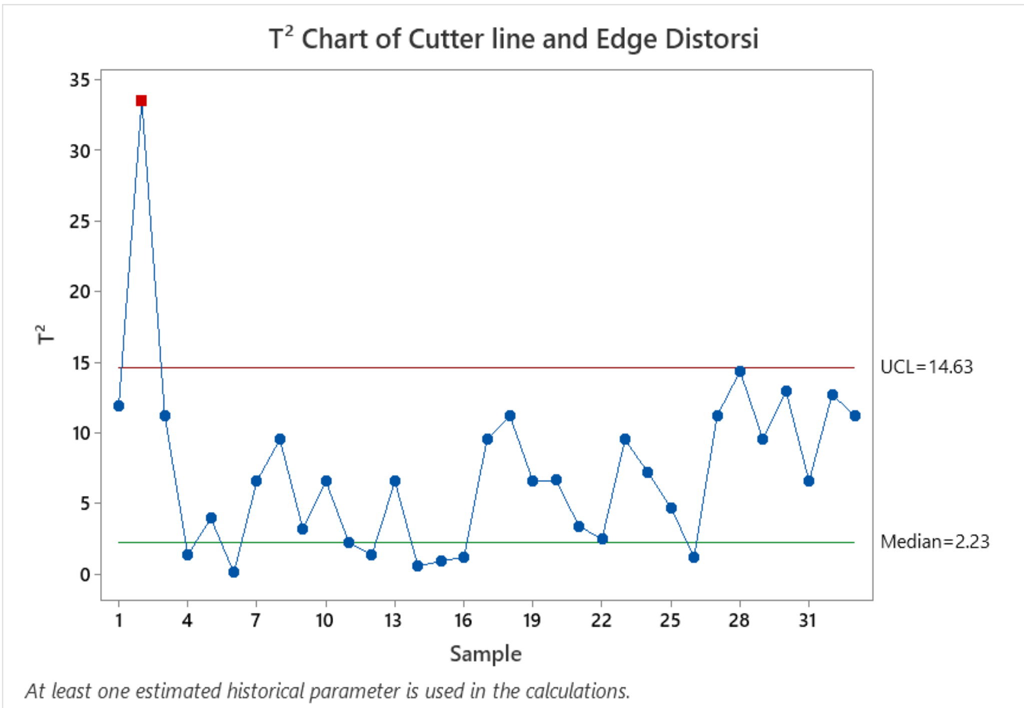

Figure 2. The Classical Hotelling T^2 control Chart

The comparison of the case study shows that T_N^2 control chart is more suitable for use than classical hotelling T^2 control chart. So the impact for the future, monitoring quality in problem in manufacture industries which has data are inaccurate or indeterminacy will be tackle with neutrosophic control chart.

D. CONCLUSION AND SUGGESTIONS

The statistics of new Hotelling T^2 monitoring scheme based on the fuzzy neutrosophic concept is obtained from a combination of neutrosophic hotelling T_N^2 statistic and classical Hotelling T^2 control chart. Hotelling T^2 chart based on the fuzzy neutrosophic is appropriate for indeterminacy data. The proposed chart consists of two charts, namely lower T_N^2 control chart and upper T_N^2 control chart. Based on the case study the proposed chart is more sensitive than the classical Hotelling T^2 control chart. The performance evaluation of T_N^2 control chart can be considered for future works. Further future research can be developed to the T_N^2 control chart that can be designed using a robust estimator or shrinkage estimator.

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