# The Four-Distance Domination Number in the Ladder and Star Graphs Amalgamation Result and Applications 

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#### Abstract

The study purpose is to determine the four-distance domination number in the amalgamation operation graph, namely the vertex amalgamation result graph of ladder graph $\operatorname{Amal}\left(L_{m}, v, n\right)$ with $m \geq 2$ and $n \geq 2$ and the vertex amalgamation result graph of a star graph with its name $\operatorname{Amal}\left(S_{m}, v, n\right)$ with $m \geq 2$ and $n>2$. In addition, the application use the Four-distance domination number on Jember Regency Covid-19 taskforce post-placement. The Importanceof this research, namely the optimal distribution of the Covid-19 task force post. It is not just doing mask surgeries every day on the streets. The optimal referred to can be in the form of integrated handlers in each sub-district or points that are considered to need fast handling so that coordination between posts can respond and immediately identify cases of transmission and potential infections due to interactions with patients who are already positive. The methods used in this research are pattern recognition and axiomatic deductive methods. The results of this study include:


$\gamma_{4}\left(\operatorname{Amal}\left(S_{\mathrm{m}}, \mathrm{v}, \mathrm{n}\right)\right)=1$; for $m \geq 2$ and $n \geq 2$,
$\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\{\begin{array}{l}1 ; \quad \text { for } 2 \leq m \leq 4 \\ \left\lfloor\frac{m}{8}\right\rfloor n+1 \text { for } m \equiv 0,1,2,3,4(\bmod 8) \\ \left\lceil\frac{m}{8}\right\rfloor n ; \quad \text { for others } m\end{array}\right.$
and based on the Indonesia Country, Jember Regency Map, 2 Covid 19 task-force posts are needed to be placed in Balung and Kalisat sub-districts using the Fourdistance domination number application.

## A. INTRODUCTION

Mathematics is a science that can be applied in various fields of life. For example, in the world of astronomy, science, health, and others. In mathematics, there is a specificity in developing it. There are several that are widely applied, such as design geometry, coding, computer science, statistics, graph theory is no exception (Knight, 2020). Graph theory is a branch of mathematics. Graph theory is also widely applied in various fields, such as agriculture, forestry, security, computers, and others but there is something interesting recently namely in the health sector. Recently, in the field of health, innovation in the graph theory field is needed. The thing that will be applied to the health and disaster management
sector is the placement method of the Covid-19 task force (Satgas) distribution post to suppress the virus spread in Jember District, Indonesia. From the data collected on the page https://www.jember.info/ (Jember, 2020) on November 21, 2020, it was found that cases of Covid-19 transmission in Jember Regency continued to increase every day. Every day, there are approximately 40-60 positive cases of Covid-19 from all sub-districts in Jember Regency. There are 3 sub-districts categorized as Covid-19 distribution. There are 11 sub-districts that are included in the red zone, 3 sub-districts are included in the yellow zone, and the remaining 17 are in the orange zone.

Thus, a breakthrough is needed, namely the optimal distribution of the Covid-19 task force post. It is not just doing mask surgeries every day on the streets. The optimal referred to can be in the form of integrated handlers in each sub-district or points that are considered to need fast handling so that coordination between posts can respond and immediately identify cases of transmission and potential infections due to interactions with patients who are already positive. In relation to graph theory, for example, task force posts are represented as points. Then as a liaison between posts, the road is represented as aside. So, if each task force post is connected to each other, which is connected by a road as an edge, then it is said to be a graph. Each post will be connected to each other, and if there are new cases, each post as a handling center will respond to immediately handle the transmission by giving disinfectant or conducting small-scale quarantine. Then, the next action is to be able to identify the potential for transmission around the area quickly. Taskforce posts that handle as points for handling Covid cases in the vicinity with a predetermined maximum distance are called domination number theory in the Covid-19 task force placement.

Number theory of domination has been studied by previous researchers, such as: (Umilasari et al., 2019) with the title Optimizing the Placement of Security Officers at the Prigen Safari Park in Pasuruan Using the Domination Association Theory. Aside from that (Poniman \& Fran, 2020) with the title eccentric domination number connected to sunlet graph and bishop graph. The domination number is the number of dominating vertices in the graph that can dominate the adjacent connected vertices and with the least number of dominating vertices. Thus, the domination number is symbolized $\gamma(G)$ (Umilasari, 2015), (Couturier et al., 2015), (Rote, 2019).

For example, it is a finite set of graphs and each graph has a fixed vertex, which is called a terminal. Amalgamation is formed by joining all the graphs at the terminal vertices $G_{i} G_{i} v_{o i} A \operatorname{mal}\left\{G_{i}, v_{o i}\right\} G_{i} v_{o i}$ (Citra et al., 2021), (Gross et al., 2014), and (Jing et al., 2021). The amalgamation operation in this study uses point amalgamation. The graph resulting from the vertex amalgamation operation is denoted where the amalgamation is composed by any $G$ graph of t copies and unites all G graphs at the terminal vertex v.Amal $(G, v, t)$ (Unnithan \& Balakrishnan, 2019), (Vargas \& Kulkarni, 2019), (Fitriani \& Salman, 2016).

The dominating set $S$ graph $G$ is a subset of $V(G)$ such that the vertices of $G$ that are not members of S are connected and have a distance of one from S (Enriquez, 2019) and (Akbari Torkestani \& Meybodi, 2012). The smallest number/cardinality among the dominant elements in the graph is called the domination number of the graph and is denoted by $\gamma(G)$
(Pino et al., 2018), (Mohanty et al., 2016), (Haddadan et al., 2016), and (Nacher \& Akutsu, 2016).

Domination Set the distance of two is denoted by $S_{2}$ which is the parts set of $V(\mathrm{G})$ such that the point G that is not a member of $S_{2}$ is connected and has the most distance 2 to $S_{2}$ (Umilasari \& Darmaji, 2017), and (Umilasari, 2015). The domination numeric distance of two from the graph is denoted by $\gamma_{2}(G)$. This means the least number/cardinality of the Domination Set is a distance of two. Determination of the domination point on any graph using a greedy algorithm (Cerrone et al., 2017), (Munir \& Rinaldi, 2012), and (Gembong et al., 2017).

## The Lemma Applied.

Lemma 1. Domination Number is a distance two on any regular graph of degree $G$ $r$ is $\gamma_{2}(\mathrm{G}) \geq\left\lceil\frac{|\mathrm{V}|}{\mathrm{r}^{2}+1}\right\rceil$
Lemma 2. The dominant number of distance two in any graph G is

$$
\gamma_{2}(\mathrm{G}) \geq\left\lceil\frac{|\mathrm{V}|}{1+\Delta(\mathrm{G})+\sum \mathrm{N}_{2}}\right\rceil .
$$

(Vikade, 2016)

## The theorem is applied.

Theorem 1. If there are $t$ copies of any connected graph $G$, then the domination number for the distance two on the graph resulting from the amalgamation operation is

$$
\gamma_{2}(\operatorname{Amal}(G, v, t))=\left\{\begin{array}{cc}
1 & ; \text { for silent }(G) \leq 2 \\
\gamma_{2}(G) t-t+1 ; \text { for silent }(G) \text { others }
\end{array}\right.
$$

(Vikade, 2016)
In this study, besides the application determined by the placement of Covid-19 task force post, it will also develop the domination number four (4) in the amalgamation operation graph, namely the vertex amalgamation result graph of the ladder graph $\operatorname{Amal}\left(L_{m}, v, n\right)$ with $m \geq 2$ and $n \geq 2$ and the vertex amalgamation result graph of the Star graph with its name $\operatorname{Amal}\left(S_{m}, v, n\right)$ with $m \geq 2$ and $n>2$. Beside that, this research can be used as a reference to suppress the spread of Covid-19 in Jember Regency or the surrounding community Thus the research raises the title "The Four Distances Domination Number in the Ladder and Star Graphs Amalgamation Result, and Applications".

## B. METHODS

The method in this study applies the method of detecting. In addition, it also uses axiomatic deductive reasoning. The method of detecting the intended pattern is by looking for a pattern, where the number of dominating points in a graph can dominate the surrounding connected points and with a minimum number of dominant points. Understanding axiomatic deductive is a method using the principles of deductive proof that apply in mathematical logic by using existing axioms or theorems to solve a problem so that the method will determine the domination number with a minimum domination point. Since this topic has not been widely studied, so that in addition to application to the spread of the Covid 19 Task Force Post in Jember Regency for Effectiveness of Emergency Response Suppressing Virus Transmission Using Dominant Number Theory. This research will also be expanded by determining the domination number of the four distances from the amalgamation operation graph, namely,
graph the result of the vertex amalgamation of the ladder graph $\operatorname{Amal}\left(L_{m}, v, n\right)$ with $m \geq$ 2 and $n \geq 2$ and the result graph of the star graph vertex amalgamation with its name $\operatorname{Amal}\left(S_{m}, v, n\right)$ with $m \geq 2$ and $n>2$. The research stages to get the domination number on the graph are shown on Figure 1 Research Flow.


Figure 1. Research Flow

## C. RESULT AND DISCUSSION

This section, it discusses the research results in the form of four distance domination numbers on the amalgamation operation graph, namely $\operatorname{Amal}\left(S_{m}, v, n\right)$ and $\operatorname{Amal}\left(L_{m}, v, n\right)$. In this study, the lower limit of the four-distance domination number on any regular graph will be shown in Lemma 3. Previously, we will show the maximum number of vertices that can be
dominated by a dominating vertex in any regular graph of $r$ degree, which can be seen in Figure 2.


Figure 2. Any Regular Graph with Black Knot as Dominant Knot

In particular, a dominating vertex in a regular $G$ graph with $r$ degree will dominate the vertex, i.e., the dominating vertex itself and all neighboring vertices are 4 . Thus, the maximum number of vertices/points that can be dominated by one vertex/dominant point on any regular G graph with r degree is $r^{4}-2 r^{3}+2 r^{2}+1$.

Lemma 3. The domination number of a four distance on any $r$ branch/degree regular graph is

$$
\gamma_{4}(G) \geq\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil
$$

Proof. G Graph is a regular/main graph with vertices and branches/degrees of each vertex is $r$. Based on the observation results, the maximum point that can be dominated by a dominating point is $r^{4}-2 r^{3}+2 r^{2}+1$. Thus, the minimum number of dominating points is $\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil$. So, $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil$. Furthermore, it will be shown that $\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil$ is the least number of dominating vertices which can dominate all vertices on a regular G graph.

$$
\text { If } \gamma_{4}(G) \geq\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil-1 \text {, the maximum number of point that can be dominated is }
$$

$$
r^{4}-2 r^{3}+2 r^{2}+1\left(\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil-1\right)=|V|-\left(r^{4}-2 r^{3}+2 r^{2}+1\right)
$$

where $|V|-\left(r^{4}-2 r^{3}+2 r^{2}+1\right)<|V|$, because the points number that is dominated is less than the points number $|V|$, it means that there are some points that cannot be dominated; thus the separation $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil-1$ is wrong. So, prove that $\gamma_{4}(G) \geq$ $\left\lceil\frac{|V|}{r^{4}-2 r^{3}+2 r^{2}+1}\right\rceil$ is the least/ minimum dominating point number of four distance that can dominate all vertices in G.
Meanwhile, the lower limit of four distance domination numbers in any G graph can be seen in Lemma 4.

Lemma 4 Four distance domination number of any $g$ graph is

$$
\gamma_{4}(G) \geq\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right\rceil
$$

Proof. G graph is any graph with many vertices as $|V|$, for example $v$ is a vertex with maximum degree $\Delta(G)$ so $v$ as a domination set $N_{2}, N_{3}, N_{4}$ are vertices that are two, three, four away from $v$. So, $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right\rceil$.

Then, it will be shown that $\left[\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right]$ is the minimum number of dominant vertices can dominate all vertices in G graph. For example, $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right\rceil-1$, so the maximum number of vertices that can be dominated is

$$
\begin{aligned}
& 1+\Delta(G)+N_{2}+N_{3}+N_{4}\left(\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right]-1\right) \\
= & |V|-\left(1+\Delta(G)+N_{2}+N_{3}+N_{4}\right)
\end{aligned}
$$

Where $|V|-\left(1+\Delta(G)+N_{2}+N_{3}+N_{4}\right)<|V|$, because the vertices number that is dominated is less than the vertices number
$|V|$, it means that there are several vertices that cannot be dominated, thus the supposition $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right\rceil-1$ is wrong. So, it is proved that $\gamma_{4}(G) \geq\left\lceil\frac{|V|}{1+\Delta(G)+N_{2}+N_{3}+N_{4}}\right\rceil$ is the minimum number of vertices dominating four distance that can dominate all vertices in G .

## 1. The Four-Distance Domination Number on Amalgamation Operation Result Graph $\operatorname{Amal}\left(S_{m}, \boldsymbol{v}, \mathrm{n}\right)$

Next, it will discuss four distance domination number on vertex amalgamation operation result graph $\operatorname{Amal}\left(S_{m}, v, n\right)$.
Theorem 2. Given a star graph $S_{m}$ as many $n$ copies, so the four distance domination number on vertex amalgamation operation result graph is

$$
\gamma_{4}\left(\operatorname{Amal}\left(S_{m}, v, n\right)\right)=1
$$

Proof. Known $\operatorname{Amal}\left(S_{m}, v, n\right)$ as vertex amalgamation operation resulting graph from $S_{m}$ graph as many $n$ copies with $v$ is a terminal vertex. If a $S_{m}$ graph has as many vertices as $m+$ 1, so $\operatorname{Amal}\left(S_{m}, v, n\right)$ has as many vertices as $m n+1 . S_{m}$ graph is a graph with two diameters and $\operatorname{Amal}\left(S_{m}, v, n\right)$ graph is an operation result graph with four diameter. So, a dominant vertex in $\operatorname{Amal}\left(S_{m}, v, n\right)$ graph can dominate all vertices that is as much as $m n+1$. The placement domination vertex can be placed at any vertex on $\operatorname{Amal}\left(S_{m}, v, n\right)$ like in Figure 3.


Figure 3. $\operatorname{Amal}\left(S_{m}, v, n\right)$ graph

## 2. The Four-Distance Domination Number on Amalgamation Operation Result Graph $\operatorname{Amal}\left(L_{m}, v, n\right)$

Next, we will discuss four distance domination numbers on the amalgamation operation result graph $\operatorname{Amal}\left(L_{m}, v, n\right)$.
Theorem 3. gives as many $n$ copies of $\mathrm{L}_{\mathrm{m}}$ star graph, then the four distance domination number on amalgamation operation result graph is

$$
\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)= \begin{cases}1 ; & \text { for } 2 \leq m \leq 4 \\ \left\lfloor\frac{\mathrm{~m}}{8}\right\rfloor \mathrm{n}+1 ; & \text { for } \mathrm{m} \equiv 0,1,2,3,4(\bmod 8) \\ \left\lceil\frac{\mathrm{m}}{8}\right\rceil \mathrm{n} ; & \text { for other } \mathrm{m}\end{cases}
$$

Proof. Known $\operatorname{Amal}\left(L_{m}, v, n\right)$ as vertex amalgamation resulting graph of from $L_{m}$ graph as many $n$ copies with $v$ is terminal vertex. If a $L_{m}$ graph has vertex as many $2 m$, then $\operatorname{Amal}\left(L_{m}, v, n\right)$ has vertex as many $2 m n-n+1$. A dominating vertex on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph can dominate the maximum as many $8 n+1$. A dominating vertex on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph for $2 \leq m \leq 4$ can dominate all vertices on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph that can be seen on Figure 4. Thus, $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=1$ for $2 \leq m \leq 4$ with $v \in S_{4}$.


Figure 4. $\boldsymbol{A m a l}\left(\mathbf{L}_{4}, \mathbf{v}, \mathbf{3}\right)$ graph with black vertex that is dominating vertex

Furthermore, to prove four distance domination numbers on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph will be divided into two parts, namely $S_{4} \in V\left(L_{m}\right)$ and $S_{4} \in v \cup V\left(L_{m}\right)$, where $v$ is a terminal vertex on vertex amalgamation result graph $S_{4}$ are four distance domination vertex set in a graph, and $V\left(L_{m}\right)$ is vertex on the ladder graph.
a. $\quad S_{4} \in v \cup V\left(L_{m}\right)$

For $S_{4} \in v \cup V\left(L_{m}\right)$ with $m \equiv 0,1,2,3,4(\bmod 8)$, every dominating vertex can dominate a maximum of as many 16 vertices on $L_{m}$ graph and add by a vertex on $v$ so $\left|S_{4}\right|=\frac{2 m}{16}+1=\frac{m}{8}+1$ means there is a dominating vertex on $v$ is terminal vertex on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph, while the other dominating vertices are on $L_{m}$ graph as many $n$ copies, so $\left|S_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)\right|=\frac{m}{8} n+1$. Because $\left|S_{4}\right|$ it must be an integer then $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lfloor\frac{m}{8}\right\rfloor n+1$. Next, it will be shown that $\left\lfloor\frac{m}{8}\right\rfloor n+1$ is the minimum dominating vertices number that can dominate all vertices on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph. If
$\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lfloor\frac{m}{8}\right\rfloor n+1-1$, then the maximum vertex number that can be dominated is

$$
16 n+1\left(\frac{2 m n-n+1}{16 n+1}-1\right)=2 m n-17 n
$$

Where $2 m n-17 n<2 m n-n+1$ because many vertices are dominated by less than many vertices $\operatorname{Amal}\left(L_{m}, v, n\right)$, means that there are several vertices that cannot be dominated, by the supposition $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lfloor\frac{m}{8}\right\rfloor n+1-1$ is wrong. So, it is proven that $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lfloor\frac{m}{8}\right\rfloor n+1$ is the least dominating vertex number of four distance that can dominate all vertices on $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph. As shown in Figure 5.


Figure 5. Amal $\left(\mathbf{L}_{\mathbf{8}}, \mathbf{v}, \mathbf{3}\right)$ graph with black vertices is an element of four distance domination set.
b. $\quad S_{4} \in V\left(L_{m}\right)$

The vertices number in ladder graph $L_{m}$ is $2 m$. For each element vertex $V\left(L_{m}\right)$ can dominate maximum as many 16 vertices, so $\left|S_{4}\right|=\frac{2 m}{16}=\frac{m}{8}$ means interval or distance each element vertex $S_{4}$ to the others is equal to 8. Because $\left|S_{4}\right|$ must be an integer then $\gamma_{4}\left(L_{m}\right)=\left\lceil\frac{m}{8}\right\rceil$, while $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lceil\frac{m}{8}\right\rceil n$ with $n$ is the number of copies. Next, we will show that $\left\lceil\frac{m}{8}\right\rceil n$ is the minimum number of vertices that can dominate all vertices on $\operatorname{graph} \operatorname{Amal}\left(L_{m}, v, n\right)$. If $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lceil\frac{\mathrm{m}}{8}\right\rceil n-1$, then the maximum number of vertices that can be dominated is

$$
16 n\left(\frac{2 m n-n+1}{16 n}-1\right)=2 m n-17 n+1
$$

where $2 m n-17 n+1<2 m n-n+1$ because many vertices are dominated by less than many vertices $\operatorname{Amal}\left(L_{m}, v, n\right)$, means there are some vertices that cannot be
dominated, the supposition $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lceil\frac{m}{8}\right\rceil n-1$ is wrong. So, it is proved that $\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\lceil\frac{m}{8}\right\rceil n$ is the vertices minimum number of four distance that can dominate all vertices of $\operatorname{Amal}\left(L_{m}, v, n\right)$ graph. As shown in Figure 6.


Figure 6. Amal $\left(L_{5}, v, 3\right)$ graph with black vertices are element of four distance set

## 3. Case study of Four Distances Dominant Number on the Distribution of Post Covid in Jember Regency

The application is discussed regarding the morphology of Jember Regency map, which can be seen in Figure 7. The first step is to represent the map into a J-Graph where a J-Graph is a way of representing a graph with the sub-district area as a node and connecting roads between sub-districts are represented as edges, as shown in Figure 7.


Figure 7. Map of Jember
The J-Graph representation of Jember Regency map can be seen in Figure 8 from the graph. The location of the Covid-19 monitoring post will be determined according to the domination number theory, in this case, using the four distance domination number. Thus, the
placement of monitoring posts will be more efficient, and the number of monitoring posts will be minimal, as shown in Figure 8.


Figure 8. A graph of Jember map
To determine the four-range domination set on J-Graph, the Jember Regency map can use the Greedy Algorithm by taking into account the following components:
a. The set of candidates are; $x_{1}, x_{2}, x_{3}, \cdots, x_{n}$;
b. The set of solutions is total $S_{4}$;
c. The selection function is to choose the maximum degree; $v \in x_{1}, x_{2}, x_{3}, \cdots, x_{n}$
d. A feasible function is to check if $S_{4}$ dominates all;
e. The objective function will create $\gamma_{4}$ minimum.

Based on the analysis and Greedy's Algorithm, there is a four-distance domination number as much as 2 , which can be seen in Figure 9. In determining the domination number, the distance of four begins by giving a label; in this case, the label $x_{1}, x_{2}, x_{3}, \cdots, x_{n}$ will be written with the name of the sub-district in Jember Regency. The completed steps are as follows:
a. Choosing a node with the maximum degree, namely the node with the label Balung District, the node can dominate a node itself, namely the District. Balung, and other nodes with a maximum distance of four, namely District of Sumberbaru, Jombang, Kencong, Gumukmas, Umbulsari, Semboro, Tanggul, Bangsal, Puger, Wuluhan, Rambipuji, Panti, Sukorambi, Patrang, Kaliwates, Ajung, Jenggawah, Ambulu, Sumbersari, Mumbulsari, and Tempurejo. It means that the Covid 19 task force is located in the Balung district.
b. Selecting a node with the next maximum degree that has not been dominated, namely a node with the label Kalisat District, this node can dominate a node itself, namely Kalisat district and other nodes with a maximum distance of four, namely Jelbuk, Sukowono, Sumberjambe, Ledokombo, Pakusari, and Silo District.
c. $S_{4}(J-G r a p h)=\{$ Balung, Kalisat $\}$.
d. $\left|S_{4}(J-G r a p h)\right|=2$.

Thus, on Jember Regency Map, 2 COVID-19 task force posts are needed, which will be placed in Balung and Kalisat District. As shown in Figure 9.


Figure 9. A graph with a dominant vertex

## D. CONCLUSION AND SUGGESTIONS

The following are the conclusions obtained from the results, and the previous discussion are:

1. Domination number for the four- distance on the vertex amalgamation operation resulting graph $\gamma_{4}\left(\operatorname{Amal}\left(S_{m}, v, n\right)\right)=1$ for $m \geq 2$ and $n \geq 2$ and the domination number for the four distance on the vertex amalgamation operation resulting graph is

$$
\gamma_{4}\left(\operatorname{Amal}\left(L_{m}, v, n\right)\right)=\left\{\begin{array}{l}
1 ; \quad \text { for } 2 \leq m \leq 4 \\
\left\lfloor\frac{m}{8}\right\rfloor n+1 ; \text { for } m \equiv 0,1,2,3,4(\bmod 8) \\
{\left[\frac{m}{8}\right\rceil n ; \text { for other } m}
\end{array}\right.
$$

2. Based on Jember Regency Map, two Covid-19 task-force posts are needed to be placed in Balung and Kalisat sub-districts using the four-distance Domination number application.

From the results of the Domination Number research, the researcher gives suggestions to other researchers in order to be able to examine the domination number with other operating graphs with different distances. And can determine applications related to Domination Number to solve solutions in everyday life.

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## REFERENCES

Akbari Torkestani, J., \& Meybodi, M. R. (2012). Finding minimum weight connected dominating set in stochastic graph based on learning automata. Information Sciences, 200, 57-77.
https://doi.org/10.1016/j.ins.2012.02.057
Cerrone, C., Cerulli, R., \& Golden, B. (2017). Carousel greedy: A generalized greedy algorithm with applications in optimization. Computers and Operations Research, 85(May 2018), 97-112. https://doi.org/10.1016/j.cor.2017.03.016
Citra, S. M., Kristiana, A. I., Adawiyah, R., Dafik, \& Prihandini, R. M. (2021). On the packing chromatic number of vertex amalgamation of some related tree graph. Journal of Physics: Conference Series, 1836(1). https://doi.org/10.1088/1742-6596/1836/1/012025
Couturier, J.-F., Letourneur, R., \& Liedloff, M. (2015). On the number of minimal dominating sets on some graph classes. Theoretical Computer Science, 562, 634-642. https://doi.org/https://doi.org/10.1016/j.tcs.2014.11.006
Enriquez, E. L. (2019). Super Fair Dominating Set In Graphs. February. Available online at http://www.jgrma.info
Fitriani, D., \& Salman, A. N. M. (2016). Rainbow connection number of amalgamation of some graphs. AKCE International Journal of Graphs and Combinatorics, 13(1), 90-99. https://doi.org/10.1016/j.akcej.2016.03.004
Gembong, A. W., Slamin, Dafik, \& Agustin, I. H. (2017). Bound of Distance Domination Number of Graph and Edge Comb Product Graph. Journal of Physics: Conference Series, 855(1). https://doi.org/10.1088/1742-6596/855/1/012014
Gross, J. L., Mansour, T., \& Tucker, T. W. (2014). Log-concavity of genus distributions of ring-like families of graphs. European Journal of Combinatorics, 42, 74-91. https://doi.org/https://doi.org/10.1016/j.ejc.2014.05.008
Haddadan, A., Ito, T., Mouawad, A. E., Nishimura, N., Ono, H., Suzuki, A., \& Tebbal, Y. (2016). The complexity of dominating set reconfiguration. Theoretical Computer Science, 651, 37-49. https://doi.org/https://doi.org/10.1016/j.tcs.2016.08.016
Jember, T. S. C.-19. (2020). Jember Information Center. Jember.info.
Jing, Y., Yang, Y., Wang, X., Song, M., \& Tao, D. (2021). Amalgamating Knowledge from Heterogeneous Graph Neural Networks. 15704-15713. https://doi.org/10.1109/cvpr46437.2021.01545
Knight, A. (2020). Risk-Assessment Frameworks. In Hacking Connected Cars. https://doi.org/10.1002/9781119491774.ch8
Mohanty, J. P., Mandal, C., Reade, C., \& Das, A. (2016). Construction of minimum connected dominating set in wireless sensor networks using pseudo dominating set. Ad Hoc Networks, 42, 61-73. https://doi.org/https://doi.org/10.1016/j.adhoc.2016.02.003
Nacher, J. C., \& Akutsu, T. (2016). Minimum dominating set-based methods for analyzing biological networks. Methods, 102, 57-63. https://doi.org/https://doi.org/10.1016/j.ymeth.2015.12.017
Pino, T., Choudhury, S., \& Al-Turjman, F. (2018). Dominating Set Algorithms for Wireless Sensor Networks Survivability. IEEE Access, 6, 17527-17532. https://doi.org/10.1109/ACCESS.2018.2819083
Poniman, B., \& Fran, F. (2020). Bilangan dominasi eksentrik terhubung pada graf sunlet dan graf bishop. BIMASTER, 09(1), 71-78.
Rote, G. (2019). The maximum number of minimal dominating sets in a tree. Proceedings of the Annual ACM-SIAM Symposium on Discrete Algorithms, 1201-1214. https://doi.org/10.1137/1.9781611975482.73
Umilasari, R. (2015). Bilangan Dominasi Jarak Dua Pada Graf-. ITS.
Umilasari, R., \& Darmaji, D. (2017). Dominating number of distance two of corona products of graphs. Indonesian Journal of Combinatorics, 1(1), 41. https://doi.org/10.19184/ijc.2016.1.1.5
Umilasari, R., Saifudin, I., \& Azhar, R. F. (2019). Optimasi Penempatan Petugas Keamanan Di Taman Safari Prigen Pasuruan Menggunakan Teori Himpunan Dominasi. JUSTINDO (Jurnal Sistem Dan Teknologi Informasi Indonesia), 4(2), 36. https://doi.org/10.32528/justindo.v4i2.2613
Unnithan, S. K. R., \& Balakrishnan, K. (2019). Betweenness centrality in convex amalgamation of graphs. Journal of Algebra Combinatorics Discrete Structures and Applications, 6(1), 21-38. https://doi.org/10.13069/jacodesmath. 508983
Vargas, J. G., \& Kulkarni, A. (2019). Spectra of infinite graphs via freeness with amalgamation. arXiv.
Vikade, W. D. (2016). Bilangan Dominasi Jarak Dua pada Graf Hasil Operasi. Universitas Jember.

