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Teachers Promoting Mathematical Reasoning in Tasks

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ABSTRACT

The phenomenon today in schools is that teachers rarely build student arguments but only accept students' answers. However, teacher activities that always make students 'arguments and support each solution and defend students' arguments without long debates are relevant and exciting to study. This study aims to explain and explore the promotion of mathematics teacher reasoning in tasks in the classroom. The author surveyed teachers who teach mathematics at the junior high level. This research is a qualitative descriptive study with an experimental research design, which begins with a survey of teachers who teach mathematics at the junior high school level in Maluku Province. First, the authors conducted initial observations for one month in several junior high schools in Maluku Province with the guide of the observation instrument for the promotion of teacher reasoning in learning. Second, the writer states that there are three groups of attractive teachers who can promote mathematical reasoning in each of their teachings based on indicators of teachers' mathematical reasoning. Third, the writer created the same math problems for the three teachers and observed how they built students' reasoning and evaluated their thinking efficiently. Finally, the results and surveys were carried out by triangulation with direct trials on seven classes at the junior high level. The results show how teacher actions promoting mathematical reasoning give generalizations or justifications. Teacher actions supporting precise rationale are discussed in more detail in this article.

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A. INTRODUCTION

Students need mathematical reasoning as a condition for carrying out proofs and using routine concepts that go beyond procedures in each task. Thus developing students' mathematical reasoning in everyday classrooms is an essential aspect of learning mathematics (Mata-Pereira & da Ponte, 2017); (Segerby & Chronaki, 2018)). Mathematical reasoning is also seen as making correct conclusions using deductive, inductive, and abductive processes (Hidayah et al., 2020; Hjelte et al., 2020; Moguel et al., 2019). So formulating questions and completing strategies, formulating and testing generalizations and other conjectures, and justifying them are also seen as mathematical reasoning processes. To promote mathematical reasoning, a teacher must provide a challenging learning environment, not only from the lesson but also from exercises requiring thinking processes using various procedures. Four important things form the basis for promoting mathematical reasoning carried out by teachers, namely teacher intervention, tasks, questioning skills, listening skills, and the mathematics community (Mueller et al., 2014).

The four basic mathematical reasoning that the teacher must do will provide direct involvement of students in arguing, which is a challenge in teaching mathematics, including ordering statements to refute mathematical claims (Lin, 2018). Many studies have emphasized the importance of argumentation (Firdausy et al., 2021; Nergård, 2021; Tavșan & Pusmaz, 2021), because effective argumentation skills are essential for good conceptual understanding and communication. Accumulated studies and reform documents have recommended that students should have an early opportunity to make conjectures, explore the veracity of their conjectures, use counter-examples, justify their conclusions, communicate them to others, and respond to the arguments of others (Simsek, 2021; Widjaja & Vale, 2021). To provoke students' arguments, teacher intervention is needed. Teacher intervention is an important component of promoting ideas and sharing solutions among students. Based on (Darling-Hammond et al., 2020), the teacher's role in increasing meaningful interventions when students solve problems will help students represent and improve their ideas to play an active role in learning. Therefore, the teacher can facilitate a more meaningful and clear explanation that leads to detail and streamlined representation so that, in the end, perfecting students' arguments (Santia et al., 2019). Teachers who are proficient in intervening can generate students' reasoning.

One form of intervention that is often planned before the teacher enters the classroom is task selection or design. Many researchers have emphasized that task promotes reasoning and understanding (Darling-Hammond et al., 2020; Mastuti et al., 2016, 2016; Mata-Pereira & da Ponte, 2017; Sølvik & Glenna, 2021). Tasks for reasoning can be designed in a challenging and open manner (Darling-Hammond et al., 2020; Johnson et al., 2017). In addition, teachers need to design tasks that can encourage students to justify their arguments and rely on themselves (Lodge et al., 2018; Van Lacum et al., 2014). After doing tasks, teachers encourage students to build their justifications and share ideas. During this phase, the teacher observes and listens carefully to expect students to think of their solutions. Then, based on the type of task posed, the teacher initiates specific movements to promote reasoning and understanding.

The specific movement made by the teacher is that when students try to improve understanding, the teacher must practice being a skilled and attentive listener. By listening, teachers can recognize if their students build solutions from their understanding of the problem. Listening also makes teachers understand students' ideas and recognize students' conceptions so that they can find problem-solving (Abdillah et al., 2022; Abramovich et al., 2019; Wilkinson et al., 2018). (Neumann, 2014) explains that teachers must listen, investigate, interpret and respond to students' thoughts.

In addition to listening, investigating, interpreting, and responding to students' thoughts, the important thing that teachers must also have is the ability to ask questions to improve students' reasoning, construct new knowledge, and share ideas (Tambunan & Naibaho, 2019). Questions are important things teachers must do to create a supportive environment in understanding mathematics and problem solving (Abdillah et al., 2022; Li & Schoenfeld, 2019; Masingila et al., 2018). (McCarthy et al., 2016) suggest that teachers should be aware of the types of questions they ask and their purpose in asking questions and divide the questions into three types: probing, guiding, and factual. Probing questions consist of questions that ask students to explain their thinking, offer justification or evidence, and use prior knowledge in dealing with tasks. Guiding questions tend to support students in creating their heuristics in obtaining mathematical concepts. In comparison, factual questions are requests for facts or definitions and answers or the next step in solving problems.

Mathematical reasoning and understanding naturally result in communication in the mathematics community (C. Wilkinson et al., 2018). The mathematics community is a classroom where students learn to speak and work in mathematical discussions, propose and defend arguments, and respond to their ideas and conjectures (Darling-Hammond et al., 2020). The formation of learning communities with social norms that promote reasoning directly promotes the development of autonomy in students.. (Pepin et al., 2017) characterizes mathematically autonomous students as individuals who are aware of their mathematical resources and can call on and use these resources to make mathematical judgments. Based on the previous explanation, the writer realized that the teacher's intervention, task, questioning ability, listening ability, and the mathematics community played an important role in promoting mathematical reasoning carried out by teachers.

This article is the author's investigative report on the behaviour of teachers in the mathematics community by assigning tasks to their students by promoting their reasoning. The research shows whether the promotion of mathematical reasoning carried out by teachers is successful or not in the mathematics community. Therefore, the results of this study are significant for the development of teaching thinking. This study provides an overview of mathematics education practices that can provide recommendations for better practice in Indonesia and the future. Mathematics teachers can use the results of this research as insight into how to make tasks for their students best and allow students to have many ideas about developing students' understanding and reasoning. In-service teacher trainers can also use the results of this study to train and improve the quality of existing mathematics teachers according to their competencies who can encourage their students to produce better student work that expresses their abilities.

B. METHODS

This research is qualitative descriptive research and experimental design research. Qualitative research describes how teachers promote students' mathematical reasoning and expect their students to use their reasoning in every math task in class. This research is an experimental design that aims to provide a means for teachers to promote mathematical reasoning so that researchers themselves are the main tool for systematically analyzing these methods. Promoting mathematical reasoning in question is a problem-solving process carried out by students involving student analysis and arguments with teacher intervention. An intervention structure is needed to carry out this design to focus on teacher tasks and actions taken from literature research and the experimental design cycle. Participants in this study were ten selected mathematics teachers who joined the Junior High School Mathematics Teachers' Community Discussion Group. The conditions to determine research participants are:

1. Teachers can teach for at least five years or teachers who have taken master education,

2. Mathematics teachers who have characteristics in making design tasks during teaching observed and proven by their portfolio,
3. Mathematics teachers who are willing to become a research participant,
4. Teachers are divide into three groups, and group 1 consists of 3 teachers who will conduct reasoning promotions in 3 classes at School A, group 2 consists of 3 teachers who teach in 3 courses at School B, group 3 consists of 4 teachers who teach in 4 classes at School C.

To find out the best way on how to promote mathematical reasoning in student tasks, the authors ask teachers to do mathematical task-based learning that involves student reasoning in its completion. In this study, the writer prepares a simple problem and proposes a group of teachers to give their students the issue. Then the process will be observed using a video camera. This problem can saw in the question below.

1. Bayu only has 50 thousand. He wants to buy seven books and three pens in shop A, But Bayu is hesitant to buy it and is worried about the lack of money. When he was watching the shop, he saw someone paying ten books that he was targeting at the cashier with a piece of 50 thousand that received a change of five thousand. Then Bayu saw another child paying the same pen with his choice of twelve thousand for two pens. Bayu realized that every time he bought five notebooks, he would automatically get a 10% discount. Help Bayu to calculate whether the money is enough to buy seven books and three pens? If it's enough to give your argument? If not enough, how many books and pens should Bayu buy?
2. The mother wants to buy T-shirts for her twin children. The first t-shirt is Rp. 85,000 with a 23% discount while the second one is Rp. 90,000 with a 15% + 15% discount. If your money is only 140,000, help me choose which shirt to buy to save more? Your decision? Why?
3. A middle school student named Abdul wants to buy a collection of junior high school mathematics UN questions from his savings. After checking with the two bookstores, he found different prices and different offers. At the "Dian Pertiwi" store, the price of the book is Rp. 150,000 with a discount of 15% + 15%. While at the NN store, the book's price is Rp 135,000, with a 17% discount. The "Dian Pertiwi" store's location is further from the "NN" store, so Abdul Abdul has to take public transportation to and from the shop to get to the store. If Abdul money is only Rp. 125,000, help Abdul to choose books from what store he should buy? Your decision? Give your argument?

This research is design to investigate how mathematical ideas and student reasoning develop over time. Attend the problems presented in mathematical tasks. In the process of promoting mathematical reasoning, the writer's focus is on the question of students generalizing and justifying (decision making). This study focuses on the characteristics of the task (Mastuti et al., 2016; Mata-Pereira & da Ponte, 2017). These characteristics include (1) a variety of questions from various levels of challenge, emphasizing problems and exploratory issues; (2) items that arise generalization; (3) questions asking for justification of answers and completion process; (4) elements that enable different settlement process.

Based on (Mata-Pereira & da Ponte, 2017), the author considers several principles regarding teacher actions to structure interventions: (a) monitoring students as they complete tasks, with the aim of not reducing the level of challenges; (b) ask students to explain why and to present alternative justifications; (c) highlighting or asking students to identify valid and invalid arguments by emphasizing what can validate them; (d) encourage the sharing of ideas; (e) accept and value false or partial contributions by deconstructing, adding to, or clarifying them; (f) support or inform students to highlight reasoning processes such as generalization and justification; (g) challenge students to formulate new questions, generalize, or justify. Each group of teachers conducts learning and tasks for 3 x 35 minutes done in 10 classes. Data collected in the form of video footage of teacher activities in class and interviews with teachers. Data analysis centers on each teacher's actions during class discussions. The author analyzes each data recorded on the video, which included in the category of teacher action, the process of mathematical reasoning delivered at each student argument. Teacher data is obtained from the learning records of each group of teachers who teach in grades VIII and IX.

C. RESULT AND DISCUSSION

1. When the Teacher Engages Students in Algebra and Arithmetic Problems at the Same Time

The first question corresponds to characteristic (1), where students are involved in various questions of various challenge levels, emphasizing problems and exploratory questions. The first question is designed by combining two problems at once, namely algebra and arithmetic. Students are placed in pair discussions and combine several ideas. The information needed is the price of books and pens by mathematical modelling, which involves a simple two-variable linear equation system, parsing prices after discounts by involving percentages in arithmetic. Questions 2 and 3 lead students to generalize the procedure according to characteristics (2). For example, students are asked to choose which items are more economical and choose a store selection decision based on the distance and price offered. Question 2 allows students to use different solving strategies, such as trial and error or looking for the right procedure, according to characteristic (4), namely questions that allow various solving processes. Question 2 also asks for justification, which is implicit in question 3 according to characteristic (3). Students are not expected to indicate which items to choose but consider the store's location to spend more efficiently. Design principles for teacher action regarding student work and whole-class discussion are considered in the lesson plans used throughout the lesson.

2. Formulate Strategies According to the Principle of Teacher's Actions

Each teacher in each group starts the lesson by introducing the task to students and asking questions while watching students work on the task. Some students observed beginning to work on problem 1, which directed students to start simple reasoning and make decisions. Then students begin to work on tasks in pairs. When they work, the teacher feels that many have difficulty with the second question. All three teachers call attention to students and invite students to be directly involved in class discussions. In contrast, the third group of teachers tries to repeat questions without reducing the level of challenges. Teacher statements following principle (a):

> "If you have never heard of the term discount, try to start with representation using more natural numbers, such as numbers 100 or 100,000. Try to imagine the solution and try to be directly involved in decision making as in the first problem."

The teacher suggests an interpretation of the second question, but some observed teachers are aware of starting from the first problem. Each teacher compares the solution to the first problem because students find it easier to solve. The teacher tries to encourage various ideas (principle d), challenges students to justify the answer (principle b), hoping that the learning objectives achieved. Student 1 tries to overcome the challenges in Question 1.

S1 : *Ma'am, I try the price of books and pens first*
G : *why should it be counted one by one?*
S2 : *Because the book's price is determined from the first payment while the pen from the second payment*
S3 : *Pay attention must be given to the first payment*

Based on interview data with teacher group 1 involved in the student discussion above, the teacher commented on student answer 1, that student 1 refers to the calculations one by one as a means to get answers, but the justification is not complete. The teacher considers students' responses 1 not to refer to the book discount paid. But the teacher in group 1 values students' answers without arguing. The teacher appreciates his partial contribution by using the answer (principle e). The teacher always asks "why" in each student's response. It aims to challenge students' answers to go one step further than their initial thought (principle b). When students step on the book discount, students begin to get close to justification. So the teacher suggests a more accurate form for modeling mathematically (principle f), so students can validate the rationale for the questions in Question 1.

S1 : *I understand ma'am after we calculate the book's price after the discount and the cost of the pen paid we will know*
G : *Okay. How do we know if the money is enough or not?*
S4 : *we will try to add them for seven books and three pens*

The teacher gives agreement to students 'answers to the responses to question 1 and validates students' explanations. After that, the teacher continues on problem 2. The teacher rechallenges students to do different strategies in solving problems (principle d). But not every teacher's challenge is immediately answered by students because there are still some students in the discussion still silent. The teacher realizes and begins to guide the discussion group. However, this guiding act is not enough to lead students in formulating strategies, and the teacher goes further in guiding. Even so, the teacher's guidance hasn't helped much. Although these actions do not result in student participation was to be sought, they open the way for students to present their ideas. S5 intervenes:

S5 : *I calculate the price of the shirt first, then second*
G : *okay, how to count it?*
S5 : 0,77×85.000×2
G : *Try to count the second shirt after* 15% *discount than* 15% *discount again*

At the beginning of their contribution, S5 did not answer the real challenge posed by the teacher, but immediately gave answers to problem 2. The teacher tried to encourage him to share his ideas (principle d) and support to complete his contribution (principle e). This teacher's action leads S5 to present the sum of discounts in question 2. Where is the price after the first discount then the results are discounted $(0{,}85 \times 90.000) = y, and\ (y\ \times 0{,}85) = z$, then compared to the price the first shirt and students asked to determine their decisions, so the teacher successfully identifies strategies to solve problem 2.

The strategy for completing problem 2 requires a generalization of the procedure sought. So the process of determining the price of each shirt is related to arithmetic thinking where students can choose to use the process $(0{,}77 \times 85.000) \times 2$ or $(\ 0{,}85 \times 90.000) = y, dan\ (y\ \times\ 0{,}85) = z, and\ z\ \times\ 2$ or $(0{,}77 \times 85.000) + z$, with this, the teacher challenges students 5 to ask for a justification for the statement (principle b).

G : *why do we choose this way?*
S5 : *To determine if I choose the first two shirts*

S5 presents invalid justification. By asking other questions, the teacher tells S5 that their statements are not sufficient justification (principle c) and guides students to get the right argument (principle b).

G : *Why do we need to know the price of each shirt?*
S6 : *To verify whether you have enough money to buy*
G : *why do we have to verify?*
S5 : *I don't agree well with student 6, because it's not just enough that is needed but how to save money.*

S6 provides invalid justification, but this time by repeating his statement, the teacher challenges students to validate the answer (principle c). The development obtained by the teacher, that S5 succeeded in identifying that the statement of S6 was less valid. Then the teacher tries again to repeat the same question until most students begin to be sure about their decision. The researcher analyzes at the stage of formulating this strategy, and the teacher feels the difficulty of students in dealing with problem 2. This leads him to repeat the question and involve students in it without reducing the challenge (principle a). Then, throughout the discussion, the teacher challenges or supports the action, guides students to clarify and deconstruct the wrong or correct incomplete contributions (principle e), and encourages sharing ideas (principle d). Principle B is rather prominent because the teacher asks students several times to lead them in the discussion. To overcome this principle, most teachers rely on challenging actions. However, when a student's justification is wrong or incomplete, he also handles this principle by guiding work. The teacher initially considers students 'mathematical thinking the same as students' fundamental knowledge (Çelik & Güzel, 2017). In this case, students need to connect their essential familiarity with the new knowledge they have gained. In terms of validating student justification (principle c), the teacher confirms or cancels the student's argument by relying on informing or asking students to express their strategies. In this last case, he also highlighted the need for justification (principle f) and suggested students in more accurate mathematical language. In the stage of formulating this strategy, the generalization of the procedure sought arises because of the student's plan 5. Even though the student or teacher does not state generalization, it becomes more visible when the teacher requests justification for the strategy student 5. So, at this stage, students have several opportunities to put forward arguments or ideas based on the principles of teacher action design and provide responses to other students. Reasons for giving explanations or arguments to explain students 'reasons based on mathematical principles they know and students' arguments also explain mathematical concepts and facts that students know (Murtafiah et al., 2018).

3. Focus on Procedures That Inadvertently Lead to Generalizations on Student Reasoning

The discussion took place intending to promote student reasoning by finding problem-solving problems 2. Students expected to make decisions on which shirts to choose concerning the amount of money to save. At the end of the discussion, S7 raised his hand and began to guess without completing the procedure like other students. S7 prefers to use his intuition in reasoning.

G : *Fine, we arrived at the last session of discussion, has anyone decided which one to choose?*

S7 : *I chose to buy the first two shirts at a discount of 23% because I saw that the cut was more significant and the shirt's price was lower.*

The teacher challenges students to share ideas to solve problems, not only because of the substantial discounts and prices (principle d). The teacher guides and supports student actions in turn, and with student participation, discussion requires a t-shirt solution that students choose with their arguments. Although the teacher expects students to solve problems by considering all things, the teacher still allows S7 to explain the solution process (principle d).

S7 : *I have just calculated if the price of the first shirt is* $2 \times (0{,}23 \times 85.000) = 39.100$*, so the price to be paid by the mother* $(2 \times 85.000) - 39.100 = 130.900$

G : *why do you multiply two at a discount.*

S7 : *Because I decided to choose the first two shirts*

Based on the above case, the teacher does not expect the student's completion strategy 7. The teacher decides to support answer S7 without arguing with it. The teacher asks for confirmation so that S7 make their arguments (principle e). In this case, S5 quickly replies to statement S7. S5 argues that it does not refer directly to the reason for S7. Still, S5 relies on generalizations that show a direct result of $0{,}77 \times 85.000$ then multiplies by two whose results will be the same as S7.

To focus the class on a new strategy for solving this problem, the teacher informs students about the ideas of S7 who chose another way from their peers (principle g). The teacher challenges students to explain why the opinions of S7 and S5 are relevant (principle b), it aims to clarify if a product of $0{,}77 \times 85.000$ will be the same as a result of $85.000 - (0{,}23 \times 85.000)$, because some students may not be aware of the generalization regarding the relationship between the two operations.

G : *Well, students all notice what your two friends are doing is very interesting. Both chose the same decision but with different ideas. Why did you (S7) make this decision?*

S7 : *I immediately saw if the first two t-shirts priced before the discount was* 170,000 *then reduced by two times the discount.*

The teacher realizes that his students do not have the tools to respond to their challenges. Still, the teacher reinforces ideas S7 and suggests clarification of the chosen procedure (principle e). S7 begins to validate and pay attention to the process, then concludes to synthesize the strategy to answer problem 2.

The researcher analyzes this stage, and the teacher tries to further developed a discussion about the strategy to solve the problem in problem 2. To do so, the teacher guides students while encouraging student participation (principle d). In general, the objectives of question 2 have achieved. The teacher supports the different answers developed by S7. The teacher lets students present their ideas, both in the process and generalizations during the discussion. Teachers provide opportunities for students to become creative individuals and provide opportunities for students to develop their concepts and develop logical arguments (Mastuti et al., 2016; Murtafiah et al., 2018). When the teacher handles generalizations that are not formulated by students, the teacher tries to involve students to justify and use challenging methods that aim to obtain their explanations (principle b), in addition to that students respond to the teacher's challenges with different answers (principle g). The teacher realizes that students do not have special procedures to return to their challenges, and interestingly, students can summarize the proposed solutions. The teacher also understands that mathematics education students' schematic representation in solving structured problems shows that they tend to use verbal descriptions to symbolic or vice versa (Santia et al., 2019). This stage allows students to focus on procedures that will be generalized.

4. Students Generalize and Interpret The Results

After discussing problems 1 and 2, students work independently to solve problem 3. During problem-solving, the teacher monitors students and asks students to explain what they are doing without reducing the problem level of challenge (principle a). With various procedures developed to solve previous problems, most students have the ease of using the generalizations given in question 3. After giving students time to work, the teacher asks one of them to write the equation on the blackboard and suggest students share ideas about problem 3 (principle d).

S5 : *more significant discount at the "Dian Pertiwi" store*
G : *please explain. Because you talk with code*
S8 *seen from the cut the price is lower at the "Dian Pertiwi" store*

The teacher reacts to contributions student 8 with challenging statements, shows student 8 thinking is more critical by using mathematical language (principle e). But the explanation that should have been received by student 5 was responded to by student 8. In this discussion, the teacher began by letting students share their ideas (principle d), but the teacher seemed to need to refocus the discussion on the procedure used to solve the problem in problem 3. The teacher realizes that the generalization has obtained in problem 2, but the teacher chooses to inform students about the procedure in problem 3 (principle f). Thus, the teacher guides students to conclude equality in the arithmetic, aiming to complete the previous answers (principle e):

G : *Problem 3 leads us to what conclusion?*
S5 : *choose what books to buy at the price, discount, and distance*

Contribution S5 gives the answer the teacher wants. But the teacher still informs students validating conclusions (principle c) and expressing their ideas with challenging answers (principle g), by asking several solutions to the equation.

G : *How many solutions can we possibly make to choose which shop we are going to?*
S5 : *a lot, ma'am*
G : *A lot, does that mean can you decide to choose a shop even though the book is more expensive?*
S5 : *Yes ma'am, because the higher price also has a more significant discount*
G : *Is it possible to choose a shop with lower book prices?*
S8 : *It could be ma'am because considering the possible distance*

Students efficiently respond to the challenges in problem 3, and the teacher guides them to build relationships with the number of solutions to the problem. The teacher also makes conclusions by informing students about the validity of this relationship (principle c).

G : *how big is the opportunity for us to choose "Dian Pertiwi" shop or "NN" shop*

S9 : *same*
G : *why is it the same?*
S9 : *Because every store has different considerations, it can be price, it can be a discount or distance*
S8 : *Right ma'am, everyone can take different answers*
S8 : *It could be ma'am because considering the possible distance*

At this stage, the teacher has confidence in students to go through several procedures because indirectly, students begin to understand the completion of problem 3. Students only need some guidance from the teacher to communicate mathematically well (principle e). With excellent mathematical communication, students get the desired knowledge. For example, when giving the opportunity, they generalize two similar arithmetic equations from two questions with more than 1 solution, the teacher indirectly promotes each student's reasoning. The teacher concludes this stage by finding, and informing students about their initial rough ideas becomes a justification (principle f).

The researcher analyzes this stage. The discussion on problem 3 is structured primarily by guiding and giving rise to challenging actions that consider or promote partial contributions from students (principle e) followed by guiding or informing activities that highlight the validity of student contributions (principle c). Teacher assistance can make students communicate well (Qohar & Sumarmo, 2013). This action line starts with a provocative act that encourages students to discuss (principle d). Apart from this stage, the teacher begins by highlighting the generalization of the procedure sought (principle f) by relying on guidelines and informing actions. The teacher also challenges students to go beyond question 3 by linking it to question 2 (principle g). The teacher provides a basis for generalizing to the procedures used in this task, which involve many solutions and representations on each student (Santia et al., 2019).

5. Use the Procedure Again and Interpret the Results Further

The discussion returns to part two of problem 3, which aims to invite students' reasoning in deciding to choose which bookstore to go to. This discussion is similar to the discussion on problem 2; most teachers provide information and guide actions. When students finish solving the equation on the board, the teacher informs all students to pay attention to the equation, $B = (0{,}85 \times 150.000) = 127.500\ then\ B1 = 0{,}85 \times B = 0{,}85 \times 127.500 = 108.375$, while $B2 = 135.000 \times 0{,}83 = 112.050$.

However, there student 3 reacting that he had different results namely$B1 = 150.000 - 2(0{,}15 \times 150.000), B2 = 135.000 - (0{,}17 \times 135.000)$. The teacher allows student 3 to share the process of completion (principle d) even though the information leads to things that are not true. The teacher invites students to see the end of student 3 while guiding the discussion (principle a).

G	:	*Is the discount* $15\% + 15\% = 2 \times 15\%$?
Some students	:	*no*
S8	:	*The discount is equivalent to buying items with the first discount of 15% then the results are given a 15% discount back*

In the discussion student 3 was still not satisfied with the student's and teacher's arguments.

S3	:	*is it not the same as the result of* $B1 = 150.000 - 2(0{,}15 \times 150.000)$
G	:	*Not the same, because you immediately multiply 2 as* $15\% + 15\% = 2 \times 15\%$?. *Try if you count one by one, whether the results are the same or not?*

The teacher provides a challenging question for student 3, and reminds student 3 of problem 2. Some students justify that the equation of student 3 is different from the solution, which is almost the same as the previous question 2. The teacher explains the assumptions of some students but does not entirely blame the students 3. The teacher gives all students challenging problems to make as many solutions as they know (principle g). But before the teacher explained the justification once again to all students (principles c and f). The teacher directs students to follow the path to the previous question.

The researcher analyzes this stage, and the teacher uses information and guides actions to engage students (principle d) in solving possible equations to many decisions. Thus, by relying on guiding actions and using informational activities mostly to repeat students' answers, the teacher does not reduce the task challenge (principle a). Due to student 3 unexpected statements, the teacher challenges and guides students by asking for further alternative justifications and justifications (principles b and e), leading the discussion towards the correct justification student 3 invalid statement. To synthesize this justification (principle f), the teacher highlights its validity (principle c), by informing action. When comparing the completion of questions 2 and 3, the teacher challenges students to go beyond the procedure (principle g) by identifying the number of equation solutions. Giving students ideas and directing them to generalize that adding a discount does not mean adding up the discount but calculating the discount once again after the initially given discount.

Regarding design principles for teacher actions, all of them seem relevant to improve students' mathematical reasoning. Moreover, certain design principles, combined with more general teacher actions, seem to lead to student generalizations and justifications. As highlighted in previous studies (Kosko et al., 2014), one teacher's activities in the class like, explaining the material is not enough to promote situations that give rise to mathematical reasoning processes. A systematic framework is needed to analyze teacher actions that distinguish between inviting, informing/suggesting, guiding/supporting, and challenging activity (da Ponte & Quaresma, 2016), It is useful to understand the path of action and help the rearrangement of principles design principles. The most visible principle in the discussion is to ask to explain alternative justifications. Often, the teacher's actions in this principle followed by activities related to validating statements, where the teacher tends to act informally and rarely uses challenging movements. Challenging works have the potential to involve students in justifying (As'ari et al., 2019). Generalization starts by inviting students and also by challenging or guiding them to participate, focusing on encouraging students to share their ideas can request unexpected generalizations. Challenging students to think more usually through challenging questions. They were then followed by guiding and informing actions, directing students to generalize. In this study, designing an intervention aims to develop students' mathematical reasoning.

D. CONCLUSION AND SUGGESTIONS

The promotion of mathematics teacher reasoning in-class tasks is done by First, the strong teacher involvement in the task discussion is an intervention in itself. Second, the class discussion provides an opportunity to improve students' reasoning processes, which allows them to make logical arguments. Third, teacher trust and respect for students 'arguments are the most effective actions in promoting students' mathematical reasoning. Fourth, student reasoning tends to emerge if the teacher follows several lines of work. Besides that, it also found that the teacher's challenging actions could influence students' answers and arguments, giving rise to generalizations or justifications. Further research that is interesting to do is to discuss new steps taken and taken by the teacher for a mathematical proof—the shift from generalization to a justification for indirect evidence for students or teachers.

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