JTAM (Jurnal Teori dan Aplikasi Matematika)
http://journal.ummat.ac.id/index.php/jtam
p-ISSN 2597-7512 | e-ISSN 2614-1175
Vol. 6, No. 3, July 2022, pp. 565-580

The Practice of Reflection Based on Didactical Design Research: An Analysis of the Geometry Transformation Material

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Article History:
Received : 18-04-2022
Revised : 16-06-2022
Accepted: 24-06-2022
Online : 16-07-2022

Keywords:
Concept Image;
Geometics
Learning Obstacles;
Transformation;

ABSTRACT

This study aims to identify the possibility of learning obstacles in the concept of Geometric Transformation based on the meaning of the Geometric Transformation concept that students have regarding their experience in obtaining the definition of the Geometric Transformation concept. This study uses a qualitative method and Didactical Design Research for methodological framework that contains three stages of analysis: prospective, metapedadidactic, and retrospective. Teachers who will carry out the learning reflection process based on didactical design research were chosen through purposive sampling as research participants. Forty-eight students took the written test, and then six students were selected by purposive sampling to participate in in-depth interviews. Data analysis was carried out descriptively by reducing data, presenting data, and drawing conclusions. The result indicates that the meaning of the concept of Geometry Transformation, according to students, was the mapping of points in a plane to a set of points in the same plane; the existence of inconsistencies and ambiguity of meaning, and the emergence of the findings of other meaning units of Geometric Transformation concepts. The experience of student meaning shows a tendency for students to get a sense from what is taught by teachers and books with more procedurally oriented concept meanings. Based on the purpose and experience of students' definition, there are learning obstacles in the Geometric Transformation concept, including ontogenic obstacles, epistemological obstacles, and didactical obstacles. These learning obstacles can be a valuable consideration for improving and developing learning designs related to the concept of Geometric Transformation.

Crossref
https://doi.org/10.31764/jtam.v6i3.8441

A. INTRODUCTION

Students, teachers, and mathematics are all part of learning mathematics (Harel, 2008; Maher et al., 2018; Suryadi, 2010a). Many studies have discovered math learning obstacles (Ali & Reid, 2012). In the past, a teacher would offer a formal definition, work on it for a while, and then spend time on a series of examples to demonstrate its usefulness (Cheng & Kong, 2017). Students struggle to understand the structural elements of mathematical concepts if they have to go by ontological characteristics between operational and structural phases, but we need to understand the structure and scientific method (Attorps, 2007a; Tekin-Sitrava, 2017). The scientific method is defined as a series of repeated steps in which one's comprehension of a mental action reveals certain cognitive qualities (Harel, 2008) Because a comprehensive understanding instils confidence in students (Sadi & Daugyar, 2015). As a result, pupils must possess complete information. Furthermore, students must grasp the algorithm for constructing meaning to support their knowledge of mathematical ideas.

Before entering first grade, the student studies and develops geometrical ideas (Clements et al., 1999). The development of concepts in geometry, like in other sciences, is a difficult task. Conceptual spaces, for example, are mathematical entities in dimensional structures that provide a geometric representation of knowledge, while abstract spaces are mathematical entities in dimensional structures (Bechberger & Kühnberger, 2019; Gärdenfors, 2019; Tsamir et al., 2015a). Geometric concepts also represent and solve other mathematical materials and real-world problems (NCTM, 2000). Then, as one of the fundamental concepts in geometry, geometric transformations reflect the process of mapping points one-to-one and onto the plane (Martin, 2012). The geometric transformations strand can evaluate the ability to combine algebraic and geometric concepts Bansilal & Naidoo, (2012) as well as give pupils the chance to ponder crucial mathematical ideas (e.g., functions, symmetry) (Hollebrands, 2003; Uygun & Akyüz, 2019). Students can build mathematical reasons and proofs through a geometric transformation while learning the features and pictures of forms (Uygun, 2020). As a result, geometry and geometric transformations are critical components of mathematics education.

According to the data, the average degree of completeness among students who met the Minimum Completeness Criteria of 70 on the Geometry Transformation Competency Test 3 on November 1, 2019, was just 60.42 per cent. As a result, low student achievement is a problem that can be rectified right away. The researchers discovered that the low accomplishment of class IX students in Geometry Transformation content was due to various issues, including students' failure to grasp the material's essential principles and awful learning experiences and learning obstacles. Furthermore, students struggle to comprehend the concepts of geometric transformation (Albab et al., 2014). Earlier studies have discovered a common misunderstanding about rotations(Ada & Kurtulucs, 2010; Edwards, 1997). Also, it was limited when creating geometric transformational proofs (Portnoy et al., 2006). Students struggled to understand rotations and identify the centre and angle of course of a figure and its age; they also struggled to identify a single process that would match a figure with its look (Yanik & Flores, 2009). Malatjie & Machaba (2019) Conducted a study based on students' lack of conceptual knowledge of coordinate and transformational geometry. As a result, it is essential to understand students' learning obstacles to construct a learning situation that will help them overcome these obstacles.

Due to the difficulty of the material, the diverse abilities of students in comprehending the material, and the ingenuity of teachers in designing minimal learning materials, mathematics learning in schools is constantly confronted with the issue of how to package mathematics understanding correctly and adequately, especially in building a complete knowledge and understanding of a mathematical concept. The emphasis of designed learning situations is on more abstract thinking about transformations, such as viewing them as mapping fields to themselves (Yanik, 2011). Introducing new mathematics concepts requires a thorough review; pupils must accept mathematical principles based on prior beliefs, values, and experiences (Rösken & Rolka, 2007; Sullivan, 2011). As a result, mathematics education in schools does not proceed as intended. Geometry material in mathematics appears straightforward at first glance. Students sketch and calculate the necessary information to solve the challenge. In reality, many students struggle to comprehend the taught geometry concepts. Geometry examples that pupils are familiar with include lines, forms, and shapes. According to studies undertaken by several specialists, pupils have limited understanding and experience with the properties of geometric shapes (Jiang, 2008). Therefore, students' fundamental knowledge is established on a shaky foundation (Bardini et al., 2014). Students have encountered problems with this geometry program, and teachers have voiced their frustration.

In light of the facts presented, this suggests a fundamental issue, particularly concerning the meaning (hermeneutics) and definition of the idea of Geometric Transformation, which is intimately linked to one's experience (phenomenology). The process of students constructing the meaning of a concept, which is the concept of Geometric Transformation, is influenced by definition from various points of view, such as other students and teachers through the learning designs they develop in the context of learning mathematics (Suryadi, 2018), produces a series of meaningful experiences related to the concept of Geometric Transformation in each student. According to (Suryadi, 2019a), This reality can lead to a conceptual chasm in Geometric Transformation, known as the Zone of Concept Image Differences (ZCID). This discrepancy can undoubtedly produce learning difficulties. Therefore it has ramifications for students' lack of excellent grasp of the concept of geometric transformation.

Teachers have only utilized classroom action research to reflect on learning so far and have done it without paying attention to student reactions. Action research in the classroom is used to improve student learning results, as is the case with what is done (Nurul'Azizah, 2019), (Priatina, 2018) and (Fendrik, 2019). So, we will need another alternative reflection technique to improve learning by paying attention to student responses. DDR is a theoretical, conceptual, and metrological paradigm for designing learning materials based on learning obstacles (Suryadi, 2010b) that can reduce learning obstacles. Based on the DDR perspective, the nature of reality in learning mathematics is manifested in several situations (Suryadi, 2019b). The design of the problem is based on the learning obstacles that arise in students (Brousseau, 2006a), and in several previous DDR studies, such as that conducted by (Rosita et al., 2020), learning obstacle is used as the basis for designing a didactic design, then research conducted by (Yusuf et al., 2017) and (Miftah et al., 2020a) *l*earning designed based on learning obstacles will create an optimal learning process. Wati et al. (2020) study aims to create a didactic design to overcome the learning obstacle through DDR. Therefore, DDR is an alternative that can improve the learning process. Previous research has given us information that the didactic design used to make the didactic design was based on learning obstacles.

The didactic design is not only based on learning obstacles. Analyzing the gap between concept images and definitions can also create didactic designs in didactical design research. The primary cognitive process model uses concept images and descriptions (Vinner, 1983a). The concept image is non-verbal in our mind and associated with the concept's name (Vinner, 2002). Concept images are primarily acquired using their purposes (Vinner, 2002). Students meet the old idea in a new context. The concept image responds to the task with all the implicit assumptions abstracted from earlier contexts (Tall & Vinner, 1981a). The concept of image is influenced by knowledge and experience related to previous studies. Constructing a formal definition is a potential indicator of a deep understanding of the concept (Gan, 1982; Ouvrier-Buffet, 2004). Researchers have reasonable arguments and empirical data that knowing the definition of a concept does not guarantee understanding the idea (Vinner, 2002). The concept image may not reflect the concept definition accurately, and the concept image continues to develop as students discover new mathematical ideas (Engelke Infante et al., 2018a; Habineza, 2013). So based on this, there is a possible gap between concept image and concept definition. Based on the explanation above, this research aims to examine the learning obstacles and describe the gap between the concepts and definitions of students, which are the basis for designing didactic designs in didactic design research.

B. METHODS

The paradigm utilized in this research is qualitative research. Qualitative techniques generally focus on how humans perceive, describe, and comprehend particular occurrences (Creswell, 2009a). This study employs the research design proposed by Didactical Design Research (DDR)(Suryadi, 2010c). DDR is a methodological framework that contains three stages of analysis: prospective, metapedadidactic, and retrospective(Suryadi, 2010b, 2015). DDR is utilized as a strategy used by teachers to reflect on learning. This research focuses on implementing prospective analysis where the teacher evaluates the concept image and learning obstacle students face.

The purposive sampling strategy is used in qualitative research where the researcher selects several individuals and situations to be studied to inform and understand the main problem in the study (Creswell & Clark, 2017). This study's participant was a teacher who conducted the learning reflection process based on DDR. The following are the actions performed by the instructor: involves junior high school students taking the Respondent Ability Test (RAT), the concept of Geometric Transformation, and interviews. The Respondent Ability Test (RAT) is an instrument to evaluate students' conceptual understanding abilities. As for the questions, they are as follows:

1. Find image A by rotating point A(4, 7) 90^0 degrees counterclockwise with the centre at O(0, 0).
2. In the jungle, a tiger is hunting deer. According to observations, the deer's coordinates are at point A, and the tiger's coordinates are at point B. The deer are making their way to C.
   a. Determine the translational numbers that transport the deer from point A to point C.
   b. Can the tiger catch the deer if it employs the exact translation of the deer?
   c. Calculate the number of translation pairs the tiger must complete to catch the deer!
3. Luminosity Make a rabbit shadow on the wall with a flashlight and your hand.
   a. Do you think your natural hand or the shadow of your hand that makes up the rabbit is more giant?
   b. What kind of flashlight is used in the experiment if it is expanded?
   c. The finger's length is 7 cm, whereas the size of the shadow on the wall is 14 cm, according to the results. What is the scale factor, and what does it mean?
   d. What do you believe will happen to the flashlight's reflection on the wall as your hand approaches it? What is the significance of this in terms of the scale factor?

Observations, discussions, interviews, public records, respondents' diaries, and researcher reflections are used to gather qualitative data (Creswell, 2012). The test is carried out through the process, and this study involved 48 students of class IX divided into two classes. Grade IX students were chosen because they were or had studied Geometry Transformation material based on a syllabus review for junior high school mathematics (Kementerian Pendidikan dan Kebudayaan, 2016). Forty-eight students took part in the RAT. Six were selected research focus and acted as participants in the interview. Students for interview participants were selected based on the representation of the Geometric Transformation concept definition category according to the Ministry of Education and Culture (2017) on each concept meaning expressed by each student at RAT and sources and other considerations according to the student's sources to research needs.

The study with student participants occurred at SMPN 1 Sindang, and the research started with learning activities from October 2019 to September 2020. Meanwhile, the data collection technique uses triangulation (combined) techniques. The stages of data analysis in this study used the synthesis of the steps of phenomenological data analysis proposed by Creswel (2009). DDR s a type of qualitative study (Suryadi, 2014). This study employs qualitative research data analysis approaches as a result of this. Data collection, data reduction, data presentation, and conclusions are the stages of this qualitative data analysis technique in general (Miles & Huberman, 1992). In this stage, the data analysis procedure for implementing the reflective practice is as follows: Analyze the total information gathered through observation, interviews, document studies, and documentation. The steps include (1) sort the information you've gathered into categories; (2) make explicit descriptions of any issues that arise during the implementation of reflection; (3) search for connections and compare multiple categories; (4) based on the original data, finds and sets patterns; (5) carry out the interpretation; (6) you are telling a story with your presentation.

C. RESULT AND DISCUSSION

1. The Meaning of Geometric Transformation Concepts According to Students

The distribution of student responses regarding the definition of the Geometric Transformation concept can be summarized as shown in Table 1.

Table 1. Distribution of Student Responses regarding the Meaning of the Concept of Geometric Transformation

Category Meaning	Description of the definition of Geometric Transformation	Many Students
1	Change of position or displacement	9
2	Change of place or Change of shape and size	8
3	Mapping points on a plane to a set of points on the same plane	13
4	A combination of several formulas for flat shapes; reflection, translation, rotation and dilation	8
5	Etc	8
6	No answer	2
Total		48

Based on Table 1, the meaning of the concept of Geometry Transformation that most students wrote down was the category meaning number (3), namely Geometry Transformation as a mapping of points in a plane to a set of points in the same plane. Other definitions of Geometric Transformation concepts written by students are changes in position or displacement, Change of place or shape and size, and a combination of several different formulas for flat conditions. There are also other definitions of Geometric Transformation concepts written by students and students who do not write down definitions of Geometric Transformation concepts. However, based on further analysis, it was found that the actual definition of the concept of Geometric Transformation written by some students still contained the meaning of the concept of Geometric Transformation, which was inconsistent and ambiguous. Therefore, the researcher chose six randomly from each category in the meaning of Table 1 as participants to explore the meaning of the Geometric Transformation concept more deeply through interviews.

Based on the research assumptions, distribution of responses, and the meaning units of the six selected students that have been constructed, four findings related to the meaning of the Geometric Transformation concept according to the students in this study were obtained. These four findings, among others:

a. The meaning concept of Geometric Transformation is a mapping point in a plane to a set of points in the same plane.
b. Inconsistency is the interpretation of the term geometric transformation.
c. The ambiguous meaning of the concept of Geometric Transformation; and
d. Other meaning units result in the idea of Geometric Transformation.

Students' inconsistency in the meaning assigned to the Geometric Transformation idea is a significant indicator of some researchers' cognitive conflict, referred to as compartmentalization or the division of meaning units. In this case, the concept of Geometric Transformation in students is due to something (Gagatsis & Kyriakides, 2000; Septyawan, 2018). According to the theory of knowledge, the meaning of the Geometric Transformation concept constructed by students is still at the information stage or has not yet arrived at an understanding (Uriarte, 2008a). The implication is that students in certain situations tend to be inconsistent in interpreting and using the meaning of Geometric Transformation. They were revealed in the previous section on the importance of Geometric Transformation, where students are inconsistent in interpreting the concept of Geometric Transformation in what is written, spoken, and done in the RAT questions.

The researcher also found limited meanings related to students' concept image of the concept of Geometric Transformation. Founding in several student responses related to the RAT question number 3 regarding reflection, followed by a translation written or expressed in interviews. Student responses related to this can be seen in Figure 1.

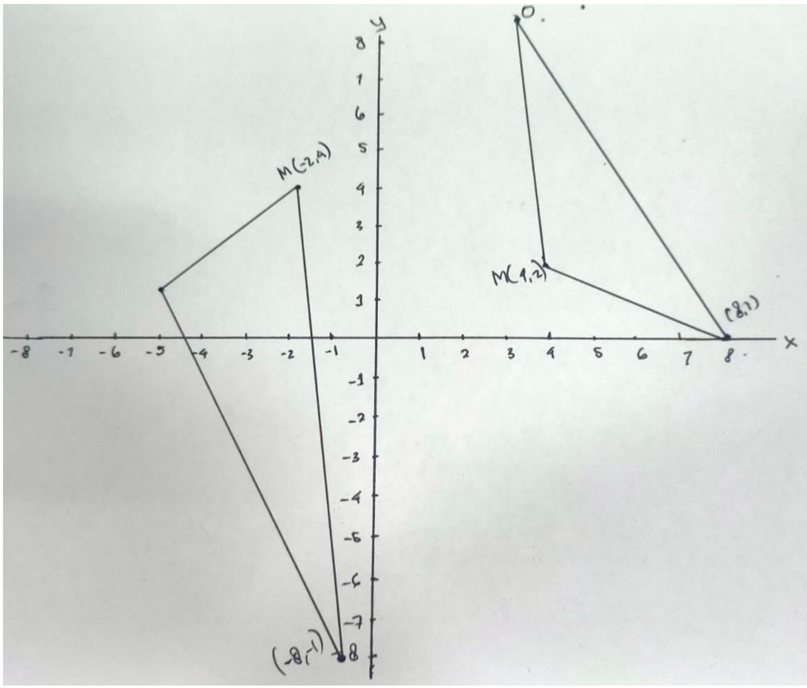

Figure 1. Answers of Student 5 (S5) related to the completion of the RAT Question number 3

Based on this interview excerpt, the researcher looked at the students' concept image of the concept of Geometric Transformation, displayed in a very narrow Cartesian field. The researchers also observed this based on the results of student interviews related to their experience in encountering, studying, and working on questions regarding the process of mapping an object with a Cartesian plane. The researcher suspects representation of the Geometric Transformation concept from the Cartesian diagram that students have received so far tends to be oriented to the overall shape and not map the points one by one so that the expected object shape is obtained. This orientation implies that students directly interpret each geometric transformation involving a particular form or object. Another finding from this problem is that students do not understand the question's meaning.

2. Student Experience Getting Meaning of Geometric Transformation Concept

The six students selected revealed implicitly or explicitly, written on the RAT or what was said in the interview came from books and teachers. Based on in-depth interviews, students understood Geometric Transformation from what was taught by the teacher and what was learned from books. Teachers and books are related to "colouring," meaning students get from the Geometric Transformation concept. Below are two excerpts from interviews in group discussions with several participants.

P: ....so why is that?

S4: Yes, because of that, remember that picture. Time taught Transformation geometry. A youngster was shifting the table, and then the mother claimed the train was going, and passengers moved along with everything in it. He.....

Q: ...why does S3 like this define the geometry transformation?

S3: At that time, I saw the back of the book or on google, yes, in the book, ma'am, when you gave group assignments. Yes, it is on the back of the book (glossary)

Some students found the meaning of the concept of Geometric Transformation as a change in position or a change in shape and size revealed that the purpose was obtained and constructed through the introduction of the concept of Geometric Transformation, which was preceded by the concept of Change in appearance: shape, nature, function (transformation literally according to KBBI). The illustration used in introducing the concept of Geometric Transformation is when a child is looking in the mirror, a student is shifting the table, the propeller is rotating, and the magnification of a photo of a natural landscape is strong enough to form a concept image in students about the meaning of the concept of Geometric Transformation.

For teachers, knowing the concept of image is very important for teaching. Not only to increase the teacher's knowledge for a better understanding of students but also to improve teaching methods that cause concept image errors(Vinner, 2002). Vinner's statement indicates that the role of the teacher is significant. Teachers' teaching that is not good can cause students' concept images to be far from scientific concepts. Investigates the influence of teachers' conceptions on teaching methods; their research shows that what teachers know about mathematics affects what teachers do in the classroom (Maulida, 2018). The same mathematical concept can be described using different representations or definitions, leading to differences in students' and teachers' concept images (Thompson, 2002).

The process of students' meaning of the Geometric Transformation concept has not yet reached the internalization process from the perspective of the theory of knowledge (Uriarte, 2008b) or has not yet reached the institutionalization situation from that point of view (Brousseau, 2006b). From the perspective of the theory of knowledge, explicit knowledge obtained by students from various sources (teachers, books, other media, the internet, and other students) related to the concept of Geometric Transformation has not been entirely 'deposited' into their respective tacit knowledge. Meanwhile, in the perspective of didactic situation theory, there has not been a situation where students have understood and further developed the meaning of the concept of Geometric Transformation to solve various problems.

3. Learning Obstacles to the Concept of Geometric Transformation

The findings of the meaning and experience of students in obtaining the importance of the Geometric Transformation concept that has been described have not fully guaranteed students to gain a complete understanding of the Geometric Transformation concept. Therefore, it is essential to explore the possibility of learning obstacles experienced by students in the concept of Geometric Transformation. Based on the explanation related to the meaning and experience of students in obtaining the importance of the Geometric Transformation concept, it was found that there were learning obstacles to the Geometric Transformation concept. Learning obstacles in the Geometry Transformation concept generally include learning obstacles that are ontogenic, epistemological, and didactical obstacles. According to the researcher's opinion, this results from a paradigm shift in one's understanding of Geometric Transformation. Therefore, we need a tool that can bridge the jump to represent students' thinking process on the concept of Geometric Transformation. The recapitulation of learning obstacles or Obstacles to the Geometric Transformation concept found in this study can be seen in Table 2.

Table 2. Recapitulation of Learning Obstacles

No.	Identify Learning Obstacles	Finding
1.	Ontogenic obstacles	Fundamental Geometric Transformation concepts students do not know, for example, the terms of rotation, the direction, of course, a centre of the process, and the size of the rotation angle. Students mistakenly use symbols for translation
2.	Ontogenic obstacles	Students do not know the true meaning of Geometric Transformation in the form of a formal definition. Error because you do not understand the meaning of the question Errors caused by not understanding how to solve the problem
3.	Ontogenic obstacles	Students show low motivation and interest in learning the concept of Geometric Transformation
4.	Epistemological Obstacles	Students have difficulty solving problems with the concept of Geometric Transformation in different shapes and concepts
5.	Didactical Obstacles	There is no introduction and integration of the benefits of the concept of Geometric Transformation in everyday life. The order of the material is not structurally appropriate. For example, several practice questions in the material before the concept of Geometric Transformation have explicitly or implicitly raised the idea of Geometric Transformation in the student book. The order of the material is not functionally appropriate. There is no formal definition of the concept of Geometric Transformation

Teachers in Indonesia are currently using classroom action research in their learning reflection process. Classroom action research is an endeavour by teachers or practitioners to improve the quality of classroom learning through various activities (Syahmani et al., 2020). The teacher's classroom action research focuses on how to design the learning process implemented by the instructor Kusmanto (2013) constructivism as a method of learning mathematics. Classroom action research by Lasmanah (2016) Think-pair-share cooperative learning is a method of learning math. Then Classroom Action research by Muah (2016) uses problem-based instruction learning. As a result, classroom action research has so far focused solely on how to build learning models rather than paying greater attention to conceptual understanding and learning obstacles faced by the students.

In contrast to didactical design research, which helps teachers construct learning designs tailored to students' needs, making the learning experience more accurate and relevant (Fauzi & Suryadi, 2020). DDR contains three analytical processes when utilized by instructors to reflect on the learning process in the classroom as shown in Figure 3.

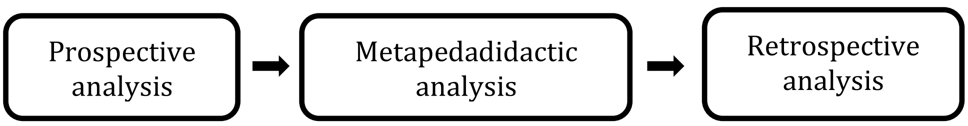

Figure 3. Three-stage DDR Analysis

The teacher studies the phenomena that underpin the process of building hypothetical learning designs found through examining concept images and learning obstacles in the prospective analysis stage (analysis of the didactic context before learning). The original didactic design was based on the findings of the learning obstacles analysis and the foresight of the pedagogical didactic in light of potential challenges (Miftah et al., 2020b). The motivation for this study stems from student learning issues. Students have learning challenges when studying the topic of circles based on identification (Rosita et al., 2020b). According to the prospective analysis results, students face learning obstacles during the learning process (Dedy & Sumiaty, 2017). The majority of them use an examination of students' learning difficulties during the prospective analysis stage of the DDR research. In this study, the teacher examines students' concept representations and learning hurdles that serve as a starting point for further investigation.

The gap analysis between concept image and concept description can also be used to decide how learning scenarios should be designed. Tall & Vinner (1981b). Consider the difference between concept image and concept definition when developing concepts. The concept image and concept definition are used to build the basic structure of cognitive processes (Vinner, 1983b). Symbols, theorems, representations, characteristics, and other elements of concepts can all be considered cognitive structures (Barnard & Tall, 1997). For the development of effective mathematical thinking, the capacity to govern the interconnections between different cognitive processes is critical (Giraldo, Tall, & Carvalho, 2003). A concept picture is a whole cognitive structure that comprises all mental representations and related qualities and processes connected with a mathematical concept formed through time. The notion is a formal definition used to describe a specific mathematical concept (Viholainen, 2008). Although the concept image may not accurately match the concept definition, it evolves as pupils discover new mathematical concepts (Engelke Infante et al., 2018a). As a result, there may be a disconnect between idea image and concept description.

Universal (definition of a concept) or vice versa (Vinner, 2002). hen reviewing learning situations, the DDR principles, as illustrated in the tripartite teacher-student-material relationship, become a frame of reference, and the process of interpreting each mathematical concept is subjective (concept image) and then leads to a formal institutionalization context. The design of learning circumstances is critical in developing concept meaning in the mathematics learning process. Ineffective teaching methods might cause students' images to diverge from scientific concepts (Maulida, 2018). Naturally, there is a gap between the concept images and the concept definition. The concept definition, on the other hand, does not guarantee that the concept will be understood (Attorps, 2007b). As a result, when pupils lack the fundamental knowledge of a subject, their understanding is based on a shaky basis.

The findings of similar research, work done by Tsamir et al. (2015b), show the gap between the concept image and the concept definition had trouble defining circles and cylinders, as well as distinguishing examples and non-examples of cylinders. It occurs as a result of the concept image's lack of coherence. The other results showed that students' conceptual representations of unit fractions were relatively constrained before instruction and mainly connected to area models (Zhang et al., 2015). According to the findings of earlier studies, using computers to improve students' conceptual representations of derived concepts has several limits (Giraldo, Tall, & Mariano Carvalho, 2003). Students who struggled to understand Newton's method included those who did not have enough images of tangent lines and those who were unable or unwilling to change such pictures after reading the text (Engelke Infante et al., 2018b). The teacher's examination of the concept image can guide how to create relevant learning activities.

The carefully designed action/consequence documents seem to have the potential to be valuable tools in providing students with the experiences they need to develop robust concept images (Burrill, 2019). According to past studies, the practice of reflection utilizing classroom action research does not result in understanding the learning process that is responsive to students' needs. DDR is a method for learning reflection that can be utilized as an alternative. However, in this study, the teacher attempted to examine the students' concept image, which might provide information on the extent to which students understand the concept of transformation geometry based on the abovementioned analysis. The prospective study is carried out by examining the learning hurdle, which was once the primary weapon for customizing learning to meet the demands of pupils.

Based on the observations, research, and discussion that the researchers conducted. Based on the findings, results of the study, and discussion that the researchers have done, the following are the solutions offered by the researcher as a teacher in addressing the problems of learning mathematics, especially the concept of Geometry Transformation:

a. Need to deepen understanding of a mathematical concept, especially Geometric Transformation with various references.
b. When teaching, pay more attention to the learning flow and how to present a concept.
c. Pay attention and focus on the critical points to be taught to students. If in the image of geometric transformation, essential things must be emphasized, for example, the term direction of rotation is why it is positive or negative, the centre point of the process, the size of the rotation angle, the axis of reflection, scale factor, a centre of dilation and others. In addition, symbols or ways of writing on translations are matrices, object symbols, and shadows resulting from the transformation.
d. Make the learning atmosphere ready, among others, by diagnosing interests and psychologically motivating students to deliver the material.
e. Check student readiness regarding mastery of prerequisite material through apperception or, if necessary, providing pre-learning meetings.
f. Provide rich examples and real-life implications related to the material to be delivered to foster student interest and curiosity about the material.

D. CONCLUSION AND SUGGESTIONS

Based on findings and discussion, it revealed the reality of the meaning of the concept of Geometry Transformation according to students was mapping points on a plane to a set of points on the same plane; there are inconsistencies and ambiguities in the meaning of the concept of Geometric Transformation and the emergence of the findings of other units of meaning in the concept of Geometry Transformation in students. The students' experience of meaning revealed shows a tendency that students get the meaning of Geometric Transformation from what is taught by the teacher and learned from books, with the meaning of Geometry Transformation, which is more procedurally oriented. Based on the meaning and experience of students' meaning of the revealed Geometric Transformation concept, it can be concluded that there are learning obstacles in the Geometric Transformation concept, which include ontogenic obstacles, epistemological obstacles, and didactical obstacles. The preceding findings can help teachers improve the learning process by tailoring it to the needs of their students. The prospective analysis step was only completed by the instructor participants in this study, which included examining learning obstacles and concept images. Continued research on metapedadidactic and retrospective analysis phases is required for future research.

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