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Triangular Fuzzy Time Series for Two Factors High-order based on Interval Variations

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ABSTRACT

Fuzzy time series (FTS) firstly introduced by Song and Chissom has been developed to forecast such as enrollment data, stock index, air pollution, etc. In forecasting FTS data several authors define universe of discourse using coefficient values with any integer or real number as a substitute. This study focuses on interval variation in order to get better evaluation. Coefficient values analyzed and compared in unequal partition intervals and equal partition intervals with base and triangular fuzzy membership functions applied in two factors high-order. The study implemented in the Shen-hu stock index data. The models evaluated by average forecasting error rate (AFER) and compared with existing methods. AFER value 0.28% for Shen-hu stock index daily data. Based on the result, this research can be used as a reference to determine the better interval and degree membership value in the fuzzy time series.

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A. INTRODUCTION

Forecasting plays an important role in making decisions such as enrolment data, stock indexes, air pollution, agriculture, economics, climatology, etc. Fuzzy time series (FTS) forecasting is a sequence of consecutive values in a particular domain to predict the future with a precise forecast to prevent losses with uncertainty, imprecision, and ambiguity that emergent research area using linguistic values (Qiang & Brad S., 1993), (Bose & Mali, 2019). Song and Chissom (1993) was laid the fuzzy time series forecasting requires complex calculations. Chen (1996) simplified using arithmetic operations in first order for enrolment data of Alabama University. There is one process in FTS forecasting regarding selection of interval very urgent because it effects in forecast results then there are many approaches proposed. One of the frequently used is random approach called manual approach to choose the interval (S.-M. Chen, 1996; S. M. Chen, 2002; S. M. Chen & Chen, 2011; Gautam et al., 2018; Jilani et al., 2007; Lee et al., 2006; F. Li et al., 2021; F. Li & Yu, 2018; Mashuri et al., 2018; Qiang

& Brad S., 1993). Huarng (2001a) investigated for FTS forecasting in one order that the length of the intervals at the fuzzification phase affects the performance. He concluded that Chen's model with distribution and average-based lengths forecast better than the same model with randomly chosen lengths for enrolment forecasting.

Actually, FTS forecasting models can calculate in first-order such as Bai et al. (2011) presented a simple heuristic time-invariant, Bisht & Kumar (2016) proposed the problem of establishing a common membership grade, Izakian et al. (2015) developed alternative for fuzzy clustering, Lu et al. (2015) proposed partition of the universe of discourse based on interval information granules, and others (Mirzaei Talarposhti et al., 2016; Peng et al., 2015; P. Singh & Borah, 2013a; L. Wang et al., 2013) and high-order (M. Y. Chen, 2014; S. M. Chen & Jian, 2017; Deng et al., 2016; P. Singh & Borah, 2014; Sun et al., 2015; W. Wang et al., 2015; Ye et al., 2016). Applied in TAIEX stock market data, the result using RMSE criteria in Lu et al. (2015) obtained 74.7 and in M. Y. Chen (2014) obtained 68.5. Based on that, it can be concluded that high order is better than first order.

In Chen's research (S. M. Chen, 2002) improved accuracy in to be high-order to reduce ambiguity for enrolment data of Alabama University. The prediction can be cause by other factors, Lee et al. (2006) increased accuracy consider more effect with construct two factors in high-order. F. Li & Yu (2020) constructed new fuzzy logical relationship (FLR) with cross association to increase accuracy in first-order and F. Li et al. (2021) in high-order. Gautam et al. (2018) approached one factor high-order took grades of membership using triangular fuzzy sets and gave a solution to the decision-making problems in intuitionistic fuzzy environment.

Huarng & Yu (2006) proposed different method in the length of intervals with named ratio-based lengths of intervals (intervals ratio algorithm) that applied in one factor first-order in grades of membership 1, 0.5, 0 more accurate forecast for enrolment, TAIEX stock price, and inventory demand data. In addition, it examines in fuzzification stage the uses of took grades of membership 1, 0.5, 0 and triangular fuzzy sets. This study focuses on interval variations of triangular fuzzy time series in order to determine the best interval and degree membership value. It is applied in the Shen-hu stock index data. The accuracy compared using average forecasting error rate (AFER).

B. METHODS

In this section discusses previous methods and modification method from previous method.

1. Previous Methods

All previous methods are applied in enrolment of the University of Alabama.

a. Chen's Method (S.-M. Chen, 1996)

Chen's Method described the first-order one factor and changed operations (Song & Chissom, 1993) to be simple arithmetic. This method included:

1) Define universe of discourse with random coefficient value D_1 and D_2.
2) Partition the universe of discourse in equal length.
3) Define fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval.
4) Fuzzification.

5) Build Fuzzy Logical Relationship (FLR) and Fuzzy Logical Relationship Group (FLRG) in first-order with one factor.
6) Calculate the forecast outputs (defuzzification).

b. Chen's Method (S. M. Chen, 2002)
Chen's Method changed (S.-M. Chen, 1996) into high-order one factor. This algorithm included:
1) Define universe of discourse with random coefficient value D_1 and D_2.
2) Partition the universe of discourse in equal length.
3) Define fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval.
4) Fuzzification.
5) Build FLR and FLRG in high-order with one factor.
6) Calculate the forecast outputs (defuzzification).

c. Gautam's Method (Gautam et al., 2018)
Gautam's Method described the high-order one factor and changed grade of membership value of (S. M. Chen, 2002) to triangular fuzzy sets. This algorithm included:
1) Define universe of discourse with random coefficient value D_1 and D_2.
2) Partition the universe of discourse in equal length.
3) Define fuzzy sets using grades of membership value corresponding to triangular fuzzy sets.
4) Fuzzification.
5) Build FLR and FLRG in high-order with one factor.
6) Calculate the forecast outputs (defuzzification).

d. Lee's Method (Lee et al., 2008)
Lee's method described high-order and modified (S. M. Chen, 2002) into two factors. This algorithm included:
1) Define universe of discourse with random coefficient value D_1 and D_2.
2) Partition the universe of discourse in equal length.
3) Define fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval.
4) Fuzzification.
5) Build FLR and FLRG in high-order with two factors.
6) Calculate the forecast outputs (defuzzification).

e. Huarng's Method (Huarng & Yu, 2006)
Huarng's method described first-order one factor and modified (S.-M. Chen, 1996) in determining and partitioning universe of discourse based on intervals ratio. This algorithm included:
1) Partition the universe of discourse based on intervals ratio algorithm and obtained unequal length interval automatically.
2) Obtained the universe of discourse.
3) Define fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval.
4) Fuzzification.
5) Build FLR and FLRG in high-order with two factors.
6) Calculate the forecast outputs (defuzzification).

2. Modification Method

In this part modify Lee's Method (Lee et al., 2008) in determining and partitioning universe of discourse using intervals ratio algorithm (Huarng & Yu, 2006) to get coefficient value D_1 and D_2 automatically and combine (Huarng & Yu, 2006), (Lee et al., 2008), and (Gautam et al., 2018). The modification method divided to be four cases to determine the better interval and degree membership value in two factors third-order fuzzy time series with lower error. The framework of this study as follow:

Step 1: Find data for experiment.

Step 2: Find maximum and minimum value of data.

Step 3: Determining and partitioning universe of discourse using intervals ratio algorithm. Where $u_1, u_2, \dots, u_n$ is the partition of the main factor and $v_1, v_2, \dots, v_m; n, m \in \mathbb{Z}^+$

Step 4: From step 3, we get coefficient value D_1 and D_2 automatically.

Step 5: Define universe of discourse with coefficient value D_1 and D_2 from intervals ratio algorithm.

Step 6: Divided into four cases as follows:

a. Case 1: automatically get unequal partition using interval ratio with grades of membership 1, 0.5, 0 for linguistic interval (combination of Lee's Method and Huarng's Method).

   Step 7.1: Define fuzzy sets with grades of membership 1, 0.5, 0 for linguistic interval.

   Step 8.1: Fuzzification.

   Step 9.1: Build FLR and FLRG in two factors high-order.

b. Case 2: automatically get unequal partition using interval ratio with grades of membership value corresponding to triangular fuzzy sets (combination of Lee's Method, Huarng's Method, and Gautam's Method).

   Step 7.2: Define fuzzy sets with grades of membership value corresponding to triangular fuzzy sets.

   Step 8.2: Fuzzification.

   Step 9.2: Build FLR and FLRG in two factors high-order.

c. Case 3: from step 5, partition universe of discourse into several intervals in equal length with grades of membership 1, 0.5, 0 for linguistic interval (combination of Lee's Method and Huarng's Method).

   Step 7.3: Define fuzzy sets with grades of membership 1, 0.5, 0 for linguistic interval.

   Step 8.3: Fuzzification.

   Step 9.3: Build FLR and FLRG in two factors high-order.

d. Case 4: from step 5, partition universe of discourse into several intervals in equal length with grades of membership value corresponding to triangular fuzzy sets (combination of Lee's Method, Huarng's Method, and Gautam's Method).

   Step 7.4: Define fuzzy sets with grades of membership value corresponding to triangular fuzzy sets.

   Step 8.4: Fuzzification.

   Step 9.4: Build FLR and FLRG in two factors high-order.

   Step 10: Defuzzification.

   There are two rules to calculate the defuzzification. Let the current state at time $t-3$ to be $\left((A_{t-3}, B_{t-3}), (A_{t-2}, B_{t-2}), (A_{t-1}, B_{t-1})\right)$, then:

Rule 1: If fuzzy logical relationship group is obtained from any dataset as $\left((A_{t-3}, B_{t-3}), (A_{t-2}, B_{t-2}), (A_{t-1}, B_{t-1})\right) \rightarrow A_k$, then the forecast value at time t is m_k, where m_k is the midpoint of u_k.

Rule 2: If fuzzy logical relationship group is obtained from any dataset as $\left((A_{t-3}, B_{t-3}), (A_{t-2}, B_{t-2}), (A_{t-1}, B_{t-1})\right) \rightarrow A_{k_1}, A_{k_2}, \dots, A_{k_n}$, then the forecast value at time t is $\frac{(m_{k_1}+m_{k_2}+\cdots+m_{k_n})}{n}$, where $\left(m_{k_1}, m_{k_2}, \dots, m_{k_n}\right)$ is the midpoint of $u_{k_1}, u_{k_2}, \dots, u_{k_n}$. as shown in Figure 1.

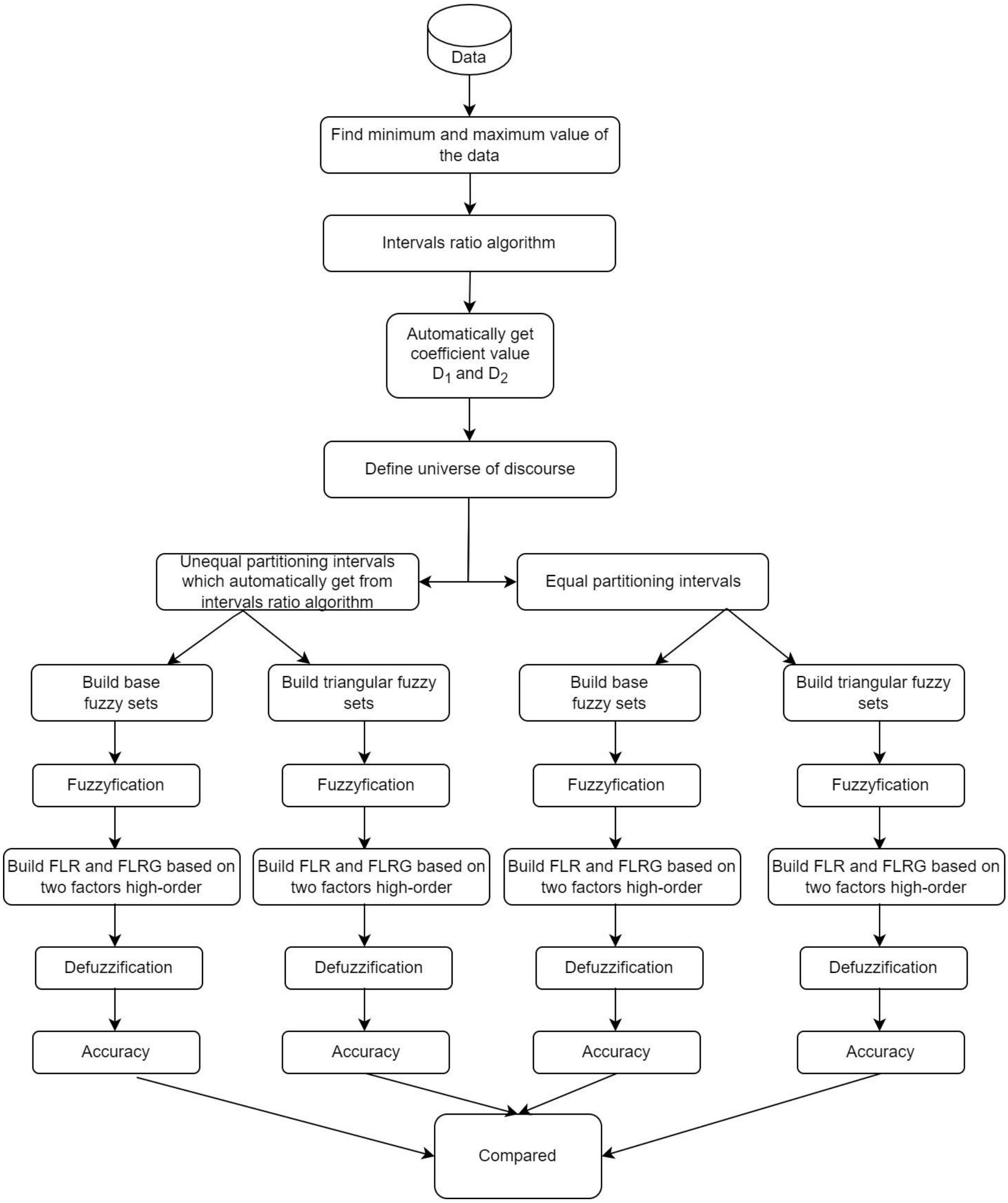

Figure 1. The framework of study

Based on previous research, forecasting model usually tested using AFER obtained from (F. Li & Yu, 2018) to know the error value. The formula of AFER as follow:

$$AFER = \frac{\sum_{j=1}^{n} \left| \frac{F_j - A_j}{A_j} \right|}{n} \times 100\% \qquad (1)$$

where n is amount of data, A_j is actual data result j-th and F_j is forecasting result j-th.

From intervals ratio algorithm, obtained coefficient values D_1 and D_2 to applied in four cases to know which one the best result between unequal length partition and equal length partition with grades of membership 1, 0.5, 0 or grades of membership value corresponding to triangular fuzzy sets. The demonstrated of four cases in the Shen-hu stock index daily data in March-September 2021 (Finance, 2021). Generally, the framework of this research can be shown in Figure 1.

C. RESULT AND DISCUSSION

In this section explained the modification of method with the simulation of the Shen-hu stock index data (Finance, 2021).

Step 1: Data for experiment in this research using the Shen-hu stock index data (Finance, 2021). This research using the Shen-hu stock index daily data with two factors. The main factor is the opening prices and the influence factor is highest prices. They are from March 11, 2021 to September 05, 2021. The data is consisting of 179 data, can be seen in the Table 1. The simulation is using Microsoft excel, as shown in Table 1.

Table 1. The Shen-hu stock index daily data (Finance, 2021)

Date	Price	
	Open (Main Factor)	**High (Influence Factor)**
11/03/2021	5024.56	5138.41
12/03/2021	5153.66	5153.66
13/03/2021	5054.41	5084.31
⋮	⋮	⋮
05/09/2021	4641.81	4660.51

Step 2: Find maximum and minimum value of data. From data shown for main factor that minimum value is D_{min} = 4641.813 and maximum value is D_{max} = 5348.340 meanwhile for influence factor minimum value is E_{min} = 4660.519 and maximum value is E_{max} = 5360.280.

Step 3: Determining and partitioning universe of discourse using intervals ratio. After known the minimum value of the data, next step is defining the ratio using intervals ratio algorithm then obtained ratio for main factor is 0.776 and second factor is 0.505. For main factor using formula in intervals ratio algorithm, the intervals are shown in Table 2 and for second factor shown in Table 3..

Table 2. Increasing interval using intervals ratio algorithm of main factor

Index	Interval
u_1	[4500, 4534.92]
u_2	[4534.92, 4570.101]
u_3	[4570.101, 4605.560]
$\vdots$	$\vdots$
u_{23}	[5334.069, 5375.455]

Table 3. Increasing interval using intervals ratio algorithm of second factor

Index	Interval
v_1	[4500, 4522.709]
v_2	[4522.709, 4545.533]
v_3	[4545.533, 4568.472]
$\vdots$	$\vdots$
v_{35}	[5340.010, 5366.958]

Step 4: From step 3, we get coefficient value D_1 and D_2 automatically. Based on Table 2, obtained coefficient value automatically which is coefficient value D_1 = 141.813, D_2 = 27.115 for main factor and coefficient value E_1 = 160.519, E_2 = 18.618 for influence factor.

Step 5: Define universe of discourse with coefficient value D_1 and D_2 from intervals ratio algorithm. The universe of discourse for main factor is U = [4500, 5375.455] and for influence factor is V = [4500, 5366.958].

Step 6: Divided into four cases as follows:

a) Case 1: automatically get unequal partition length using interval ratio with grades of membership 1, 0.5, 0 for linguistic interval.

Step 7.1: Define fuzzy sets.

We have the universe of discourse in step 5 and automatically get unequal partition. After we have the universe of discourse and the partition, the next step is defining fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval corresponding to the linguistic intervals of Table 2 and Table 3 as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{21}} + \frac{0}{u_{22}} + \frac{0}{u_{23}}$$
$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{21}} + \frac{0}{u_{22}} + \frac{0}{u_{23}}$$
$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{21}} + \frac{0}{u_{22}} + \frac{0}{u_{23}}$$
$$\vdots$$
$$A_{23} = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{21}} + \frac{0.5}{u_{22}} + \frac{1}{u_{23}}$$

where $A_1, A_2, \ldots, A_{23}$ are linguistic terms to describe the values of the main factor.

$$B_1 = \frac{1}{v_1} + \frac{0.5}{v_2} + \frac{0}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \cdots + \frac{0}{v_{33}} + \frac{0}{v_{34}} + \frac{0}{v_{35}}$$
$$B_2 = \frac{0.5}{v_1} + \frac{1}{v_2} + \frac{0.5}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \cdots + \frac{0}{v_{33}} + \frac{0}{v_{34}} + \frac{0}{v_{35}}$$
$$B_3 = \frac{0}{v_1} + \frac{0.5}{v_2} + \frac{1}{v_3} + \frac{0.5}{v_4} + \frac{0}{v_5} + \cdots + \frac{0}{v_{33}} + \frac{0}{v_{34}} + \frac{0}{v_{35}}$$

$$\vdots$$

$$B_{35} = \frac{0}{v_1} + \frac{0}{v_2} + \frac{0}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \cdots + \frac{0}{v_{33}} + \frac{0.5}{v_{34}} + \frac{1}{v_{35}}$$

where $B_1, B_2, \ldots, B_{35}$ are linguistic terms to describe the values of the influence factor.

Step 8.1: Fuzzification.

Fuzzified the historical data described as shown in Table 4.

Table 4. Fuzzification of case 1

Date	Open Price (Main Factor)	Fuzzified Open Price	High Price (Influence Factor)	Fuzzified High Price
11/03/2021	5024.56	A_{15}	5138.41	B_{27}
12/03/2021	5153.66	A_{18}	5153.66	B_{27}
13/03/2021	5054.41	A_{16}	5084.31	B_{25}
⋮	⋮	⋮	⋮	⋮
05/09/2021	4641.81	A_5	4660.51	B_7

Example. The actual open price data in 11/03/2021 is 5024.56. The grade of membership value is $\frac{0.5}{A14} + \frac{1}{A_{15}} + \frac{0.5}{A_{16}}$ so it is fuzzified by A_{15}. Analog for the remaining data.

Step 9.1: Build FLR and FLRG in two factors high-order, as shown in Table 5.

Table 5. Fuzzy Logic Relationship Two Factor Third-Order

Date	Fuzzified Open Price	Fuzzified High Price	Fuzzy Logic Relationship
11/03/2021	A_{15}	B_{27}	NA
12/03/2021	A_{18}	B_{27}	NA
13/03/2021	A_{16}	B_{25}	NA
14/03/2021	A_{17}	B_{28}	$\big((A_{15}, B_{27}), (A_{18}, B_{27}), (A_{16}, B_{25})\big) \to A_{17}$
15/03/2021	A_{14}	B_{24}	$\big((A_{18}, B_{27}), (A_{16}, B_{25}), (A_{17}, B_{28})\big) \to A_{14}$
16/03/2021	A_{16}	B_{24}	$\big((A_{16}, B_{25}), (A_{17}, B_{28}), (A_{14}, B_{24})\big) \to A_{16}$
17/03/2021	A_{12}	B_{19}	$\big((A_{17}, B_{28}), (A_{14}, B_{24}), (A_{16}, B_{24})\big) \to A_{12}$
⋮	⋮	⋮	⋮
05/09/2021	A_5	B_7	$\big((A_8, B_{13}), (A_6, B_{10}), (A_6, B_9)\big) \to A_5$

The FLRG is built by gathering equal parts on the left side of the FLR. Because the left side of the FLR is not the same, then obtained the FLR is FLRG. As shown in Table 6.

Table 6. Fuzzy Logic Relationship Group Two Factor Third-Order

Number	Fuzzy Logic Relationship Group
1	$\big((A_{15}, B_{27}), (A_{18}, B_{27}), (A_{16}, B_{25})\big) \to A_{17}$
2	$\big((A_{18}, B_{27}), (A_{16}, B_{25}), (A_{17}, B_{28})\big) \to A_{14}$
3	$\big((A_{16}, B_{25}), (A_{17}, B_{28}), (A_{14}, B_{24})\big) \to A_{16}$
4	$\big((A_{17}, B_{28}), (A_{14}, B_{24}), (A_{16}, B_{24})\big) \to A_{12}$
⋮	⋮
176	$\big((A_8, B_{13}), (A_6, B_{10}), (A_6, B_9)\big) \to A_5$

b) Case 2: automatically get unequal partition using interval ratio with grades of membership value corresponding to triangular fuzzy sets.

Step 7.2: Define fuzzy sets with grades of membership value corresponding to triangular fuzzy sets as follows.

$$\tilde{A}_1 = (4500,4534.91,4570.10)$$
$$\tilde{A}_2 = (4534.91,4570.10,4605.55)$$
$$\tilde{A}_3 = (4570.10,4605.55,4641.29)$$
$$\tilde{A}_4 = (4605.55,4641.29,4677.30)$$
$$\vdots$$
$$\tilde{A}_{23} = (5334.06,5375.45,5375.45)$$

$$\tilde{B}_1 = (4500,4522.70,4545.53)$$
$$B_2 = (4522.70,4545.53,4568.47)$$
$$\tilde{B}_3 = (4545.53,4568.47,4591.52)$$
$$\tilde{B}_4 = (4568.47,4591.52,4614.69)$$
$$\vdots$$
$$\tilde{B}_{35} = (5340.01,5366.95,5366.95)$$

Step 8.2: Fuzzification.

The grades of membership for each datum corresponding to the triangular fuzzy sets.

$A_\mathrm{k} = \emptyset\ (\mathrm{k} = 1,2,3), A_4 = \frac{0.98}{4641.813}, A_5 = \frac{0.02}{4641.813} + \frac{0.45}{4697.1} + \frac{0.15}{4708.44}, A_6 = \frac{0.55}{4697.10} + \frac{0.85}{4708.44} + \frac{0.98}{4714.29} + \frac{0.17}{4743.91} + \frac{0.58}{4728.75}, \dots, A_{22} = \frac{0.80}{5326.12}, A_{23} = \frac{0.11}{5338.73} + \frac{0.34}{5348.34}.$

$B_k = \emptyset\ (k = 1,2,3,4,5), B_6 = \frac{0.037}{4660.51}, B_7 = \frac{0.963}{4660.51}, B_8 = \frac{0.005}{4708.43}, B_9 = \frac{0.995}{4708.43} + \frac{0.56}{4718.99}, B_{10} = \frac{0.44}{4718.99}, B_{11} = \frac{0.67}{4764.04} + \frac{0.20}{4775.24},\ B_{12} = \frac{0.33}{4764.04} + \frac{0.80}{4775.24} + \frac{0.77}{4785.66} + \frac{0.13}{4801.10} + \frac{0.95}{4781.39},\ \dots,\ B_{35} = \frac{0.15}{5344.27} + \frac{0.75}{5360.27} + \frac{0.46}{5352.64},$

Example. The actual data for open price in 05/09/2021 is 4641.813. Subsequently, the grades of membership were obtained for data point 4641.813 as 0.98 and 0.02, respectively, corresponding to the triangular fuzzy set A_4 and A_5. This data has reached maximum membership in the triangular fuzzy set A_4. Therefore, the open price in 05/09/2021 is fuzzified by a triangular fuzzy set A_4.

Step 9.2: Build FLR and FLRG in two factors high-order.

Table 7 showed the fuzzification of the time series data by triangular fuzzy sets, as shown in Table 7, Table 8 and Table 9.

Table 7. Fuzzification of case 2

Date	Open Price (Main Factor)	Fuzzified Open Price	High Price (Influence Factor)	Fuzzified High Price
11/03/2021	5024.56	A_{14}	5138.41	B_{26}
12/03/2021	5153.66	A_{18}	5153.66	B_{27}
13/03/2021	5054.41	A_{15}	5084.31	B_{24}
⋮	⋮	⋮	⋮	⋮
05/09/2021	4641.81	A_4	4660.51	B_7

Table 8. Fuzzy Logic Relationship Two Factor Third-Order

Date	Fuzzified Open Price	Fuzzified High Price	Fuzzy Logic Relationship
11/03/2021	A_{14}	B_{26}	NA
12/03/2021	A_{18}	B_{27}	NA
13/03/2021	A_{15}	B_{24}	NA
14/03/2021	A_{17}	B_{27}	$\left((A_{14},B_{26}),(A_{18},B_{27}),(A_{15},B_{24})\right) \rightarrow A_{17}$
15/03/2021	A_{14}	B_{24}	$\left((A_{18},B_{27}),(A_{15},B_{24}),(A_{17},B_{27})\right) \rightarrow A_{14}$
16/03/2021	A_{15}	B_{23}	$\left((A_{15},B_{24}),(A_{17},B_{27}),(A_{14},B_{24})\right) \rightarrow A_{15}$
17/03/2021	A_{11}	B_{19}	$\left((A_{14},B_{24}),(A_{14},B_{24}),(A_{15},B_{23})\right) \rightarrow A_{11}$
⋮	⋮	⋮	⋮
05/09/2021	A_4	B_7	$\left((A_8,B_{13}),(A_6,B_{10}),(A_6,B_9)\right) \rightarrow A_5$

Table 9. Fuzzy Logic Relationship Group Two Factor Third-Order

Number	Fuzzy Logic Relationship Group
1	$\left((A_{14},B_{26}),(A_{18},B_{27}),(A_{15},B_{24})\right) \rightarrow A_{17}$
2	$\left((A_{18},B_{27}),(A_{15},B_{24}),(A_{17},B_{27})\right) \rightarrow A_{14}$
3	$\left((A_{15},B_{24}),(A_{17},B_{27}),(A_{14},B_{24})\right) \rightarrow A_{15}$
4	$\left((A_{14},B_{24}),(A_{14},B_{24}),(A_{15},B_{23})\right) \rightarrow A_{11}$
⋮	⋮
176	$\left((A_8,B_{13}),(A_6,B_{10}),(A_6,B_9)\right) \rightarrow A_5$

c) Case 3: from step 5, partition universe of discourse into several intervals in equal length with grades of membership 1, 0.5, 0 for linguistic interval. As shown in Table 10 and Table 11.

Table 10. Partitioning universe of discourse in equal length of main factor

Index	Interval
u_1	[4500, 4597.27]
u_2	[4597.27, 4694.54]
u_3	[4694.54, 4791.81]
u_4	[4791.81, 4889.09]
u_6	[4889.09, 5083.63]
u_7	[5083.63, 5180.90]
u_8	[5180.90, 5278.18]
u_9	[5278.18, 5375.45]

Table 11. Increasing interval using intervals ratio algorithm of second factor

Index	Interval
v_1	[4500, 4623.85]
v_2	[4623.85, 4747.70]
v_3	[4747.70, 4871.55]
v_4	[4871.55, 4995.40]
v_5	[4995.40, 5119.25]
v_6	[5119.25, 5243.10]
v_7	[5243.10, 5366.95]

Step 7.3: Define fuzzy sets with grades of membership 1, 0.5, 0 for linguistic interval.

We have the universe of discourse in step 5 and we divide it in equal length. The universe of discourse of the main factor divided by 9 and the influence factor divided by 7. After we have the universe of discourse and the partition, the next step is defining fuzzy sets using grades of membership 1, 0.5, 0 for linguistic interval corresponding to the linguistic intervals of Table 10 and Table 11 as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8} + \frac{0}{u_9}$$
$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8} + \frac{0}{u_9}$$
$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0}{u_8} + \frac{0}{u_9}$$
$$\vdots$$
$$A_9 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7} + \frac{0.5}{u_8} + \frac{1}{u_9}$$

where $A_1, A_2, \dots, A_9$ are linguistic terms to describe the values of the main factor.

$$B_1 = \frac{1}{v_1} + \frac{0.5}{v_2} + \frac{0}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \frac{0}{v_6} + \frac{0}{v_7}$$
$$B_2 = \frac{0.5}{v_1} + \frac{1}{v_2} + \frac{0.5}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \frac{0}{v_6} + \frac{0}{v_7}$$
$$B_3 = \frac{0}{v_1} + \frac{0.5}{v_2} + \frac{1}{v_3} + \frac{0.5}{v_4} + \frac{0}{v_5} + \frac{0}{v_6} + \frac{0}{v_7}$$
$$\vdots$$
$$B_7 = \frac{0}{v_1} + \frac{0}{v_2} + \frac{0}{v_3} + \frac{0}{v_4} + \frac{0}{v_5} + \frac{0.5}{v_6} + \frac{1}{v_7}$$

where $B_1, B_2, \dots, B_7$ are linguistic terms to describe the values of the influence factor.

Step 8.3: Fuzzification.

Fuzzified the historical data described as shown in Table 12.

Table 12. Fuzzification of case 3

Date	Open Price (Main Factor)	Fuzzified Open Price	High Price (Influence Factor)	Fuzzified High Price
11/03/2021	5024.56	A_6	5138.41	B_6
12/03/2021	5153.66	A_7	5153.66	B_6
13/03/2021	5054.41	A_6	5084.31	B_5
⋮	⋮	⋮	⋮	⋮
05/09/2021	4641.81	A_2	4660.51	B_2

Step 9.3: Build FLR and FLRG in two factors high-order.

Table 13. Fuzzy Logic Relationship Two Factor Third-Order

Date	Fuzzified Open Price	Fuzzified High Price	Fuzzy Logic Relationship
11/03/2021	A_6	B_6	NA
12/03/2021	A_7	B_6	NA
13/03/2021	A_6	B_5	NA
14/03/2021	A_7	B_6	$((A_6, B_6), (A_7, B_6), (A_6, B_5)) \rightarrow A_7$
15/03/2021	A_6	B_5	$((A_7, B_6), (A_6, B_5), (A_7, B_6)) \rightarrow A_6$
16/03/2021	A_6	B_5	$((A_6, B_5), (A_7, B_6), (A_6, B_5)) \rightarrow A_6$

Date	Fuzzified Open Price	Fuzzified High Price	Fuzzy Logic Relationship
17/03/2021	A_5	B_4	$((A_7,B_6),(A_6,B_5),(A_6,B_5)) \to A_5$
⋮	⋮	⋮	⋮
05/09/2021	A_2	B_2	$((A_3,B_3),(A_3,B_2),(A_3,B_2)) \to A_2$

Table 14. Fuzzy Logic Relationship Group Two Factor Third-Order

Number	Fuzzy Logic Relationship Group
1	$((A_3,B_3),(A_3,B_3),(A_3,B_2)) \to A_3,A_2$
2	$((A_3,B_3),(A_3,B_3),(A_3,B_3)) \to A_3$
3	$((A_3,B_3),(A_3,B_3),(A_4,B_3)) \to A_3$
4	$((A_3,B_3),(A_4,B_3),(A_3,B_3)) \to A_3,A_4$
⋮	⋮
90	$((A_9,B_7),(A_9,B_7),(A_9,B_7)) \to A_8$

d) Case 4: from step 5, divide universe of discourse into several intervals in equal length with grades of membership value corresponding to triangular fuzzy sets.

Step 7.4: Define fuzzy sets with grades of membership value corresponding to triangular fuzzy sets as follows.

$$\tilde{A}_1 = (4500,4597.27,4694.54)$$
$$\tilde{A}_2 = (4597.27,4694.54,4791.81)$$
$$\tilde{A}_3 = (4694.54,4791.81,4889.09)$$
$$\tilde{A}_4 = (4791.81,4889.09,4986.36)$$
$$\vdots$$
$$\tilde{A}_9 = (5278.18,5375.45,5375.45)$$

$$\tilde{B}_1 = (4500,4623.85,4747.70)$$
$$B_2 = (4623.85,4747.70,4871.55)$$
$$\tilde{B}_3 = (4747.70,4871.55,4995.40)$$
$$\tilde{B}_4 = (4871.55,4995.40,5119.25)$$
$$\vdots$$
$$\tilde{B}_7 = (5243.10,5366.95,5366.95)$$

Step 8.4: Fuzzification.

The grades of membership for each datum corresponding to the triangular fuzzy sets.

$$A_1 = \frac{0.54}{4641.813}, A_2 = \frac{0.46}{4641.813} + \frac{0.79}{4714.29} + \frac{0.14}{4777.72} + \frac{0.21}{4770.72} + \frac{0.49}{4743.91} + \frac{0.64}{4728.75} + \frac{0.25}{4766.77} + \frac{0.13}{4778.25} + \frac{0.38}{4753.93} + \frac{0.30}{4761.95} + \frac{0.97}{4697.10} + \frac{0.86}{4708.10}, A_3 = \frac{0.21}{4714.29} + \frac{0.86}{4777.72} + \frac{0.79}{4770.72} + \frac{0.51}{4743.91} + \frac{0.36}{4728.75} + \frac{0.75}{4766.77} + \frac{0.87}{4778.25} + \frac{0.62}{4753.93} + \frac{0.70}{4761.95} + \frac{0.03}{4697.10} + \frac{0.14}{4708.10}, \dots, A_8 = \frac{0.50}{5326.12} + \frac{0.37}{5338.72} + \frac{0.27}{5348.33} + \frac{0.93}{5248.89}, A_9 = \frac{0.50}{5326.12} + \frac{0.63}{5338.72} + \frac{0.73}{5348.33} + \frac{0.07}{5248.89}.$$

$$B_1 = \frac{0.23}{4697.10} + \frac{0.31}{4708.10} + \frac{0.70}{4641.81}, B_2 = \frac{0.77}{4697.10} + \frac{0.69}{4708.10} + \frac{0.30}{4641.81} + \frac{0.69}{4714.29} + \frac{0.10}{4845.75} + \frac{0.37}{4825.70} + \frac{0.29}{4817.49} + \frac{0.22}{4777.72} + \frac{0.06}{4794.97} + \frac{0.18}{4847.02} + \frac{0.40}{4803.08} + \frac{0.10}{4797.10} + \frac{0.32}{4770.72} + \frac{0.02}{4846.02} + \frac{0.10}{4835.81} + \frac{0.04}{4841.14} +$$

$$\frac{0.006}{4858.25}+\frac{0.31}{4832.79}+\frac{0.001}{4871.41}+\frac{0.10}{4820.00}+\frac{0.002}{4857.18}+\frac{0.21}{4831.89}+\frac{0.03}{4839.91}+\frac{0.11}{4842.16}+\frac{0.12}{4824.31}+\frac{0.21}{4812.22}+$$
$$\frac{0.16}{4840.00}+\frac{0.15}{4818.89}+\frac{0.14}{4853.02}+\frac{0.86}{4743.91}+\frac{0.77}{4728.75}+\frac{0.36}{4766.77}+\frac{0.34}{4815.81}+\frac{0.21}{4778.25}+\frac{0.42}{4808.77}+\frac{0.56}{4753.93}+$$
$$\frac{0.72}{4761.95},\ldots,B_6=\frac{0.18}{5326.12}+\frac{0.05}{5338.72}+\frac{0.11}{5348.33}+\frac{0.26}{5228.27}+\frac{0.64}{5284.89}+\frac{0.91}{5224.95}+\frac{0.54}{5236.47}+\frac{0.93}{5159.79}+$$
$$\frac{0.85}{5253.60},B_7=\frac{0.82}{5326.12}+\frac{0.95}{5338.72}+\frac{0.89}{5348.33}+\frac{0.74}{5228.27}+\frac{0.36}{5284.89}+\frac{0.09}{5224.95}+\frac{0.46}{5236.47}+\frac{0.07}{5159.79}+\frac{0.16}{5253.60}$$

Example. The actual data for open price in 05/09/2021 is 4641.813. Subsequently, the grades of membership were obtained for data point 4641.813 as 0.54 and 0.46, respectively, corresponding to the triangular fuzzy set A_1 and A_2. This data has reached maximum membership in the triangular fuzzy set A_4. Therefore, the open price in 05/09/2021 is fuzzified by a triangular fuzzy set A_1.

Step 9.4: Build FLR and FLRG in two factors high-order. As shown in Table 15, Table 16 and Table 17.

Table 15. The Shen-hu stock index daily data (Finance, 2021)

Date	**Open Price (Main Factor)**	**Fuzzified Open Price**	**High Price (Influence Factor)**	**Fuzzified High Price**
11/03/2021	5024.56	A_5	5138.41	B_5
12/03/2021	5153.66	A_7	5153.66	B_5
13/03/2021	5054.41	A_6	5084.31	B_5
⋮	⋮	⋮	⋮	⋮
05/09/2021	4641.81	A_1	4660.51	B_1

Table 16. Fuzzy Logic Relationship Two Factor Third-Order

Date	**Fuzzified Open Price**	**Fuzzified High Price**	**Fuzzy Logic Relationship**
11/03/2021	A_5	B_5	NA
12/03/2021	A_7	B_5	NA
13/03/2021	A_6	B_5	NA
14/03/2021	A_6	B_5	$((A_5,B_5),(A_7,B_5),(A_6,B_5)) \to A_6$
15/03/2021	A_5	B_5	$((A_7,B_5),(A_6,B_5),(A_6,B_5)) \to A_5$
16/03/2021	A_6	B_5	$((A_6,B_5),(A_6,B_5),(A_5,B_5)) \to A_6$
17/03/2021	A_5	B_4	$((A_6,B_5),(A_5,B_5),(A_6,B_5)) \to A_4$
⋮	⋮	⋮	⋮
05/09/2021	A_1	B_1	$((A_3,B_2),(A_2,B_2),(A_2,B_2)) \to A_1$

Table 17. Fuzzy Logic Relationship Group Two Factor Third-Order

Number	**Fuzzy Logic Relationship Group**
1	$((A_2,B_2),(A_4,B_3),(A_3,B_3)) \to A_4$
2	$((A_2,B_2),(A_3,B_3),(A_3,B_3)) \to A_3$
3	$((A_3,B_2),(A_2,B_2),(A_2,B_2)) \to A_1$
4	$((A_3,B_2),(A_2,B_2),(A_3,B_3)) \to A_3$
⋮	⋮
93	$((A_9,B_7),(A_9,B_7),(A_7,B_7)) \to A_8$

Step 10: Defuzzification.

Example. In the table 5, we have $\left((A_{15}, B_{27}), (A_{18}, B_{27}), (A_{16}, B_{25})\right) \rightarrow A_{17}$ at 14/03/2021. Then the defuzzification is the midpoint of u_{17} which is $\frac{5092.356+5131.867}{2} = 5112.11$. The remaining defuzzification result is presented in Table 18.

Table 18. Defuzzification (Forecasting Result)

Number	Lee's Method	Case 1	Case 2	Case 3	Case 4
14/03/2021	5088.88	5112.11	5112.11	5132.27	5035.00
15/03/2021	5000	4994.94	4994.94	5229.54	5010.68
16/03/2021	5088.88	5072.75	5033.69	5035.00	5035.00
17/03/2021	4911.11	4918.32	4880.45	5132.27	4840.45
18/03/2021	5088.88	5033.69	5033.69	4986.36	5035.00
19/03/2021	5088.88	5072.75	5072.75	5132.27	5035.00
20/03/2021	5177.77	5191.74	5151.77	5132.27	5132.27
⋮	⋮	⋮	⋮	⋮	⋮
05/09/2021	4644.44	4659.29	4623.42	4645.90	4548.63

Then compared using AFER (equation 1). Table 19 presented the comparison of four cases and Lee's method.

Table 19. Comparison of four cases in Shen-hu stock index data with Lee's Method

Evaluated criteria	Lee's method	Case 1	Case 2	Case 3	Case 4
AFER	0.95%	0.28%	0.39%	0.76%	1.06%

Based on Table 19, case 1 which is unequal length partition using interval ratio is more accurate than other cases with AFER value for Shen-hu stock index daily data 0.28%. From the differences in the existing cases, we can see a comparison of the effectiveness of each case. Case 1 in addition to getting the smallest error, it is also more effective than the other cases because it immediately gets a lot of partitions so that the left side of the FLR is not the same, or in other words FLR is the same as FLRG and does not cause ambiguity.

D. CONCLUSION AND SUGGESTIONS

In this study discusses modification method to determine the better length of interval and degree membership value in the fuzzy time series two factors high-order. Based on four cases in modification method, the result is case 1 which is unequal partition using interval ratio with grades of membership 1, 0.5, 0 for linguistic interval has the lowest error or in other words, the modification method with case 1 is more accurate than others. AFER value 0.28% for Shen-hu stock index daily data. The suggestion for the future work is applying the case 1 which is unequal partition using interval ratio with grades of membership 1, 0.5, 0 for linguistic interval in other data.

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