Laminar Viscous Fluid Flow with Micro-rotation Capabilities through Cylindrical Surface

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ABSTRACT

Viscous fluid can micro-rotate due to collisions between particles that affect viscous fluid's velocity and temperature.This study aims to determine the effect of viscosity parameters, micro-rotation materials, and heat sources on fluid velocity and temperature. The model of the laminar flow equation for viscous fluid in this study uses the laws of physics, namely, the law of conservation of mass, Newton II, and Thermodynamics I. The formed dimensional equations are converted into non-dimensional equations by using non-dimensional variables. Then, the non-dimensional equations are converted into similarity equations using stream function and similarity variables. The formed similarity equation was solved numerically by using the Gauss-Seidel method. The results of this study indicate that the velocity and temperature of the viscous fluid flow can be influenced by the parameters of viscosity, micro-rotation material, and heat source. The presence of collisions between particles causes heat to cause an increase in the variance of viscosity parameters, micro-rotation materials, and heat sources. Therefore, the viscous fluid's velocity decreases and its temperature increases.

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A. INTRODUCTION

Fluids are divided into two characters, namely non-viscous and viscous fluids (Jafeer & Mustafa, 2021). Non-viscous fluids do not experience internal friction like water, ethanol, and benzene (Norasia & Zulaikha, 2019). In contrast, viscous fluids experience internal friction between particles that change when exposed to forces (Norasia et al., 2021). Examples of viscous fluids are mud, blood, paint, and oil. The viscosity of a fluid depends on its constituent particles. Particles capable of moving randomly in a viscous fluid depend on the viscosity parameter and the micro-rotation material parameter. The movement of particles in the fluid forms a boundary layer due to friction between particles. The boundary layer divides fluid flow into laminar, transitional, and turbulent flow (Hattori et al., 2022). The regular and parallel movement of particles is called laminar flow (Committee, 2019). The change from laminar to turbulent flow is called transitional flow (Simoni et al., 2019). Then, the particles with unstable particle movement cause the flow to be shaped like a vortex.

The control of flow movement can be done through temperature and magnetic field in the fluid, which was first proposed by Shercliff (Ningtyas, 2016). Fluid temperature is based on the

convection character of the observed fluid. Convection in fluids is divided into free, forced, and mixed convection (Ur Rehman et al., 2022). Free convection occurs due to temperature differences in the fluid (Mahir & Altaç, 2019). Meanwhile, forced convection occurs due to external forces (Prameela et al., 2022). Mixed convection occurs through free and forced convection (Patel & Singh, 2019). Meanwhile, the influence of a magnetic field on a flow results in the appearance of an electric current called magneto-hydro-dynamic (MHD). MHD can increase the fluid flow temperature (Abbas & Hayat, 2008). Some of the applications of MHD in industrial fields such as MHD accelerators, power generation, and radioactive waste disposal (Mittal & Patel, 2020).

The problem of viscous fluids with the influence of free convection has been observed (Tafrikan et al., 2015). The research shows that the viscosity parameter influences velocity and temperature of the fluid increase.. The presence of mixed convection on fluid flow past a yawed cylinder shows that the temperature profiles have decreased significantly (Patil et al., 2020). The effect of viscosity and Chandrasekhar number can increase fluid flow through the flat sheet studied by Abel (Abel & Mahesha, 2008). The effect of MHD on viscous fluids can increase the flow velocity (Tafrikan & Ghani, 2020). The influence of the Hartman and Schmidt number parameters on viscous fluid flow causes the velocity profile and fluid concentration to decrease (Imtiaz et al., 2019). Another study regarding the effect of magnetic fields on polar micro fluids can increase the flow velocity (Ningtyas, 2016). Research on energy rates in polar micro fluids was carried out by Javed, it was found that material parameters increase the strength of streamlined circulations (Javed & Siddiqui, 2018). The viscous dissipation parameter has more impact on the temperature profile than the velocity profile observed in (Jenifer et al., 2021) over a Sphere. Research related to increasing thermal buoyancy on the MHD of micropolar fluids can also increase the flow velocity (Khader & Sharma, 2021). Furthermore, research on the effect of material parameters and Prandtl number on fluid temperature over a sphere has been observed. The study showed that the material parameters could increase the fluid temperature (Fauziyah et al., 2022). On the other hand, the Prandtl number causes the fluid temperature to decrease (Fauziyah et al., 2022). The Growth of Prandtl number on micropolar fluid past a stretching surface causes temperature decreases deliberated by Shankar (Shankar Goud & Nandeppanavar, 2021). Research analysis on the micro-rotation parameter increases with increasing the viscosity parameters and decreasing the magnetic parameter over a horizontal surface of a parabola (Reddy & Anki, 2022).

Fluid mechanics has developed quite rapidly. The study of fluid mechanics is known as Computational Dynamic Fluids (CFD) (Zawawi et al., 2018). Compared to fluid analysis using experimental methods, CFD can reduce the amount of time required to analyzed fluids. The study of the real problem of viscous fluid flow in the industrial and engineering fields was analyzed using mathematical modeling and solved numerically using CFD. This research can be used as reference material in engineering and industry regarding velocity and temperature analysis of viscous fluids with micro-rotation. Therefore, the movement of viscous fluids is very interesting to develop. In this study, we developed a viscous fluid model with the ability to micro-rotate through cylindrical surface. Micro-rotation occurs when particles form bonds between each other. The velocity and temperature of the viscous fluid flow that has the ability to micro-rotate can be seen from the influence of viscosity parameter, micro-rotation material

parameter, and heat source parameter. Mathematical models are built based on the corresponding laws of Physics. The mathematical solution model uses the Gauss-Seidel method numerical approach with the help of MATLAB software.

B. RESEARCH METHODS

The research method in this study is as follows:

1. Literature Study

This research uses a literature study in viscous fluid flow modeling with the influence of viscosity parameters, micro-rotation material parameters, and heat source parameters, that have been observedd by previous researcher. This study uses a literature study on the modeling of viscous fluid flow that has been carried out by previous researchers. In 2008, Abel et al conducted research on fluid viscosity that can affect fluid flow. Research on Viscous fluid modeling with the influence of free convection and mixed convection can affect fluid flow (Tafrikan et al., 2015; (Patil et al., 2020). Further research on flow control in the form of increasing MHD in viscous fluid flow can accelerate the fluid flow rate (Tafrikan & Ghani, 2020). Because this study discusses the combination of viscous fluids with micro-rotation capabilities, the next literature study is about micro-rotation. Flow control in the form of a magnetic field and the presence of micro-rotation can increase the flow velocity. Jayed et al. (Javed & Siddiqui, 2018) conducted a case study on energy velocity in micropolar fluids (Javed & Siddiqui, 2018). Sustainability research on MHD micropolar fluids can increase fluid velocity (Khader & Sharma, 2021).

2. Mathematical Modeling Development

The following steps are included to get mathematical modelling of the problem:

a. The construction of mathematical models on the flow of viscous fluids is obtained from the derivation of the Laws of Physics: the Law of Conservation of Mass, Newton's Second Law, and the First Law of Thermodynamics to get the dimensional equations are defined as follows.

The Law of Conservation of Mass

$$\frac{D\,M_{system}}{Dt} = 0$$

The Newton's Second Law

$$\rho_f \left(\frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{V}.\nabla \boldsymbol{V} \right) = \sum F$$

The First Law of Thermodynamics

$$\Delta E = Q - W$$

b. The formed dimensional equations are simplified to build the equations of continuity, momentum, and energy with the effect of viscosity parameters, micro-rotation material parameters, and heat source parameters on the velocity and temperature profile of a viscous laminar flow with micro-rotation capability through a cylindrical surface.

c. Determination of boundary conditions on viscous laminar fluid flow with micro-rotation capability through a cylindrical surface.

3. Mathematical Modeling Solutions

The following steps are included to get mathematical modelling of the problem:

a. The formed dimensional equations are converted into non-dimensional equations by using non-dimensional variables $(\bar{u}, \bar{v}, \bar{p}, \overline{u_e}, \bar{x}, \bar{y}, g_{\bar{x}}, \bar{T}$,and $\bar{T}_\infty)$ and non-dimensional parameters (Prandtl number (Pr), viscosity parameters (Vs), material micro-rotation parameters (K), magnetic parameters (M), Grashof numbers (Gr), Reynold numbers (Re), and convection parameter (λ)).

b. Then, the non-dimensional equations are converted into similarity equations by using stream function and similarity variables as follows.
stream function (Norasia et al., 2021)
$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x} \tag{1}$$
similarity variables (Prameela et al., 2022)
$$\Psi = xf(x, y)$$
$$T = T(x, y)$$
$$N = h(x, y) \tag{2}$$

c. The formed similarity equations is solved numerically by using Gauss-Seidel method. The momentum and energy similarity equations obtained are discretized using the central difference scheme. Next, the discretization obtained is substituted into the momentum and energy equations. Then the boundary conditions are carried out at each point using Gauss-Seidel iteration assisted by MATLAB software.

4. Analysis and Discussions

Analysis and discussion is carried out on the velocity and temperature profile of viscous fluid flow with the influence of viscosity parameters, micro-rotation material parameters, and heat source parameters.

C. RESULT AND DISCUSSION

1. Mathematical Model Development

The mathematical model of viscous fluid flow is built based on the law of conservation of mass, Newton's second law, and the second law of thermodynamics (Anjum et al., 2018). The equations for dimensional continuity, linear momentum, and energy are obtained.

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + v\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \frac{k_0}{\rho}\left[\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial \bar{N}}{\partial \bar{y}}\right]$$
$$+ (\bar{T} - \bar{T}_\infty)g_{\bar{x}} - \frac{1}{\rho}\sigma B_o^2(\bar{u} - \overline{u_e})$$
$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} + Q_o(\bar{T} - \bar{T}_\infty) \tag{3}$$

with non-dimensional variables (Jenifer et al., 2021)

$$x = \frac{\bar{x}}{a}; y = Re^{1/2}\frac{\bar{y}}{a}; u = \frac{\bar{u}}{U_\infty}; v = Re^{1/2}\frac{\bar{v}}{U_\infty}$$

$$T = \frac{\bar{T}-\bar{T}_\infty}{T_w-\bar{T}_\infty}; p = \frac{\bar{p}}{\rho U_\infty^2}; u_e(x) = \frac{\bar{u}_e(\bar{x})}{v_\infty}$$

And non-dimensional parameters based on viscosity and material micro-rotation parameters (Khan et al., 2020) as follows.

$$Pr = \frac{v}{\alpha}; V_S = \frac{k_o U_\infty}{a\rho v}; K = \frac{k_o}{\mu}.$$

Description

$\bar{u},\bar{v}$	: Dimensional velocity component in $x-$ and $y-$
u,v	: Non-dimensional velocity component in $x-$ and $y-$
$\overline{u_e}$	: Dimensional free flow
u_e	: Non-dimensional free flow
$\bar{p}$	: Dimensional pressure
p	: Non-dimensional pressure
$\bar{x},\bar{y}$	: Dimensional axial coordinate of the cylinder
x,y	: Non-dimensional axial coordinate of the cylinder
$g_{\bar{x}}$	: Dimensional gravitational acceleration
$\bar{T}$	: Dimensional temperature
$\bar{T}_\infty$	: Dimensional ambient temperature
T_w	: Cylinder surface temperature
v	: Kinematic viscosities
μ	: Dynamic viscosities
k_0	: Chemical reaction rate
B_o	: Magnetic field flux
ρ	: Density
σ	: Electrical conductivity
α	: Thermal diffusivity
Q_o	: Initial heat sources
Re	: Reynold numbers
U_∞	: Free stream velocity
Pr	: Prandtl numbers
Vs	: Viscosity parameter
K	: Micro-rotation parameter
a	: Length of the cylinder

Meanwhile, the magnetic parameters (Mittal, 2021) are given as follows.

$$M = \frac{\sigma B_o a}{\rho U_\infty}.$$

Grashof numbers (Gr), Reynold numbers (Re), convection parameter (λ) (Arifuzzaman et al., 2018) and the heat source parameter (Hassan, 2019) are defined by.

$$Gr = \frac{g\beta(T_w - T_\infty)a^3}{v^3}; Re = \frac{U_\infty a}{v}; \lambda = \frac{Gr}{Re^2}; Q = \frac{a^{\frac{1}{2}}Q_0}{(g(T_w - T_\infty)\beta)^{\frac{1}{2}}(\rho c_p)}$$

Substituting non-dimensional variables and parameters into Equation (3), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - V_S\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right] + \lambda\theta\sin x$$

$$- (M)(u - u_e) + K\frac{\partial N}{\partial y}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + QT \tag{4}$$

In this research, there are two velocity components, to connect two functions we use stream function (1), by substituting the stream functions in Equation (4), we get

$$\frac{\partial \Psi}{\partial x\partial y} = \frac{\partial \Psi}{\partial x\partial y}$$

$$\frac{\partial \Psi}{\partial y}\left(\frac{\partial^2 \Psi}{\partial x\partial y}\right) - \frac{\partial \Psi}{\partial y}\left(\frac{\partial^2 \Psi}{\partial y^2}\right)$$

$$= u_e\frac{\partial u_e}{\partial x} + \frac{\partial^3 \Psi}{\partial y^3} - V_S\left[\frac{\partial}{\partial x}\left(\frac{\partial \Psi}{\partial y}\left(\frac{\partial^3 \Psi}{\partial y^3}\right)\right) - \frac{\partial \Psi}{\partial x}\left(\frac{\partial^4 \Psi}{\partial y^4}\right) - \left(\frac{\partial^2 \Psi}{\partial y^2}\right)\left(\frac{\partial^3 \Psi}{\partial x\partial y^2}\right)\right]$$

$$+ \lambda\theta\sin x - (M)\left(\frac{\partial \Psi}{\partial y} - u_e\right) + K\frac{\partial N}{\partial y}$$

$$\frac{\partial \Psi}{\partial y}\left(\frac{\partial T}{\partial x}\right) - \frac{\partial \Psi}{\partial x}\left(\frac{\partial T}{\partial y}\right) = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + QT \tag{5}$$

because continuity equation $\frac{\partial \Psi}{\partial x\partial y} = \frac{\partial \Psi}{\partial x\partial y}$, there are two equation namely momentum and energy equations. The result of the stream function obtained then transformed into similarity equation. By substituting the similarity variables $(\Psi, T, N)(2)$ in Equation (5) and with free flow $u_e = \sin x$ (N. Abbas et al., 2018), we get

$$\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + 1 - V_S\left[2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3} - f\frac{\partial^4 u}{\partial y^4} - \left(\frac{\partial^2 f}{\partial y^2}\right)^2\right] + \lambda T - (M)\left(\frac{\partial f}{\partial y} - \frac{\sin x}{x}\right)$$

$$+ K\frac{\partial h}{\partial y} = 0$$

$$\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + f\frac{\partial T}{\partial y} + QT = 0 \tag{6}$$

with the following boundary conditions

$$f = \frac{\partial f}{\partial \eta} = 0, h = -nf'', T = 1 \text{ for } y = 0$$

$$\frac{\partial f}{\partial y} = 1, h = T = 0 \text{ for } y \to \infty$$

2. Analysis and Discussions

The formed similarity equations are solved numerically by using the Gauss-Seidel method to examine the effect of the viscosity parameter (V_s), the micro-rotation material parameter (K), and the heat source parameter (Q) on the laminar flow of a viscous fluid that can micro-rotate through a cylindrical surface. Numerical outcomes are computed and displayed with figure illustrations to obtain the effect of V_s, K, and Q of the velocity and temperature distributions.

The effect of variations in viscosity parameters on the velocity and temperature of the laminar flow of viscous fluid with micro-rotation capability is shown in Figure 1 and Figure 2. The parameter value used provides magnetic parameter (M), Prandtl number (Pr), micro-rotation material parameter (K), and heat sources parameter (Q) are equal to $M = 1, Pr = 5, K = 1, Q = 0.00085$. Figure 1 shows that the velocity has increased from $f' = 0$ to $f' \approx 1$. Meanwhile, Figure 2 shows that the temperature has decreased from $s' = 1$ to $s' \approx 0$. If observed with the variation of viscosity parameters, it can be seen that the greater the viscosity, the fluid velocity will decrease. On the other hand, the greater the viscosity, the higher the fluid temperature. The reason behind this is the increase of viscosity parameter, which takes density also increases, causes particles to collide then fluid flow slower and heat increased, as shown in Figure 1 and Figure 2.
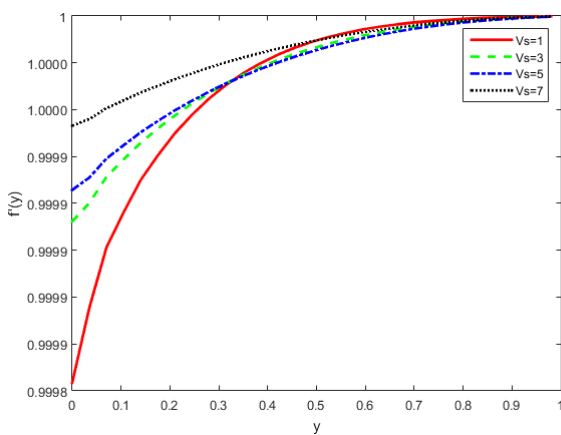

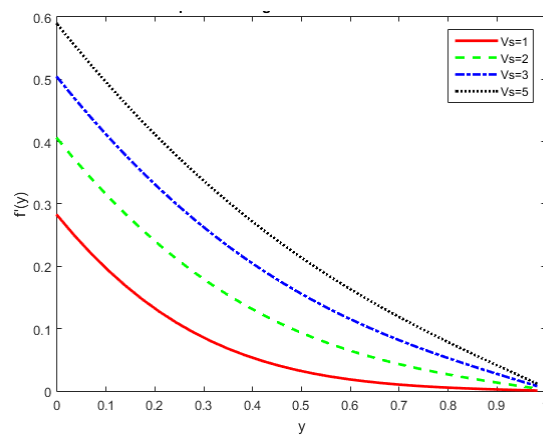**Figure 1**. Impact V_s on Velocity outlines

Figure 2. Impact V_s on Temperature outlines

The effect of variation in the micro-rotation material parameters is shown in Figure 3 and Figure 4. The parameter value used is provides magnetic parameter (M), Prandtl number (Pr), viscosity parameter (V_s), and heat sources parameter (Q) are equal to $M = 1, Pr = 5, V_s = 1, Q = 0.00085$. The variation parameters of the micro-rotation material used in this study are $K = 1, 2, 3$, and 4. Figure 3 shows that the larger the parameter of the micro-rotation material, the slower the flow movement fluid. On the other hand, Figure 4 shows that the greater the micro-rotation material parameter, the higher the temperature. This result is because of to the relationship between the micro-rotation material parameters and the density of particles. The greater the micro-rotation material parameters, the greater the density particles. Therefore, there are particle collisions that slow down the velocity of fluid flow. The slower the fluid movement, the greater the friction of fluid flow. It caused the temperature of fluid also increases, as shown in Figure 3 and Figure 4.
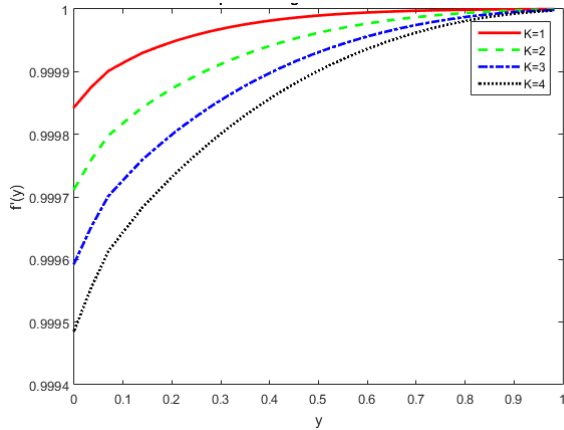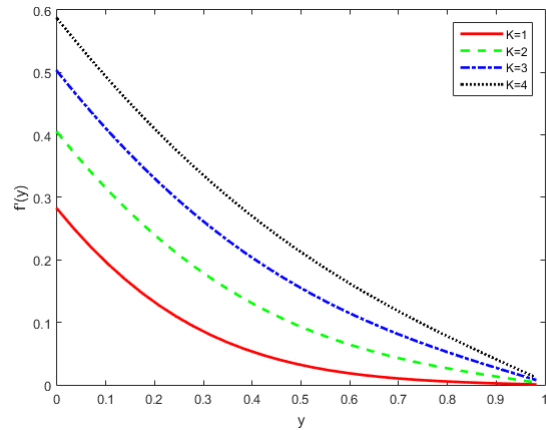
Figure 3. Impact K on Velocity outlines

Figure 4. Impact K on Temperature outlines

The effect of variation in the heat source parameters is shown in Figure 5 and Figure 6. The parameter value used is provided magnetic parameter (M), Prandtl number (Pr), viscosity parameter (V_s), and micro-rotation material parameter (K) are equal to $M = 1, Pr = 5, V_s = 1, K = 1$. The heat source parameters used are $Q = 0.00085, 0.0015, 0.0025$, and 0.012, as shown in Figure 5 and Figure 6.
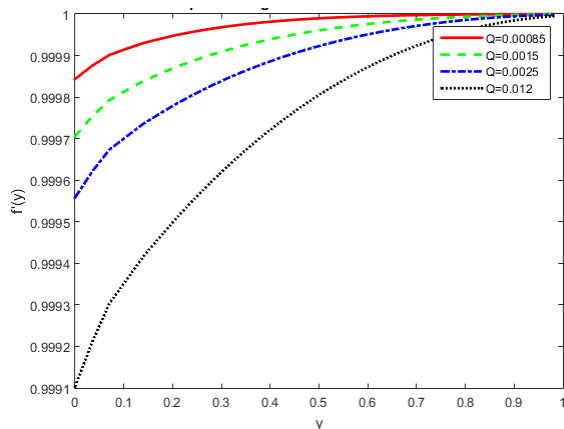

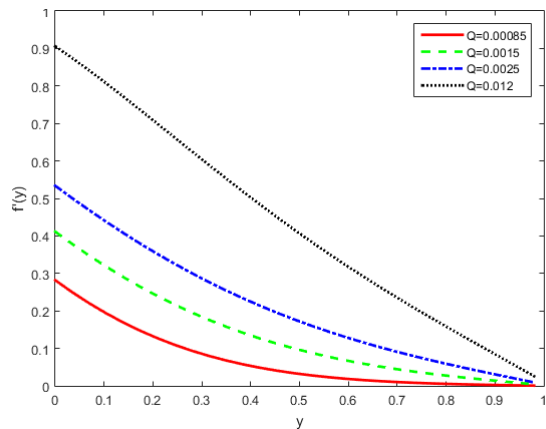**Figure 5**. Impact Q on Velocity outlines

Figure 6. Impact Q on Temperature outlines

Figure 5 shows that the larger the parameter of the heat sources, the slower the flow movement fluid. Figure 5 show that the greater the heat sources parameter, the higher the temperature. It caused the heat sources parameter is directly proportional to the initial heat source $\left(Q = \dfrac{a^{\frac{1}{2}} Q_o}{\left(g(T_w - T_\infty)\beta \right)^{\frac{1}{2}} (\rho c_p)} \right)$. The greater Q_o, the greater heat source parameter. The greater heat, the greater friction in viscous fluid. This causes the velocity of fluid decreased and the temperature of fluid increased.

The results showed that the viscosity parameters, micro-rotation material parameters, and heat source parameters were able to increase the flow temperature and decrease the fluid flow velocity. As a result of an increase in variation of the magneto-hydrodynamic (MHD) parameter, temperature also decreases, according to Tafrikan et al. On the other hand, when the MHD parameter is increased, the flow velocity of the viscous fluid increases. This is because the effect

of the density of the viscous fluid is decreasing, so the fluid velocity can move faster (Tafrikan & Ghani, 2020). Meanwhile, in the research conducted by Patil et al, it was found that the fluid temperature decreased due to the influence of mixed convection. This is due to an increase in the angle of the yawed cylinder. The greater the angle on the bent cylinder, the faster the fluid moves, resulting in more heat moving from the surface to the fluid (Patil et al., 2020).The decrease in the heat transfer rate due to the increase in the parameters of the micro-rotation material in the micropolar fluid was carried out by Javed et al (Javed & Siddiqui, 2018). The results of the research we have done are different from the research conducted by Khader et al. The study showed that the fluid temperature decreased and the fluid velocity increased (Khader & Sharma, 2021). This result is becuse of increasing the micro-rotation material parameters and thermal buoyancy parameters (Khader & Sharma, 2021). However, the research conducted by Javed et al and Khader et al has something in common, namely the temperature decreases due to the influence of the micro-rotation material parameter.

D. CONCLUSION

The mathematical model of viscous fluid flow is built based on the law of conservation of mass, Newton's second law, and the second law of thermodynamics. The law obtained the equations for dimensional continuity, linear momentum, and energy. Using stream function and similarity variables, we get similarity equations for linear momentum and energy formed as follows.

$$\frac{\partial^3 f}{\partial y^3} + f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + 1 - V_S \left[2 \frac{\partial f}{\partial y} \frac{\partial^3 f}{\partial y^3} - f \frac{\partial^4 u}{\partial y^4} - \left(\frac{\partial^2 f}{\partial y^2}\right)^2\right] + \lambda T - (M)\left(\frac{\partial f}{\partial y} - \frac{\sin x}{x}\right)$$
$$+ K \frac{\partial h}{\partial y} = 0$$
$$\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + f \frac{\partial T}{\partial y} + QT = 0$$

The formed similarity equations are solved numerically by using Gauss-Seidel method to examine the effect of the viscosity parameter(V_s), the micro-rotation material parameter (K), and the heat source parameter(Q) on the laminar flow of a viscous fluid that can micro-rotate through a cylindrical surface. The viscosity parameters, micro-rotation materials parameters, and heat sources parameters increased the flow temperature and decreased the fluid flow velocity. This condition is due to viscosity's influence and micro-rotation ability. So that the particles in the fluid collision each other and cause the temperature to increase. The next interesting research to be developed is regarding the effect of free convection and mixed convection on viscous-fluid with micro-rotation capability flow models.

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