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Systematic Literature Review Robust Graph Coloring on Electric Circuit Problems

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ABSTRACT

Graph Coloring Problem (GCP) is the assignment of colors to certain elements in a graph based on certain constraints. GCP is used by assigning a color label to each node with neighboring nodes assigned a different color and the minimum number of colors used. Based on this, GCP can be drawn into an optimization problem that is to minimize the colors used. Optimization problems in graph coloring can occur due to uncertainty in the use of colors to be used, so it can be assumed that there is an uncertainty in the number of colored vertices. One of the mathematical optimization methods in the presence of uncertainty is Robust Optimization (RO). RO is a modeling methodology combined with computational tools to process optimization problems with uncertain data and only some data for which certainty is known. This paper will review research on Robust GCP with model validation to be applied to electrical circuit problems using a systematic review of the literature. A systematic literature review was carried out using the Preferred Reporting Items for Systematic reviews and Meta Analysis (PRISMA) method. The keywords used in this study were used to search for articles related to this research using a database. Based on the results of the search for articles obtained from PRISMA and Bibliometric R Software, it was found that there was a relationship between the keywords Robust Optimization and Graph Coloring, this means that at least there is at least one researcher who has studied the problem. However, the Electricity keyword has no relation to the other two keywords, so that a gap is obtained and it is possible if the research has not been studied and discussed by other researchers. Based on the results of this study, it is hoped that it can be used as a consideration and a better solution to solve optimization problems.

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A. INTRODUCTION

Graph coloring is a fundamental combinatorial optimization problem for coloring the vertices of a given graph with a minimum number of colors so that adjacent vertices are colored differently. In graph coloring, there is a problem that is one of the challenges to get a solution, namely determining the right number or number of colors so that all vertices can be given a different color. (van Hoeve, 2020). In graph coloring, labels on graph elements are given based on some constraints or conditions and the label is a color. Colors in a graph are usually assigned to integers at the edges, vertices, or both, that is to the edges and the vertices of the graph. In graph theory, several colors are used to label edges or vertices with a constraint on using their color. If there is a color, it must be able to determine the vertex to be colored so that no two adjacent vertices have the same color. There are also some other graph coloring problems, for example, Edge Coloring and Face Coloring. In edge coloring, none of the vertices connected by two edges have the same color, and face coloring is related to Geographic map coloring (Elumalai, 2020). There are several previous studies that discuss the optimization model for graph coloring problems, namely (Diaz et al., 2004) discusses the Branch & Cut algorithm based on polyhedral studies of integer linear programming models to solve coloring problems, (Nickel, 2005) discusses the ellipsoid method approach that is used as a tool to prove the solvency of the polynomial-time of combinatorial optimization problems, (Hansen et al., 2009) discusses the consideration of two linear programming formulations of the graph coloring problem by using branch-and-cut-and-price algorithm computational experiments, (Burke et al., 2010) discusses vertex coloring in combinatorial optimization problems with integer programming formulations to survey seven known vertex coloring formulations and introduces a new formulation for vertex coloring with appropriate partitions on graphs, (Jabrayilov & Mutzel, 2018) discusses the formulation of integer linear programming with partial ordering for vertex coloring problems in graphs for a large set of benchmark graphs and randomly generated graphs of various sizes and densities, (Jovanović et al., 2020) discusses the development of optimization models using weighted node coloring combinatorial problems for strategic decision making, and so on.

Graph coloring is a traditional method and may not be able to produce an optimal solution so in graph coloring problems it often requires an approach with other optimization methods to maximize/minimize the objective function by considering the existing constraints (Marx, 2004). The problem for graph coloring can be related to optimization problems, namely determining a vertex to be colored correctly, and also the number of colorings given is minimal.

In its application, not all optimization problems can be obtained with objective functions and constraint functions with certain conditions (certain), there are also optimization problems with uncertain conditions (uncertain). In the graph coloring problem, it can be assumed that the number of vertices to be colored is not certain. To obtain optimal results, an optimization method is needed that considers uncertain circumstances where the data obtained are unknown or incomplete, while the solution must be determined. One of the optimization methods with uncertainty is the Robust Optimization (RO) method. Robust Optimization (RO) is a methodology for solving mathematical problems by assuming that the uncertain data is in a defined set of indeterminate parameter values which is called the uncertainty set (Yanıkoğlu et al., 2019).

This article discusses a systematic review that uses optimization models for graph coloring problems. The purpose of this research is to find out the methods that have been used previously in overcoming the problem of graph coloring so that gaps are obtained to solve the problem of graph coloring using the Robust Optimization method and as a validation the model is applied to the Electricity problem. The discussion of this systematic review is supported by problems related to the Graph Coloring Problem, Robust Optimization and Electrical Circuit Problems. This study applies a systematic review using the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) method and can be done with the help of RStudio software to see the relationship between one keyword and another, so that gaps can be obtained for future research.

B. METHODS

In this article, we discuss a systematic literature review on Robust Optimization for Graph Coloring. Systematic literature review using the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) method. A systematic literature review was conducted to find a research gap so that something new could be obtained for future research. Some steps will be carried out in this research, namely (1) Selecting articles to be reviewed based on the appropriate topics in the database; (2) Determining research gaps using PRISMA based on articles that have been obtained in the first stage; and (3) The author presents conclusions from the results of the literature review systematically in the Conclusion section. In this section, several materials related to the research topic, namely Graph Coloring, Graph Coloring Problem Optimization Model, Graph Coloring in The Electricity Sector, Robust Optimization, Robust Counterpart, and Robust Counterpart for Polyhedral Uncertainty Sets.

1. Graph Coloring

Graph coloring is one type of graph labeling and is a branch of graph labeling in special cases. In graph coloring, labeling is given based on existing constraints or conditions. The label used is a color. In the graph, labeling can usually be done by assigning a value (weight) to the edges, vertices, or both the edges and vertices of the graph (Elumalai, 2020). Graph coloring is a fundamental combinatorial optimization problem to color the vertices of a particular graph with the minimum number of colors so that adjacent vertices can be colored differently (van Hoeve, 2020). Based on this, an optimization problem arises to determine the minimum number of colors for nodes, this problem is usually referred to as the Graph Coloring Problem (GCP). There are three types of graph coloring, namely vertex coloring, edge coloring, and region coloring (Kabang et al., 2020). Node coloring is the assignment of color to the vertices with every two adjacent vertices having a different color. Side coloring is the assignment of colors to the sides with every two adjacent sides having a different color. Regional coloring is giving color to an area with every two neighboring areas having a different color (Arsyad et al., n.d.), as shown in Figure 1, Figure 2 and Figure 3.

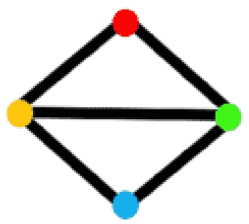

Figure 1. Example of Node Coloring

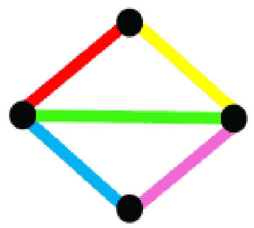

Figure 2. Example of Edge Coloring

Figure 3. Example of Region Coloring
Source: Personal Documents and
http://www.openjournal.unpam.ac.id/index.php/sm/article/view/4284/3236

2. Graph Coloring Problem Optimization Model

Jovanović, et al., in 2020 developed an optimization model for strategic decision making using well-known combinatorial problems, such as the weighted node coloring problem and the Traveling Salesman Problem. In this study, an optimization model for the vertex coloring problem in the graph will be developed as follows (Jovanović et al., 2020):

$$\beta = \min \sum_{j} y_j \tag{1}$$

Where β is the minimum number of colored points (y_j).

There are four constraints from the optimization model of the node coloring problem above, the first constraint defines if all nodes must be colored. These constraints are formulated as follows:

$$\sum_{j} x_{ij} = 1, i \in V \tag{2}$$

where x_{ij} is the node variable i which is colored j and $V = \{v_1, v_2, \dots, v_n\}$.

The second constraint defines that there is at most one of the pair of vertices and edges that receive a certain color. The second constraint is formulated as follows:

$$x_{ij} + x_{kj} \leq y_j, \forall (i,k) \in E, j = 1, .., n \tag{3}$$

where x_{ij} and x_{kj} are node variables i and k which are colored j and $E = \{e_1, e_2, \dots, e_n\}$.

The third and fourth constraints are conditions if the variables x_{ij} and y_j are binary variables. The third and fourth constraints are formulated as follows:

$$x_{ij} \in \{0,1\}, i \in V, j = 1, .., n \tag{4}$$

$$y_j \in \{0,1\}, j = 1, .., n \tag{5}$$

The description of the set, parameters, and objective variables used in the model is as follows:

a. Set
   - V : Set of vertices that are not empty
   - E : The set of edges that connect a pair of nodes
b. Decision Variables
   - $x_{ij} = 1$ Declares variable to have value 1 if and only if color j is assigned to node i;
   - x_{ij} : Node variable i which is colored j
   - $y_j = 1$ declares variable to have value 1 if and only if color j is set to at least one of the vertices;
   - y_j : The variable that determines whether the sign (color) of j used

3. Graph Coloring in The Electricity Sircuit

Ananda Maiti and Balakrushna Tripathy in 2012 conducted a study on the application of graph coloring for matching electrical circuits in electrical repositories. In this article, we discuss a colored graph isomorphism-based model for matching two electrical circuits. The procedure for matching two electrical circuits consists of two steps, first, saving the circuit in the database in the form of a colored graph and then matching the input circuit using an isomorphism graph. This study aims to determine the compatibility of two circuits using Luellau's Algorithm and Ohlrich's Algorithm (Maiti & Tripathy, 2012).

An electrical circuit consists of a set of different components and is connected by wires or conducting materials. Electrical circuits can be converted into graphs where the vertices are the components of the electrical circuit and the edges are the wires or conductors. In this model, the series is represented as a color-weighted undirected graph:

a. Nodes based on each component or terminal section such as a diode consists of 2 terminals so that it can be represented by two D+ and D- nodes connected with W as the connection node.
b. The color of the node is determined based on the electrical component it represents. For example, Resistors are always red in Figure 5.
c. The weight of the node is the value of the component itself. For example, a resistor is weighed by its resistance value. However, this is optional if the component is generic like Ground.
d. The nodes are connected in the graph if they are connected in the circuit, as shown in Figure 4.

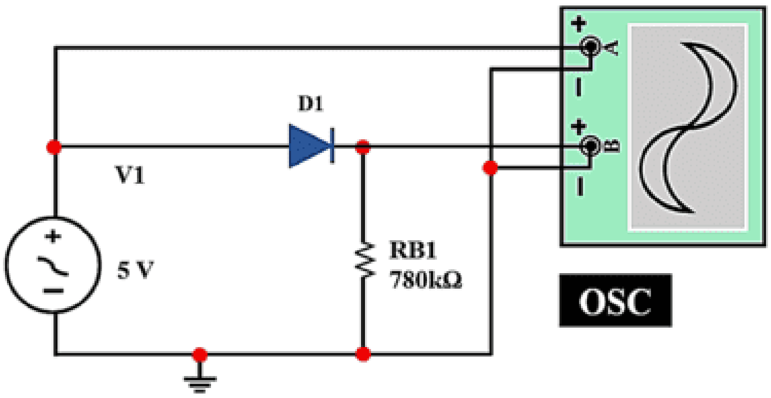

Figure 4. Half-wave Rectifier Circuit Diagram (Maiti & Tripathy, 2012)

The half-wave rectifier circuit diagram is converted into a graph consisting of 5 components, Ground (G), Oscilloscope (O), Signal Generator (V), Diode (D), and Resistor (R), as shown in Figure 5.

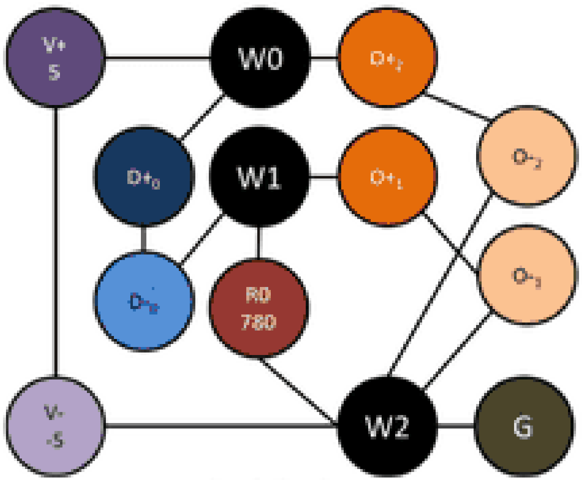

Figure 5. Electrical Circuit Graph (Maiti & Tripathy, 2012)

Each component in the circuit is represented as a vertex in the graph. For example, for diodes D+ and D-, an oscilloscope with two channels is represented by O+1, O+2, and O-1 and O-2. Resistor and Ground have orientations that are not important so they are simply represented by one node. The anode of the oscilloscope channel has the same components and functions so that it has the same color. The sides in the graph represent the relationship between each component in the series.

4. Robust Optimization

In real-world problems, optimization problems in the presence of uncertainty in their parameters can be solved in a way to find out the result of the computed solution being highly inappropriate, suboptimal, or both (potentially worthless). The optimization related to the uncertainty parameter is Robust Control. Robust optimization focuses on the theoretical concepts of traditional optimization, especially algorithms, geometry, and tractability, in addition to using modeling will obtain results that can generally be used to calculate Robustness (Bertsimas et al., 2011).

Robust Optimization (RO) is a model, combined with computational tools, to solve optimization problems with uncertain data and multiple uncertainty sets (Ben-Tal & Nemirovski, 2002). RO considers a deterministic uncertainty model consisting of sets with the ability to change the parameters of a function to obtain the best decision (Xu et al., 2010). Several things can cause optimization problems to have indeterminate parameters, including measurement/rounding errors, prediction/estimation errors, implementation errors (Hertog, 2015).

A linear optimization model, in general, can be expressed in the following equation

$$\min_x \{\boldsymbol{c}^T \boldsymbol{x} : A\boldsymbol{x} \leq \boldsymbol{b}\} \tag{6}$$

where $\boldsymbol{c} \in \mathbb{R}^n, \boldsymbol{x} \in \mathbb{R}^n,\ A \in M_{m\times n}(\mathbb{R}), \boldsymbol{b} \in \mathbb{R}^m$.

Referring to (Ben-Tal & Nemirovski, 2002) three parameters can be assumed to be indeterminate, namely parameters $\boldsymbol{c}, A, \boldsymbol{b}$, so that model (6) can be expressed in the general form of an indefinite linear optimization model as follows

$$\min_x \{\boldsymbol{c}^T \boldsymbol{x} : A\boldsymbol{x} \leq \boldsymbol{b}\ \mid (\boldsymbol{c}, A, \boldsymbol{b}) \in \mathcal{U}\} \tag{7}$$

There are three basic assumptions in Robust Optimization (RO), namely (Gorissen et al., 2015).

a. All decision variables are "here and now" namely the acquisition of numerical values as a result of solving problems before the actual value of the original data is known.
b. The decision-maker is responsible for the decision to be made with the actual data in the uncertainty set $\mathcal{U}$.
c. All constraints of the problem are not necessarily "hard", that is, the decision-maker cannot violate the existing constraints even though all data are in the uncertainty set $\mathcal{U}$.
In addition to the three basic assumptions above, there are several additional assumptions, namely:

1) If the uncertainty is in the objective function, then the problem can be changed by adding the variable t into the objective function so that the uncertainty can appear in the constraint function. From the general model (7), the objective function $\boldsymbol{c}^T\boldsymbol{x}$ is replaced by an additional variable with $t \geq \boldsymbol{c}^T\boldsymbol{x}$ and $t \in \mathbb{R}$ so that it becomes
$$\min_x\{t: \boldsymbol{c}^T\boldsymbol{x} \leq t, A\boldsymbol{x} \leq \boldsymbol{b} \mid (\boldsymbol{c}, A, \boldsymbol{b}) \in \mathcal{U}\} \quad (8)$$
or
$$\min_x\{t: \boldsymbol{c}^T\boldsymbol{x} - t \leq \boldsymbol{0}, A\boldsymbol{x} \leq \boldsymbol{b} \mid (\boldsymbol{c}, A, \boldsymbol{b}) \in \mathcal{U}\} \quad (9)$$
Therefore, the uncertainty has disappeared from the objective function.
2) If there is any uncertainty on the right side, it can be translated by adding variable $x_{n+1} = -1$.
3) The set of uncertainty $\mathcal{U}$ can be replaced with convex hull $\mathcal{U}$, which is the smallest convex set of $\mathcal{U}$. To obtain a feasible solution to $\mathcal{U}$, you can take the supremum of the left-hand side constraints $\mathcal{U}$, so that the objective function will produce an optimal value if the convex hull $\mathcal{U}$ has a maximum value.
4) Robustness to $\mathcal{U}$ can be illustrated constraint-wise.

5. Robust Counterpart

The robust Counterpart (RC) is part of the uncertainty problem. A feasible/optimal solution for RC is called a Robust feasible/optimal solution for an indeterminate problem. RC is used as a decision based on "real life" problems to obtain the Robust optimal solution (Ben tal et al., 2009). Assuming that $\boldsymbol{c} \in \mathbb{R}^n$ and $\boldsymbol{b} \in \mathbb{R}^m$ are definite, so that model (7) can be reformulated and called the RC problem is as follows (Gorissen et al., 2015).
$$\min_x\{\boldsymbol{c}^T\boldsymbol{x} : A(\zeta)\boldsymbol{x} \leq \boldsymbol{b} \mid \forall\, \zeta \in \mathcal{Z}\} \quad (10)$$
$$\min_x\{\boldsymbol{c}^T\boldsymbol{x} : \overline{\boldsymbol{a}}_{\boldsymbol{i}}^T(\zeta)\boldsymbol{x} \leq \boldsymbol{b}_{\boldsymbol{i}} \mid i = 1, \dots, m, \forall\, \zeta \in \mathcal{Z}\} \quad (11)$$
Where $\mathcal{Z} \in \mathbb{R}^L$ which denotes a user-defined set of primitive uncertainties. Solution $\boldsymbol{x} \in \mathbb{R}^n$ is a feasible solution which is called Robust feasible if it satisfies the indefinite constraint $A(\zeta)\boldsymbol{x} \leq \boldsymbol{b}$ for all realizations $\zeta \in \mathcal{Z}$. Based on the previous assumptions, namely $\boldsymbol{c} \in \mathbb{R}^n$ and $\boldsymbol{b} \in \mathbb{R}^m$ is certain, then the indefinite parameter is $A \in M_{m\times n}(\mathbb{R})$. Because Robust is assumed to be constraint-wise, the single constraint in the model (11) becomes
$$(\overline{\boldsymbol{a}} + \boldsymbol{P}\zeta)^T\boldsymbol{x} \leq \boldsymbol{b}, \forall\, \zeta \in \mathcal{Z} \quad (12)$$
Where $(\overline{\boldsymbol{a}} + \boldsymbol{P}\zeta)$ is an affine function of the primitive uncertainty parameter $\zeta \in \mathcal{Z}$, $\overline{\boldsymbol{a}} \in \mathbb{R}^n$ and $\boldsymbol{P} \in M_{n\times L}(\mathbb{R})$.

Furthermore, there are two ways to determine ζ, namely first, by applying the Robust reformulation technique to exclude for all quantifications. The second is to apply an adversarial approach. In this study, only the first method is used, which consists of three steps and produces a computationally tractable RC with a limited number of constraints.

6. Robust Counterpart for Polyhedral Uncertainty Sets

The set of uncertainty in the determination of ζ consists of the set of box, ellipsoidal, and polyhedral uncertainties. In this study, we only assume that the uncertainty parameter is in the polyhedral uncertainty set. According to (Gorissen et al., 2015) the set of polyhedral indeterminacy can be defined as follows:

$$\mathcal{Z} = \{\zeta : D\zeta + \boldsymbol{q} \geq \boldsymbol{0}\} \tag{13}$$

Where $D \in \mathbb{R}^{m \times L}, \zeta \in \mathbb{R}^L$ and $\boldsymbol{q} \in \mathbb{R}^m$. So for the set of uncertainty $\mathcal{U}$ can be formulated as follows:

$$\mathcal{U} = \{\boldsymbol{a} | \boldsymbol{a} = (\overline{\boldsymbol{a}} + P\zeta), D\zeta + \boldsymbol{q} \geq \boldsymbol{0}\} \tag{14}$$

To obtain an RC formulation with a polyhedral uncertainty set, equation (13) can be applied to equation (12) so that it becomes

$$(\overline{\boldsymbol{a}} + P\zeta)^T \boldsymbol{x} \leq \boldsymbol{b}, \forall\, \zeta : D\zeta + \boldsymbol{q} \geq \boldsymbol{0} \tag{15}$$

There are three steps to the approach using the first method, namely:

a. Worst Case Reformulation

$$\begin{gathered} \max_{\zeta : \boldsymbol{D}\zeta + \boldsymbol{q} \geq \boldsymbol{0}} (\overline{\boldsymbol{a}} + P\zeta)^T \boldsymbol{x} \leq \boldsymbol{b} \\ \overline{\boldsymbol{a}}^{\boldsymbol{T}} \boldsymbol{x} + \max_{\zeta : \boldsymbol{D}\zeta + \boldsymbol{q} \geq \boldsymbol{0}} (P^{\boldsymbol{T}} \boldsymbol{x})^{\boldsymbol{T}}\, \zeta \leq \boldsymbol{b} \end{gathered} \tag{16}$$

b. Duality

Convert the primal shape of the model (16) to the dual shape. To make it easier to convert the primal to dual form, the rules of the reflection transformation or the reflection of the primal problem to the dual problem can be used as shown in Table 1 which refers to (Rao, 2009), as shown in Table 1.

Table 1. Correspondence Rules for Primal-Dual Relations

	Primal	**Dual**
Objective Function	min $\boldsymbol{cx}$	max $\boldsymbol{b^T y}$
Variable	$x_i \geq 0$	i th constraint $A_i^T \boldsymbol{y} \leq c_i$
Variable	x_i unrestricted	i th constraint $A_i^T \boldsymbol{y} = c_i$
Constraints	j th constraint $A_j \boldsymbol{x} = b_j$	j th variable y_j unrestricted
Constraints	j th constraint $A_j \boldsymbol{x} \geq b_j$	j th variable $y_j \geq 0$
Coefficient Matrix	$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix}$	$A^T = [A_1 \quad A_2 \quad \cdots \quad A_m]$
Right-hand-side	Vector b	Vector c
Cost Coefficient	Vector c	Vector b

The primal form of the model (16) is

$$\max_{\zeta : D\zeta + \boldsymbol{q} \geq \boldsymbol{0}} (P^{\boldsymbol{T}} \boldsymbol{x})^{\boldsymbol{T}}\, \zeta \tag{17}$$

So the dual problem form for model (17) is

$$\min_{\boldsymbol{w}} \{\boldsymbol{q^T w} : -D^{\boldsymbol{T}} \boldsymbol{w} = P^{\boldsymbol{T}} \boldsymbol{x}, \boldsymbol{w} \geq \boldsymbol{0}\}$$

or

$$\min_{\boldsymbol{w}} \{\boldsymbol{q^T w} : D^{\boldsymbol{T}} \boldsymbol{w} = -P^{\boldsymbol{T}} \boldsymbol{x}, \boldsymbol{w} \geq \boldsymbol{0}\} \tag{18}$$

by substituting model (18) into the model (16) it becomes

$$\overline{\boldsymbol{a}}^{\boldsymbol{T}} \boldsymbol{x} + \min_{\boldsymbol{w}} \{\boldsymbol{q^T w} : D^{\boldsymbol{T}} \boldsymbol{w} = -P^{\boldsymbol{T}} \boldsymbol{x}, \boldsymbol{w} \geq \boldsymbol{0}\} \leq \boldsymbol{b} \tag{19}$$

c. Robust Counterpart (RC) Formulation

After obtaining the minimization form in the model (19), the constraint applies to at least one $\boldsymbol{w}$. The final formulation of RC was obtained

$$\exists \boldsymbol{w}: \overline{\boldsymbol{a}}^T \boldsymbol{x} + \boldsymbol{q}^T \boldsymbol{w} \leq \boldsymbol{b}, D^T \boldsymbol{w} = -P^T \boldsymbol{x}, \boldsymbol{w} \geq \boldsymbol{0} \quad (20)$$

The constraint in equation (20) is in the form of Linear Programming (LP). If the set can be described well with linear constraints then it can be traced computationally tractable (Chaerani & Roos, 2013). For the Tractable Formulation with Polyhedral Uncertainty Set, it can be seen in Table 2.

Table 2. Formulation of Tractable Polyhedral Uncertainty Set

Uncertainty Set	$\mathcal{Z}$	Robust Counterpart	Tracktibility
Polyhedral	$D\zeta + \boldsymbol{q} \geq \boldsymbol{0}$	$\begin{cases} \overline{\boldsymbol{a}}^T \boldsymbol{x} + \boldsymbol{q}^T \boldsymbol{w} \leq \boldsymbol{b} \\ D^T \boldsymbol{w} = -P^T \boldsymbol{x} \\ \boldsymbol{w} \geq \boldsymbol{0} \end{cases}$	Linear Programming (LP)

The RC procedure in its application is shown in Figure 6.

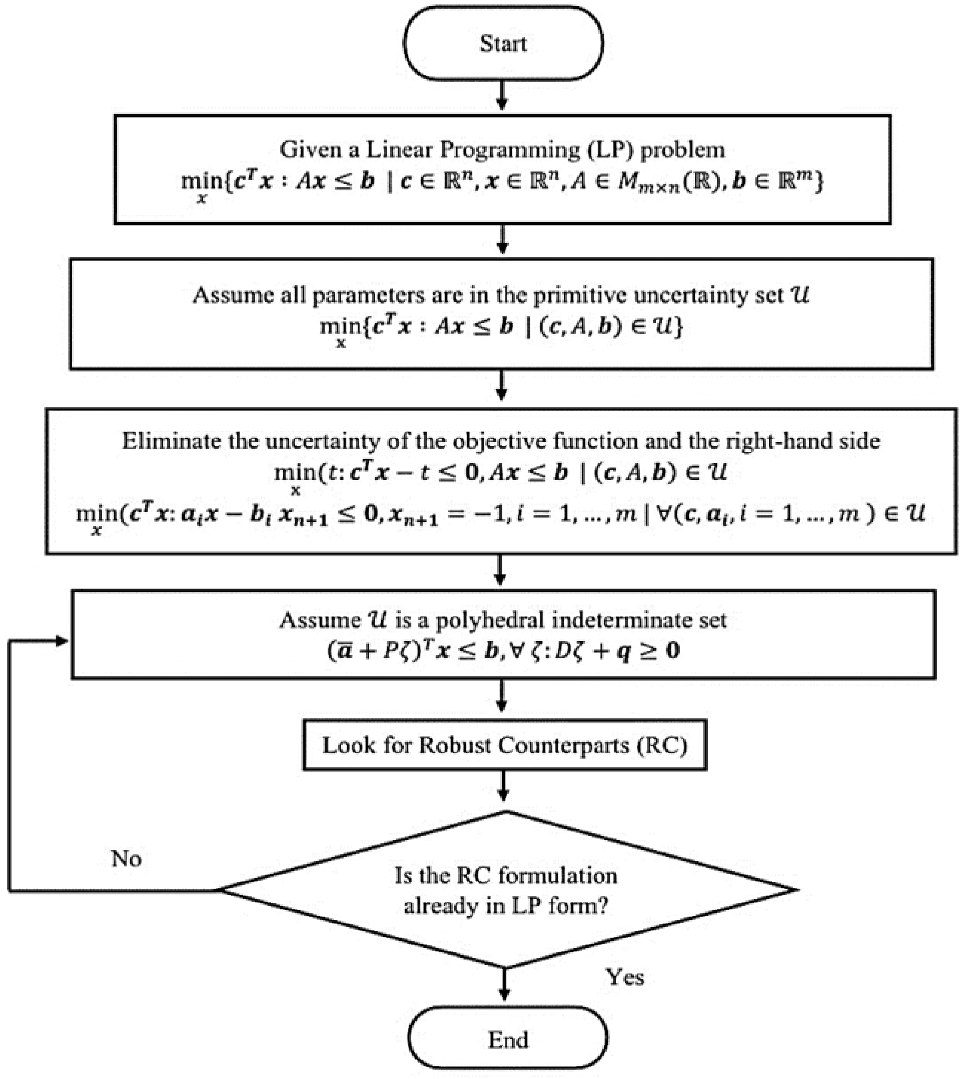

Figure 6. Flowchart of Robust Counterpart Algorithm

There are several steps to discussing a systematic literature review on Robust Optimization for Graph Coloring Problems. A literature review is an important feature of academic research conducted by keyword searches because initial relevance is determined by the title (Xiao & Watson, 2019). The literature review is carried out with several objectives, including summarizing the development of practice or technology, identifying gaps that can be used for further research, can help position new research activities in the future, and testing hypotheses with existing empirical evidence (Budgen & Brereton, 2006). A systematic literature review can be carried out using the Preferred Reporting Items for Systematic reviews and Meta-Analysis (PRISMA) and the PRISMA method that will be used in this research is PRISMA 2020. PRISMA is used to identify, select, assess and synthesize studies (Page et al., 2021). The PRISMA guideline consists of four flowchart phases, namely identification, screening, eligibility, and inclusion criteria. In addition, PRISMA consists of a checklist of 27 items on topics such as title, abstract, introduction, methods, results, discussion, and financing. Based on this flowchart and checklist, PRISMA items can serve as a guide for writers, reviewers, and editors (Selcuk, 2019). PRISMA consists of four stages, as shown in Figure 7.

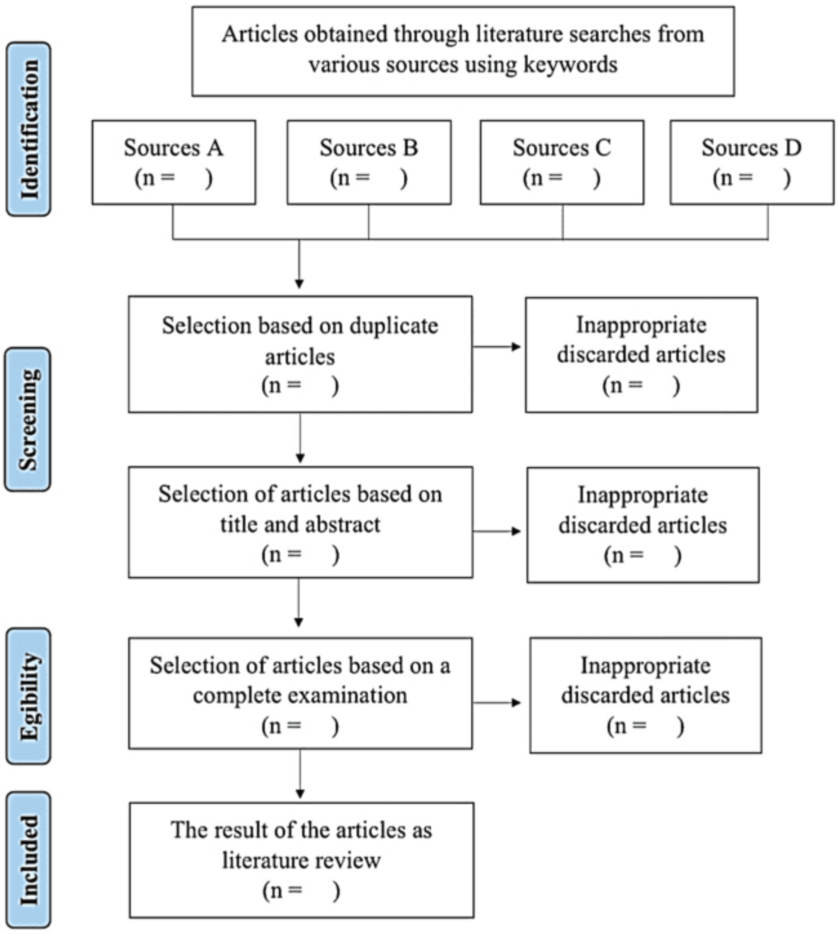

Figure 7. PRISMA Method Algorithm
Source: https://guides.lib.unc.edu/prisma

The first step that needs to be taken to conduct a systematic literature review is to collect articles that are relevant to the research topic in the form of a database. In this study, four sources of database search will be used, namely Google Scholar, Science Direct, Dimensions, and SAGE Journals. The articles obtained are in the form of BibTex for further use in the PRISMA method. There are several conditions used to obtain articles in the database, namely the database of articles used published in 2000-2022 and the database of articles used in English.

This paper focuses on the Robust Optimization Model for Graph Coloring and as a form of model validation it will be applied to electrical circuit problems. Therefore, the keywords used in this research are "Robust Optimization" AND "Graph Coloring" AND "Electricity". These keywords aim to show the relationship between Robust Optimization, Graph Coloring, and Electricity so that it can be seen whether or not there is research that examines these problems. Furthermore, after getting material from keywords based on the database, the PRISMA method is applied.

At the identification stage, an article search is carried out in the form of a database obtained by using keywords that are used in certain sources. Based on the articles that have been obtained, a screening stage will then be carried out, namely selecting articles and eliminating duplicate articles. The next screening stage is to select articles based on the title and abstract with the specified article criteria. Articles that do not meet the criteria will be excluded so that the screening stage is completed. The next stage is the eligibility, namely conducting an overall selection of the articles obtained from the previous stage. Articles that do not meet eligibility in the study will be excluded. Next is the last stage, namely the inclusion criteria (included), at this stage is the final result of the examination of all articles, and these articles can be used as material for literature review.

Based on the PRISMA results that have been obtained, an overview of the previously studied research topics is obtained. In addition, gaps are obtained that can be used as studies for further research. To see the novelty in this research, it will be done with the help of bibliometric maps using R software, namely by seeing whether or not there is a relationship between the keywords used in this study.

C. RESULT AND DISCUSSION

1. The Result of a Literature Review Using the PRISMA Method

This section discusses the application of the PRISMA method to keywords which is a part to see the development of graph coloring optimization with Robust optimization on electrical circuit problems. The explanation of the four steps in the PRISMA method is as follows:

a. First Stage (Identification)

The selected keywords will be used at this stage. Keywords are inputted into different database sources, namely Google Scholar, Science Direct, Dimensions, SAGE Journals so that the results are presented in Table 3.

Table 3. Systematic Literature Review Database

Google Scholar	Science Direct	Dimensions	SAGE Journals
0 article	0 article	0 article	9 articles

Based on Table 3, a total of 9 articles related to the keywords entered were obtained. The keywords used already contain all the topics that will be studied in this study and the results of the acquisition will be used for the next step using the PRISMA method.

b. Second Stage (Screening)
The 9 database articles obtained will be applied to the second stage, namely the screening stage. At the screening stage, it will go through two stages, namely the selection of duplicate articles so that articles indicated as duplicates will be removed, based on the results of the first screening there are no articles indicated as duplicates. Then for the second screening, namely the selection based on the title and abstract, at this stage the articles in the title and abstract that have nothing to do with the topic of this research will be deleted. In addition, study materials that are not in the form of articles will be excluded, so that at this stage 7 articles are obtained. The 7 articles in the database enter the next stage of the PRISMA method, namely the eligibility stage.

c. Third Stage (Eligibility)
At this stage, the author makes a selection by looking at the articles one by one and grouping articles that are likely to follow the research topic. The 7 articles obtained at this stage are articles that contain discussions and are in accordance with the research topic.

d. Fourth Stage (Included)
This last stage is the result of the last database obtained based on the previous stage. By using the PRISMA method, 7 article databases were obtained that can be used as reference material for literature review, as State of The Art, bibliometric mapping material for R software to see if there is any novelty. State of The Art table of the seven articles is presented in Table 4.

Tabel 4. State of The Art

Author	Summary	Objective Function	Method	Graph Coloring Problem (Yes/No)	Uncertainty Set (Yes/No)	Electric Circuit (Yes/No)
Xiong et al (2011)	Discusses power grid problems by making mathematical representations and using computational tools.	-	-	No	No	No
Hassan et al (2016)	Discussed Dynamic Spectrum Assignment (DSA) issues by using Simulated Annealing (SA) for mobile operators which integrates with Smart Grid (SG) for less energy usage and proposed strategies adapting to system dynamics, electricity pricing and end-user traffic.	Maximize	Simulated Annealing (SA)	No	No	No
Dunning et al (2017)	Discusses about open source modeling language as an application of	Minimize	JuMP	Yes	Yes	No

Author	Summary	Objective Function	Method	Graph Coloring Problem (Yes/No)	Uncertainty Set (Yes/No)	Electric Circuit (Yes/No)
	optimization problems with Julia programming and development to overcome optimization models under uncertainty.					
Shehab et al (2017)	Discusses a comprehensive overview of the Cuckoo Search Algorithm (CSA) regarding modifications and applications of CSA.	Maximize or Minimize	Cuckoo Search Algorithm (CSA)	Yes	No	No
Froger et al (2017)	Discusses scheduling maintenance tasks over different and limited planning horizons while maximizing revenue from wind turbine electricity production.	Maximize	Branch and Check (B&C)	No	No	No
Porumbel (2018)	Discusses the projection of interior points swith cutting planes in polytops by solving problems using Robust Optimization or Benders Decomposition.	Maximize	Cutting Plane	No	Yes	No
Yang et al (2022)	Reviewing the game theory-based Machine Learning (ML) method approach to solve the Combinatorial Optimization Problem (COP) in the energy sector	Maximize or Minimize	Machine Learning (ML)	No	No	No

Based on Table 4, it can be seen that there is not a single article that fully discusses research on Robust Optimization, Graph Coloring, and Electricity. From Table 4, each article has a different discussion. The difference from one article to another is that it can be seen from the presence or absence of research that examines electricity problems and none of the seven articles examines electrical circuit problems. However, in this study, the topics used were Robust Optimization, Graph Coloring, and Electricity and the results for each article did not meet the criteria for the topics chosen in this study. Furthermore, to see a novelty in this research will be done using the help of bibliometric maps sourced from seven articles. This is explained further in the result of Bibliometric maps using RStudio software section.

2. The Result of Bibliometric Maps Using RStudio Software

In this section, a analysis will be carried out using a bibliometric map based on keywords from seven previously obtained articles. In doing bibliometric mapping, it can be done using Software R by first installing the bibliometric packages, namely install.packages("bibliometrix") and after that using the "biblioshiny()" command. The bibliometric mapping uses metadata from the seven articles in BibTex format. Mapping results are presented in Figure 8.

Figure 8. Bibliometrix Map in PRISMA
Source: Bibliometrix Software R

Based on Figure 8, four keyword clusters are obtained based on the resulting color. The cluster provides information on grouping previous research topics in seven selected articles. The four clusters are presented in Table 5.

Table 5. Keyword Cluster

Cluster	Keywords
Cluster 1 (Green)	Cuckoo Search, Cuckoo Breeding, CSA Versions, CSA CSA, CSA Belong, CSA Applications, Algorithm CSA, Data Mining, Survey Paper, Intensive Research, Engineering Medical, Extended Versions, Future Directions, Approach Established, Exhaustive Overview, Cons Main, Comprehensive Review, Conducting Intensive, Breeding Behavior, Article Displays, Behavir Owing
Cluster 2 (Red)	Robust Optimization, Graph Coloring, Hessian-vector products, Jumps unique, Driven idots, Coloring Heuristic, easy-to-develop Addons, Benders Decompositions, Column Generation, Aggresive Kit, Electricity Scheduling, Electricity System, Efficiency Idots, Idots Based, Improve Efficiency
Cluster 3 (Purple)	Carbon Emissions, Cellular Network, Cellular Operators, Energy Consumption, Grid Sg-aware, End-user Requirement, Global Carbon
Cluster 4 (Blue)	Backslashnphotoelectronic Properties, Backslashnto Organic, Applied Backslashntom Applications Backslashnof, Photoredox Catalysis, Backslashnof Photoredox, Addition Recent

The results of the bibliometric mapping search above show that there are four clusters, namely cluster 1, cluster 2, cluster 3, and cluster 4. For research topics regarding Robust Optimization, Graph Coloring, and Electricity are located in the 2nd cluster. the same cluster, but related topics, namely Robust Optimization and Graph Coloring; Graph Coloring and Electricty. Based on these results, research containing the three topics can provide opportunities for future research regarding the application of Robust Optimization Graph Coloring on electrical problems, especially problems with electrical circuits.

D. CONCLUSION AND SUGGESTIONS

We have presented supporting theory and systematic literature review on Graph Coloring Robust Optimization Model on electrical circuit problems. Our findings indicate that several studies have examined the problem of graph coloring. Keywords are used as a description of the research to be carried out and filter articles that are relevant to the research topic. After using the PRISMA method, seven articles were selected as materials to determine gaps and contributions. The results are obtained if the Robust Coloring Graph problem in electricity problems has not been widely studied by other researchers. Therefore, it is new for future research to develop Graph Coloring Optimization Model using Robust Optimization on electricity problems. The identified gaps are expected to motivate and become study material for future researchers.

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