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Pattern Generation for Three Dimensional Cutting Stock Problem

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ABSTRACT

We consider the problem of three-dimensional cutting of a large block that is to be cut into some small block pieces, each with a specific size and request. Pattern generation is an algorithm that has been used to determine cutting patterns in one-dimensional and two-dimensional problems. The purpose of this study is to modify the pattern generation algorithm so that it can be used in three-dimensional problems, and can determine the cutting pattern with the minimum possible cutting residue. The large block will be cut based on the length, width, and height. The rest of the cuts will be cut back if possible to minimize the rest. For three-dimensional problems, we consider the variant in which orthogonal rotation is allowed. By allowing the remainder of the initial cut to be rotated, the dimensions will have six permutations. The result of the calculation using the pattern generation algorithm for three-dimensional problems is that all possible cutting patterns are obtained but there are repetitive patterns because they suggest the same number of cuts.

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A. INTRODUCTION

Raw materials are essential in the production process. Generally, the raw materials can be paper, cloth, wood, etc. Then the raw material will be cut according to the length and width of the consumer's request. In the cutting process, there will usually be new pieces left over. The remainder of the cut is called cut loss and must be minimized so as not to suffer losses. Laying the cutting pattern is one of the causes of the cut loss resulting in inefficient use of raw materials. This problem is known as the Cutting Stock Problem (CSP), one of the problems in a linear program that aims to determine the cutting pattern of raw materials so that the results obtained are optimal with minimum cut loss (Macedo et al., 2008).

In the cutting problem, if the cutting is only done by paying attention to the length or width side, then this problem is called one-dimensional CSP (1DCSP). If the raw material is cut by paying attention to two sides, this problem is called two-dimensional CSP (2DCSP). In the CSP cutting process, it is known as guillotine and non-guillotine cutting. Guillotine cutting is a cutting pattern that starts from one side of the rectangle and then continues on the other side. Meanwhile, non-guillotine cutting is when the item's size does not cut from one side to the end of the opposite side (De Queiroz et al., 2012).

Gilmore and Gomory were the first to propose a solution procedure for 1DCSP with an exponential number of variables based on the concept of a truncation pattern (Gilmore & Gomory, 1963). They then offered two models based on the pattern for 2DCSP (Gilmore & Gomory, 1965). The problem for 1DCSP has been discussed in many models and algorithms, such as linear programming based heuristic algorithm based on full pattern model (Alvarez-Valdes et al., 2002) using dynamic programming for procedure column generation and heuristic column generation based on GRASP (Greedy Randomizes Adaptive Search Procedure) and Tabu Search (Beasley, 1985); the use of the arc-flow model for one-dimensional bin packing problems (Valério de Carvalho, 2002); modification of the branch and bound algorithm (N. Rodrigo et al., 2015); the least loser for the Bi-objective (Alfares & Alsawafy, 2019); mera-heuristics for one dimension (Ravelo et al., 2020); the greedy heuristic that governs (Cerqueira et al., 2021); pattern-set creation algorithm for a stock problem with cost setup (Cui et al., 2015); the arc flow and one-cut models can be used for one-dimensional CSP problems (Martinovic et al., 2018); a mathematical model to solve the one-dimensional stock reduction problem using continuous trim by comparing the Residual Greedy Rounding (RGR) and CUT models (Vishwakarma & Powar, 2021). Then for the one-dimensional two-stage CSP, there are several proposed methods to determine the solution, namely the first method based on intelligent enumeration of intermediaries using the dominance relation specified for the backpack problem, the second method with the branch and bound algorithm, and the third method is a hybrid algorithm (Muter & Sezer, 2018).

Further research is continued by adding dimensions, namely for the case of the two-dimensional cutting stock problem (2DCSP). Like the 1DCSP research, there are many examples of research on 2DCSP. For example are: the arc flow model used in 1DCSP was developed for the 2DCSP problem with guillotine cutting (Macedo et al., 2010); the full-pattern model and the staged-pattern model are compared, and the results show that it is one of the staged-pattern models, which has not been well studied, offering competitive theoretical and computational performance (Kwon et al., 2019); the column generation heuristic (Furini et al., 2012) and the mixed three integer programming model (Furini & Malaguti, 2013) for 2DCSP were used for the two-dimensional two-stage cutting stock problem with multiple stock sizes; pattern-based diving heuristics for the two-dimensional guillotine stock problem with food scraps (Clautiaux et al., 2019); and Gilmore and Gomory's knapsack function-based algorithm improvement for the issue of two-dimensional guillotine cutting without constraints (Russo et al., 2013).

The application of three-dimensional cutting stock problem (3DCSP) is comprehensive in the industry, but it is still rare for 3DCSP issues to be discussed in the literature (Bortfeldt & Wäscher, 2013). Some literature that discusses 3DCSP: an exact algorithm based on the Gilmore and Gomory algorithm extension and graph search (Hifi, 2004); dynamic programming algorithm extension and column generation (Cintra et al., 2008) and combined with reduced raster points (De Queiroz et al., 2012); truncation constraints with MILP formulations and a bottom-up algorithm (Martin et al., 2021) by extending from a two-dimensional problem (Martin et al., 2020), three-dimensional CSP to mattress production (Altın et al., 2019).

In determining the cutting pattern, there is a pattern generation algorithm used in 1DCSP (Suliman, 2001). The problem was developed from 1DCSP to 2DCSP cases (N. Rodrigo, 2012). Pattern generation algorithm is used by creating a modified branch and bound algorithm to find cutting patterns. A large number of paper-cutting designs can be determined using this algorithm. Then the pattern generation can be continued by adding a cutting location where in the sheet, the raw material is given in the Cartesian coordinate plane (W. Rodrigo et al., 2012). Many studies have been carried out on cutting stock problems for 1DCSP and 2DCSP, this research will provide 3DCSP cutting algorithms for guillotine cutting by modifying the pattern generation algorithm that has been successfully used on 1DCSP and 2DCSP.

B. METHODS

1. Cutting Stock Problem

Leonid Kantorovich is a Russian scientist who introduced the Cutting Stock Problem (CSP) in 1939. CSP is a problem related to cutting items that can be seen as placing the space occupied by small objects into the space of large objects and the rest is called cut loss (Karelahti, 2002). In general, CSP is a problem related to fulfilling the demand and optimizing cutting items on raw materials by minimizing cutting waste. In the cutting process, raw materials can be guillotine or non-guillotine cut. An illustration of guillotine and non-guillotine cut is given as shown in Figure 1.

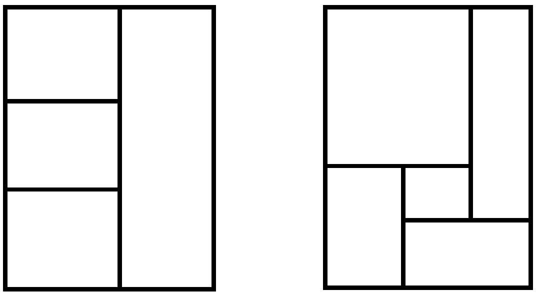

Figure 1. An illustration of guillotine and non-guillotine cut

2. Pattern Generation

The cutting pattern using Pattern Generation (PG) for CSP is formed by cutting raw materials that have length L_k, $k = 1, 2, \dots, h$. The raw material is then cut into m sizes with width w_i and length l_i where $(i = 1,2, \dots, m)$, to fulfill the demand. The purpose of this cut is to determine a cutting pattern that can minimize cut loss so that demand can be met (Suliman, 2001). CSP can be formulated as follows.

Minimize

$$z = \sum_{k=1}^{h}\sum_{j=1}^{n_k} c_{jk}x_{jk} + \sum_{i=1}^{m} w_i s_i$$

Subject to

$$\sum_{k=1}^{h}\sum_{j=1}^{n_k} a_{ijk}x_{jk} - s_i = l_i \quad \text{for all } i = 1,2, \dots, m,$$

$$x_{jk}, c_{jk}, s_i, a_{ijk} \geq 0 \quad \text{for all } i, j, k.$$

where:

a_{ijk} = Number of items with width w_i obtained according to the j^{th} pattern from k^{th} stock $(i = 1, 2, \dots, m; j = 1, 2, \dots, n_k; k = 1, 2, \dots, h)$,

x_{jk} = Length of the k^{th} stock cut according to the j^{th} pattern,

c_{jk} = Cutting loss from the k^{th} stock which is cut according to the j^{th} pattern,

s_i = Excess length which will result in a slice with width w_i.

The generation of feasible truncation patterns is obtained through a search tree. The demanded widths are reflected with the tree level. They are set in decreasing sequence with the biggest size at the 1st level and the smallest size is at the bottom level of the tree. The starting vertex of the 1st level represents the standard width of the k^{th}stock piece used to generate the pattern. Therefore, a separate search tree generates patterns tailored to each standard width.

The branch from the i^{th} level of the search tree represents the multiplication of the number of products with the width w_i obtained according to the j^{th} truncation pattern. The vertices from the second level to the m-th level represent the remaining width after fulfilling specific cuts from the previous $i-1$ branch. The final vertex of the search tree shows the remainder of the cut resulting from the different cutting patterns.

C. RESULT AND DISCUSSION

1. Three Dimensional Cutting Stock Problem

The three-dimensional cutting problem (3DCSP) is a cutting problem of a raw material B which has sides of length (L), width (W), and height (H). The raw material will be cut into m sizes with length l_i, width w_i, height h_i, and fulfill every request d_i, where $i = 1,2,\dots,m$. The small pieces that will be produced are called items. Each possible way of cutting raw materials is called a cutting pattern, as shown in Figure 2.

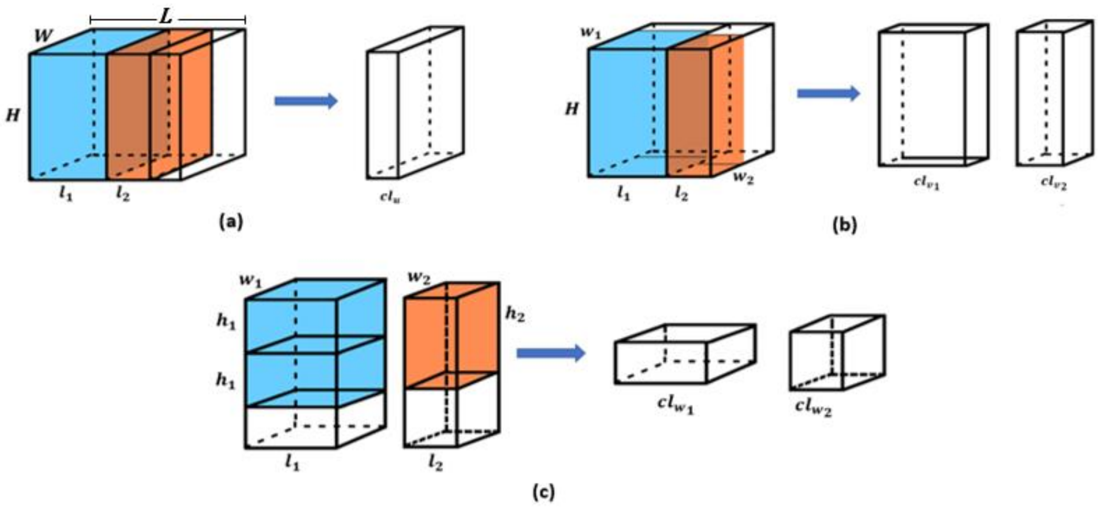

Figure 2. Examples of three-dimensional cutting patterns

For three-dimensional cutting problems, the first cut is based on the length of the raw material, the second cut is based on the width of the raw material, and the third cut is based on the height of the raw material. Each cut will have a remaining cut which is called a cut loss. The cut loss of each cut will be re-cut whenever possible (see Figure 2). For the 3DCSP problem, the variant in which orthogonal rotation is allowed is considered. By allowing the i^{th} item to be rotated, the dimensions have six permutations of $l_i \times w_i \times h_i$ (De Queiroz et al., 2012) as depicted as shown in Figure 3.

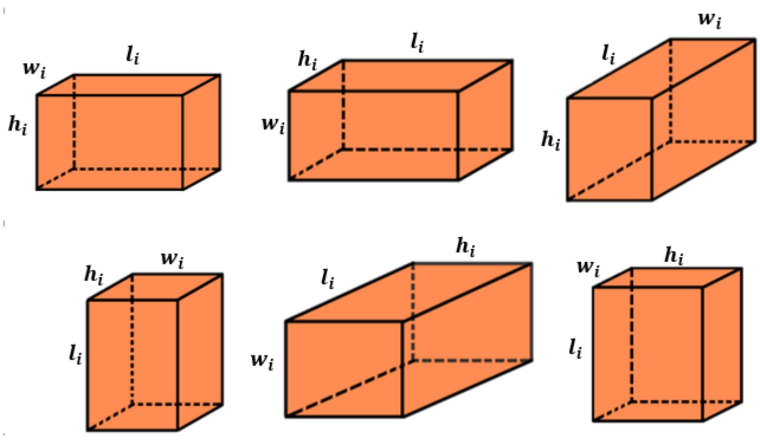

Figure 3. Six permutations of item

By developing Suliman's model (Suliman, 2001) ,the mathematical model for minimizing the residual truncation from 3DCSP by allowing rotation of items can be formulated as follows:

$$\min\left\{\sum_{k=1}^{q} Cl_k x_k\right\} \tag{1}$$

subject to:

$$\sum_{k=1}^{q} P_k x_k \geq d_i, \qquad i = 1,2,\dots,m$$

$$x_j, p_{ij} \geq 0$$

where P_k is the number of i^{th} item resulting from cutting, x_k is the number of raw materials cut, and Cl_k is the cut loss of all cutting stages.

2. Pattern Generation for 3DCSP

The steps in determining the cutting pattern for a three-dimensional problem are as follows:

a. Sorts the length of all items to be cut in descending order from largest to smallest ($l_1 > l_2 > \cdots > l_m$). The order for the width (w_i) and height (h_i) of the item is adjusted to the order of the predefined length sizes.

b. For each i^{th} item, the truncation pattern $j = 1$ is carried out following steps 3 to 6.

c. Specifies the number of cuts based on the length of the i-item based on the j-pattern (a_{ij}).

$$a_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{i-1} a_{zj} l_z}{l_i} \right\rfloor \tag{2}$$

d. If $a_{ij} > 0$, then determine the number of cuts based on the width of the i^{th} item based on the j^{th} pattern (b_{ij}).

$$b_{\text{ij}} = \left\lfloor \frac{W}{w_i} \right\rfloor \tag{3}$$

e. If $b_{ij} > 0$, then determine the number of cuts based on the height of the i^{th} item based on the j^{th} pattern (c_{ij}).

$$c_{\text{ij}} = \left\lfloor \frac{H}{h_i} \right\rfloor \tag{4}$$

f. Summing the pieces of i^{th} item that are cut based on the j^{th} pattern of raw materials.

$$p_{ij} = a_{ij} b_{ij} c_{ij} \tag{5}$$

g. Determine the cut loss of each remaining cutting based on the length, width, and height of the raw material.

1) Cut loss based on the length of the raw material

$$cl_u = \left(L - \sum_{i=1}^{m} a_{ij} l_i \right) \times W \times H \tag{6}$$

For $i = 1,2, \dots, m$ and $z = 1,2, \dots, m$, where z is the i^{th} item included in the cutting of raw materials.

Rotation-1:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq l_i$ and $W \geq w_i$ and $H \geq h_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{l_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{w_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{h_i} \right\rfloor. \tag{7}$$

Rotation-2:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq l_i$ and $W \geq h_i$ and $H \geq w_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{l_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{h_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{w_i} \right\rfloor. \tag{8}$$

Rotation-3:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq w_i$ and $W \geq l_i$ and $H \geq h_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{w_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{l_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{h_i} \right\rfloor. \tag{9}$$

Rotation-4:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq w_i$ and $W \geq h_i$ and $H \geq l_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{w_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{h_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{l_i} \right\rfloor. \tag{10}$$

Rotation-5:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq h_i$ and $W \geq l_i$ and $H \geq w_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{h_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{l_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{w_i} \right\rfloor. \tag{11}$$

Rotation-6:

If $\left(L - \sum_{z=1}^{m} a_{zj} l_z\right) \geq h_i$ and $W \geq w_i$ and $H \geq l_i$, then:

$$A_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{m} a_{zj} l_z}{h_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{W}{w_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H}{l_i} \right\rfloor. \tag{12}$$

For each rotation, if $A_{ij} > 0$, then update the cutting pattern and cut loss based on the length of the raw material as follows:

$$P_{u_{yij}} = p_{ij} + A_{ij} B_{ij} C_{ij}, \tag{13}$$

$$Cl_{u_{yij}} = \left(\left(L - \sum_{i=1}^{m} a_{ij} l_i\right) \times W \times H\right) - \left(A_{ij} B_{ij} C_{ij} \times l_{ij} w_{ij} h_{ij}\right). \tag{14}$$

2) Cut loss based on the width of the raw material

For $z = 1,2, \dots, m$

$$cl_{v_z} = a_{zj} l_z \times \left(W - b_{zj} w_z\right) \times H \tag{15}$$

For $i = 1,2, \dots, m$

Rotation-1:

If $l_z \geq l_i$ and $\left(W - b_{zj} w_z\right) \geq w_i$ and $H \geq h_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{l_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{w_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{h_i} \right\rfloor. \tag{16}$$

Rotation-2:

If $l_z \geq l_i$ and $\left(W - b_{zj} w_z\right) \geq h_i$ and $H \geq w_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{l_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{h_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{w_i} \right\rfloor. \tag{17}$$

Rotation-3:

If $l_z \geq w_i$ and $\left(W - b_{zj} w_z\right) \geq l_i$ and $H \geq h_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{w_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{l_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{h_i} \right\rfloor. \tag{18}$$

Rotation-4:

If $l_z \geq w_i$ and $\left(W - b_{zj} w_z\right) \geq h_i$ and $H \geq l_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{w_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{h_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{l_i} \right\rfloor. \tag{19}$$

Rotation-5:

If $l_z \geq h_i$ and $\left(W - b_{zj} w_z\right) \geq l_i$ and $H \geq w_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{h_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{l_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{w_i} \right\rfloor. \tag{20}$$

Rotation-6:

If $l_z \geq h_i$ and $\left(W - b_{zj} w_z\right) \geq w_i$ and $H \geq l_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{h_i} \right\rfloor, \quad B_{ij} = \left\lfloor \frac{W - b_{zj} w_z}{w_i} \right\rfloor, \quad C_{ij} = \left\lfloor \frac{H}{l_i} \right\rfloor. \tag{21}$$

For each rotation, if $A_{ij} > 0$, then update the cutting pattern and cut loss based on the width of the raw material as follows:

$$P_{v_{zyij}} = P_{u_{yij}} + A_{ij} B_{ij} C_{ij}, \tag{22}$$

$$Cl_{v_{zyij}} = \left(l_z \times \left(W - b_{zj} w_z\right) \times H\right) - \left(A_{ij} B_{ij} C_{ij} \times l_{ij} w_{ij} h_{ij}\right). \tag{23}$$

3) Cut loss based on the height of the raw material

For $z = 1,2,\dots,m$

$$cl_{w_z} = a_{zj}l_z \times b_{zj}w_z \times \left(H - c_{zj}h_z\right) \quad (24)$$

For $i = 1,2,\dots,m$

Rotation-1:

If $l_z \geq l_i$ and $w_z \geq w_i$ and $\left(H - c_{zj}h_z\right) \geq h_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{l_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{w_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{h_i} \right\rfloor. \quad (25)$$

Rotation-2:

If $l_z \geq l_i$ and $w_z \geq h_i$ and $\left(H - c_{zj}h_z\right) \geq w_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{l_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{h_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{w_i} \right\rfloor. \quad (26)$$

Rotation-3:

If $l_z \geq w_i$ and $w_z \geq l_i$ and $\left(H - c_{zj}h_z\right) \geq h_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{w_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{l_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{h_i} \right\rfloor. \quad (27)$$

Rotation-4:

If $l_z \geq w_i$ and $w_z \geq h_i$ and $\left(H - c_{zj}h_z\right) \geq l_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{w_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{h_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{l_i} \right\rfloor. \quad (28)$$

Rotation-5:

If $l_z \geq h_i$ and $w_z \geq l_i$ and $\left(H - c_{zj}h_z\right) \geq w_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{h_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{l_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{w_i} \right\rfloor. \quad (29)$$

Rotation-6:

If $l_z \geq h_i$ and $w_z \geq w_i$ and $\left(H - c_{zj}h_z\right) \geq l_i$, then

$$A_{ij} = \left\lfloor \frac{l_z}{h_i} \right\rfloor, \qquad B_{ij} = \left\lfloor \frac{w_z}{w_i} \right\rfloor, \qquad C_{ij} = \left\lfloor \frac{H - c_{zj}h_z}{l_i} \right\rfloor. \quad (30)$$

For each rotation, if $A_{ij} > 0$, then update the cutting pattern and cut loss based on the height of the raw material as follows:

$$P_{w_{zyij}} = P_{v_{zyij}} + A_{ij}B_{ij}C_{ij}, \quad (31)$$

$$Cl_{w_{zyij}} = \left(l_z \times w_z \times \left(H \times c_{zj}h_z\right)\right) - \left(A_{ij}B_{ij}C_{ij} \times l_{ij}w_{ij}h_{ij}\right). \quad (32)$$

All cutting patterns and cut losses that occur in cutting j^{th} pattern are as follows:

$$P_k = P_{w_{zyij}}, \quad (33)$$

$$Cl_k = Cl_{uzyij} + Cl_{vzyij} + Cl_{wzyij}. \quad (34)$$

h. Set index level $r = m - 1$.

i. If the vertex has a value equal to zero ($a_{rj} = 0$) go to step 12. Otherwise generate a new column $j = j + 1$.

j. A new pattern will be generated by following the conditions:

1) If $a_{rj} \geq b_{rj}$ and $a_{rj} \geq c_{rj}$

For $z = 1, 2, \dots, r - 1$

$$b_{zj} = b_{z\,j-1}, \qquad c_{zj} = c_{z\,j-1}. \tag{35}$$

For $z = r$

$$a_{zj} = a_{z\,j-1} - 1, \qquad b_{zj} = \left\lfloor \frac{W}{w_z} \right\rfloor, \qquad c_{zj} = \left\lfloor \frac{H}{h_z} \right\rfloor. \tag{36}$$

For $\boldsymbol{z = r + 1, \dots, m}$, calculate $\boldsymbol{a_{zj}}$, $\boldsymbol{b_{zj}}$, and $\boldsymbol{c_{zj}}$ using Equations (2), (3), and (4). Go back to Step 6.

2) If $a_{rj} < b_{rj}$ and $b_{rj} \geq c_{rj}$

For $z = 1, 2, \dots, r - 1$, calculate a_{zj}, b_{zj}, and c_{zj} using Equations (35).

For $z = r$

$$a_{zj} = a_{z\,j-1}, \qquad b_{zj} = b_{z\,j-1} - 1, \qquad c_{zj} = \left\lfloor \frac{H}{h_z} \right\rfloor. \tag{37}$$

For $\boldsymbol{z = r + 1, \dots, m}$, calculate $\boldsymbol{a_{zj}}$, $\boldsymbol{b_{zj}}$, and $\boldsymbol{c_{zj}}$ using Equations (2), (3), and (4). Go back to Step 6.

3) If $a_{rj} < c_{rj}$ and $b_{rj} < c_{rj}$

For $z = 1, 2, \dots, r - 1$, calculate a_{zj}, b_{zj}, and c_{zj} using Equations (35).

For $z = r$

$$a_{zj} = a_{z\,j-1}, \qquad b_{zj} = b_{z\,j-1}, \qquad c_{zj} = c_{z\,j-1} - 1.$$

For $\boldsymbol{z = r + 1, \dots, m}$, calculate $\boldsymbol{a_{zj}}$, $\boldsymbol{b_{zj}}$, and $\boldsymbol{c_{zj}}$ using Equations (2), (3), and (4). Go back to Step 6.

k. Set index level $r = r - 1$, if $r > 0$ repeat step 9. Otherwise, stop.

The following is an illustration related to the three-dimensional cutting stock problem so that the total cut loss produced is as minimal as possible. The raw materials used have a length (L) of 100 cm, a width (W) of 65 cm, and a height (H) of 40 cm. There are two types of items produced, namely 15 cm × 10 cm × 18 cm and 30 cm × 15 cm × 20 cm which has a demand of 65 and 34 pieces, respectively. By following the Pattern Generation algorithm for the three-dimensional cutting stock problem, the following calculations are carried out:

$$l_1 = 30, \qquad w_1 = 15, \qquad h_1 = 20,$$
$$l_2 = 15, \qquad w_2 = 10, \qquad h_2 = 18.$$

For $\boldsymbol{i = 1, 2}$ and $\boldsymbol{j = 1}$, then

$$a_{11} = 3, \qquad b_{11} = 4, \qquad c_{11} = 2,$$
$$a_{21} = 0, \qquad b_{21} = 0, \qquad c_{21} = 0.$$

The number of item 1 obtained from cutting pattern 1 is

$$p_{11} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

From the cuts that have been made, the cut loss appears from the cuts based on the length, width, and height of the raw material. The three cut losses are cut back if possible by changing the cut loss position following step 7 as follows:

1) Cut loss based on the length of the raw material

$$2)\ cl_u = \left(100 - (3 \times 30)\right) \times 65 \times 40 = 26.000 \text{ cm}^3$$

In the cut loss result of cutting based on length, there is no cut back for item $\boldsymbol{i} = \mathbf{1}$ for all possible rotations because it does not meet the specified conditions. Meanwhile, for item $\boldsymbol{i} = \mathbf{2}$, it can be cut by changing the item to rotations 3 and 4.

Rotation-3:

$A_{21} = 1,$ $B_{21} = 4,$ $C_{21} = 2.$

Because $\boldsymbol{A_{21}} > \mathbf{0}$, the value of $\boldsymbol{Cl_{u321}}$ and $\boldsymbol{P_{321}}$ are updated to

$Cl_{u321} = 4.400 \text{ cm}^3,$ $P_{u321} = \begin{bmatrix}24\\0\end{bmatrix} + \begin{bmatrix}0\\8\end{bmatrix} = \begin{bmatrix}24\\8\end{bmatrix}.$

Rotation-4:

$A_{21} = 1,$ $B_{21} = 3,$ $C_{21} = 2.$

Because $\boldsymbol{A_{21}} > \mathbf{0}$, the value of $\boldsymbol{Cl_{u421}}$ and $\boldsymbol{P_{421}}$ are updated to

$Cl_{u421} = 9.800 \text{ cm}^3,$ $P_{u421} = \begin{bmatrix}24\\0\end{bmatrix} + \begin{bmatrix}0\\6\end{bmatrix} = \begin{bmatrix}24\\6\end{bmatrix}.$

2) Cut loss based on the width of the raw material

For the cut loss of cutting by width to item 1:

$$cl_{v_1} = 90 \times \left(65 - (4 \times 15)\right) \times 40 = 18.000 \text{ cm}^3$$

For $\boldsymbol{i} = \mathbf{1}$ and $\mathbf{2}$, $\boldsymbol{A_{11}} = \mathbf{0}, \boldsymbol{B_{11}} = \mathbf{0}, \boldsymbol{C_{11}} = \mathbf{0}$ and $\boldsymbol{A_{21}} = \mathbf{0}, \boldsymbol{B_{21}} = \mathbf{0}, \boldsymbol{C_{21}} = \mathbf{0}$ because it is not eligible to cut rotation 1 to rotation 6. There is no cut loss for width-based cut loss, so the values of $\boldsymbol{Cl_{v1}}$ and $\boldsymbol{P_{v_{zyij}}}$ can be updated to

$$Cl_{v_{zyij}} = 18.000 \text{ cm}^3,$$

$P_{v1321} = \begin{bmatrix}24\\8\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}24\\8\end{bmatrix},$ $P_{v1421} = \begin{bmatrix}24\\6\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}24\\6\end{bmatrix}.$

3) Cut loss based on the height of the raw material

For the cut loss of cutting by height to item 1:

$$cl_{w1} = (3 \times 30) \times (4 \times 15) \times \left(40 - (2 \times 20)\right) = 0$$

Since the cut loss of cutting by height to item-1 is 0, no remaining cuts.

$$Cl_{w_{zyij}} = 0,$$

$P_{w1321} = P_1 = \begin{bmatrix}24\\8\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}24\\8\end{bmatrix},$ $P_{w1421} = P_2 = \begin{bmatrix}24\\6\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}24\\6\end{bmatrix}.$

After obtaining the cutting pattern for pattern 1, proceed to step 8 by setting a new $\boldsymbol{r}$ level index $\boldsymbol{r} = \mathbf{1}$. Since $\boldsymbol{r} > \mathbf{0}$ then generate a new column $\boldsymbol{j} = \mathbf{2}$. A new cutting pattern is generated by following the following conditions:

$$a_{11} < b_{11} \text{ dan } b_{11} \geq c_{11}$$

$a_{12} = 3,$ $b_{12} = 3,$ $c_{12} = 2,$

$a_{22} = 0,$ $b_{22} = 0,$ $c_{22} = 0.$

By following the steps of the 3DCSP Pattern Generation algorithm, 261 possible cutting patterns can be generated. However, from the 261 data generated, patterns that have many pieces of item-1 and item-2 will be ignored. Thus, the cutting pattern obtained from the Pattern

Generation algorithm produces 90 patterns for raw materials measuring 100 cm × 65 cm × 40 cm into the required items. The following is one of the cutting patterns as shown in Figure 4.

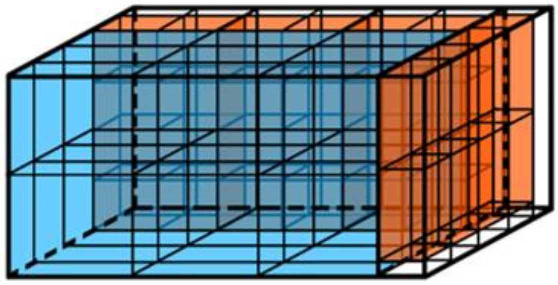

Figure 4. Cutting pattern base on 5^{th} pattern

The cutting patterns are then used to form the model (1) and solved using the LINGO 19.0 program to get the optimal solution. With the help of the LINGO 19.0 program, the optimal solution is $\boldsymbol{Z_{min} = 23,56 \times 10^3}$ $\mathbf{cm^3}$ with $\boldsymbol{x_5}$ = 2,0312. The decision variables obtained produce non-integer numbers so that they will be rounded using the Branch and Bound method. By rounding the solution, the optimal result is $\boldsymbol{Z_{min} = 34,8 \times 10^3}$ $\mathbf{cm^3}$with $\boldsymbol{x_5 = 3}$ where $\boldsymbol{x_5}$ gives a cutting pattern by producing 18 items-1 and 32 items-2. This means that to meet the demand for item-1 and item-2, it can be done with as many as raw materials with the 5th cutting pattern three times and the remaining cutting produced is $\boldsymbol{34,8 \times 10^3}$ $\mathbf{cm^3}$.

From these results, we can obtain a cutting pattern based on a pattern generation algorithm for three dimensions. In constrast to the pattern generation made from previous studies, ini this study it was made according to paying attention to each remaining cutting so as to allow the cuts obstained to be very minimal.

D. CONCLUSION AND SUGGESTIONS

The pattern generation algorithm for the 3DCSP problem is an algorithm development from the 1DCSP and 2DCSP cases. In three-dimensional problems, the resulting cut loss is obtained from cutting the length, width, and height of the raw material. The cut loss is cut back by considering the rotation of the initial position of the item so that there are six permutations performed. Due to the consideration of item rotation in cutting cut loss, many patterns emerge from the algorithm. However, from all these patterns there is the same pattern where the recommended number of i^{th} item is the same but with different cutting positions.

Further research is recommended to make a program from the pattern generation algorithm so that it can produce cutting patterns more quickly and can avoid the occurrence of the same patterns. It is also advisable to use higher dimensions with different forms of raw materials and items.

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