Students' Intuitive Thinking Process in Solving Geometry Tasks from the Van Hiele Level

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ABSTRACT
Thinking is a cognitive way of generating ideas for problem-solving decision-making strategies. This study aims to analyze and describe intuitive thinking processes in solving students' mathematical assignments about geometry based on Van Hiele's theory. The method used is descriptive qualitative. The research subjects were students of class VIIA. The research location is one of the Madrasah Tsanawiyah in Pasuruan Regency. Credibility uses source data collection techniques using the VHGT test to determine the level of the subject, geometry problem tests, document studies, and interviews. The main instruments are researchers and supporters of geometry tests and interviews. The results of this study indicate that students who think intuitively through catalytic inference are obtained spontaneously and suddenly in completing the math tasks they face without using prior knowledge, and intuitively appear globally and use shortcuts. Whereas students who think intuitively with common sense are obtained directly and directly using the steps to complete mathematical tasks neatly and neatly, intuitively, the sequence of completing tasks appears by their experience and knowledge. Based on Van Hiele's level, students who think intuitively with catalytic inference are included in level 0 (visualization), and students who think intuitively with common sense are included in level 1 (analysis). Students who think intuitively with common sense can directly and immediately use the steps to complete math tasks neatly and neatly, intuitively appearing in the sequence of completing tasks according to their experience and knowledge. Based on the Van Hiele level, students who think intuitively with catalytic inference are included in level 0 (visualization).

Keywords:
Solving Geometry; Intuitive Thinking Process; Van Hiele level.

A. INTRODUCTION
Mathematics as a basic science is essential in educating students because it can foster the necessary reasoning abilities in developing science and technology (Csapó, 2022; Wijaya et al., 2022; Yorkovsky & Levenberg, 2022). The role of geometry in mathematics is decisive; Geometry not only builds ways of thinking but also dramatically influences other subject matter in mathematics (Abdussakir, 2012; Novak & Tassell, 2017).
Studying mathematics must understand the mathematical concepts that are being explored and use these concepts adequately and appropriately when solving the problems encountered (Clem et al., 2021; Saragih & Habeahan, 2014). Teachers can develop learning methods, strategies, and models to improve basic geometry skills and student learning designs, especially in geometry subject matter, to help reduce misconceptions that occur in students (Nurani et al., 2016; Suharto & Csapó, 2022; Yang & Lu, 2021).

Thinking is the process of producing behavior directly in a solution. Someone implements his ideas in solving mathematical problems and is processed to have thinking activities whose processes are divided into three types: intuitive, analytical, and procedural (Abdillah et al., 2018; Yusuf & Sutiarso, 2017). One of the materials that are often encountered in everyday life in mathematics is geometry. Based on Serin (2018); Siyepu & Mtonjeni (2014), geometry needs to be taught because this subject is the only field of mathematics that can connect mathematics with natural physical forms, allows mathematical ideas to be visualized, and can provide examples that are not just one about mathematical systems (Serin, 2018; Siyepu & Mtonjeni, 2014).

The purpose of learning geometry is for students to gain confidence in their mathematical abilities, become good solution seekers, communicate mathematically, and reason mathematically (Abu & Abidin, 2013; Muhassanah et al., 2014; Özerem, 2012). Meanwhile, Seah (2015) states that the purpose of learning geometry is to develop the ability to think logically, develop intuition, instill knowledge to connect with other material and be able to read and interpret opinions mathematically (Seah, 2015).

To carry out geometry learning according to students’ thinking levels, it is necessary first to identify the students’ geometric thinking levels at school (Jabar & Noor, 2015). One of the well-known methods for identifying the level of geometric thinking was put forward by Van Hiele. Van Hiele divides the five levels of students’ thinking in learning geometry (Jabar & Noor, 2015; Muhassanah & Mulyatna, 2020; Putri & Nopriana, 2019): 1) the visualization stage, in this stage students recognize images through their appearance alone by comparing familiar forms. At this level, students make decisions based solely on perception, 2) In the analysis stage, students see images as a collection of their properties. They can recognize and name the properties of geometric shapes. When describing an object, students who operate at this stage may know all the properties of geometric shapes but cannot distinguish which properties are needed and which are sufficient to describe the object, 3) At the abstraction stage, at this stage students can understand the relationship between properties and between images. Students can create meaningful definitions and provide informal arguments to justify their reasoning. Logical implications include a square, a type of rectangle, and 4) the deduction stage; at this stage, students can build evidence, understand roles, axioms, and definitions, and know the meaning of necessary and sufficient conditions. Students can also build mathematical evidence as found in the material in high school, 5) rigor stage, at this stage, students can understand aspects of formal deduction, such as forming and comparing mathematical systems.

Based on its development, intuitive thinking is a cognitive process of the subject that generates ideas to make decisions that are considered correct to produce spontaneous answers in problem-solving and hypotheses to develop further knowledge (Ethics et al., 2016). Intuition is a unique process of conceptualizing information with conscious and subconscious
information processing systems. In this case, intuition is needed in learning geometry because subconscious processing allows them to learn geometry from their own experience. Intuition has the main characteristic, namely speed in processing information that cannot be explained through certain stages. Observable intuition is a response that is often correct even though it seems that it takes no time to process it, and no visible effort is put into the thought process (Kamandoko & Suherman, 2017; Muniri, 2018).

Based on several research results by Abdillah et al. (2018); Muniri (2018), it was found that intuitive thinking is a way of thinking in solving mathematical problems where students sometimes experience pauses in thinking so that they spontaneously get answers based on assumptions from facts, even though this intuition is not absolute and not always trustworthy (Abdillah et al., 2018; Muniri, 2018). The results of the study revealed that students who showed intuitive thinking after reading the questions once, then continued with analytical or procedural thinking, then in analyzing situations, sometimes students got stuck so that intuitive thinking emerged then continued with analytical or procedural thinking (Abdillah et al., 2018; Muniri, 2018). Research on intuition only guides mathematical activities, even though the results of activities based on intuition do not necessarily get the right solution. Sa'o (2016) shows that the intuitive thinking process of junior high school students has the same basis but has different answers to questions (Sa'o, 2016).

This study aims to analyze and describe students' intuitive thinking processes in solving mathematical problems. This study focuses on exploring the occurrence of students' intuition in solving geometric tasks based on studies of intuitive thinking that several experts have studied. In connection with the description above to reveal more about students' intuitive thinking processes in completing a geometry task based on Van Hiele's level of geometric thinking, the problem studied in this study is to analyze and describe "how students' intuitive thinking processes in solving geometric tasks in terms of the level of geometric thinking Van Hiele."

B. METHOD

The approach used in this research is a qualitative approach with a descriptive research type. The study was conducted at a Madrasah Tsanawiyah in Pasuruan District, East Java, Indonesia. Selection of research subjects by leading the Van Hiele Geometry Test (VHGT) geometric thinking test to determine the level of each problem to be studied, followed by working on geometry test questions and conducting interviews. The test questions have a validation test beforehand. The validity of the test questions has been declared.

Furthermore, the selection of research subjects is based on predetermined criteria. The criteria that have been determined are the provision of standard Van Hiele geometry test questions regarding the Catalytic Inference character indicator. The subject answers questions directly, suddenly, or using formula and algorithm skills. Meanwhile, the character of common sense is the subject of solving problems now, suddenly, or immediately, using straightforward steps or rules, suddenly or immediately, using shortcuts, short answers, not in detail, and not giving logical reasons. The character of the Power of Synthesis is a subject who answers rules based on his knowledge and experience.
Four subjects were selected for deepening the completion of students' mathematics assignments, two subjects each at level 0 (visualization) with codes V1 and V2, while two questions at level 1 (analysis) were coded A1 and A2. Research subjects are selected based on the highest score at each level. The topics in this study were then given a mathematical problem-solving test on flat-sided flat shapes, then analyzed according to the Van Hiele geometric thinking level.

The data in this study were obtained from tests of completing geometry assignments and recordings of semi-structured interviews to dig deeper into the process of generating ideas in completing given geometry tasks and collected with recording aids. The primary data source in this study was class VIIIA students, while the additional data source was the mathematics teacher in the research class. Data collection techniques in this study were standard VHGT tests and written tests, documentation, and interviews. Documentation in the form of test results reports is used to see the emergence of intuitive thinking in solving geometric tasks faced by the subject. Then the interview was conducted after the written test was completed.

The instruments in this study consisted of two instruments, namely the main instrument and supporting instruments. The main instrument in this study is the researcher, while the supporting instruments are geometry problem-solving tests, semi-structured interviews, and tape recorders. This study uses data credibility as seen from the validity of the data using source triangulation. Source triangulation to test the credibility of the data is done by checking the data obtained from various sources. Data from several sources are then described, which are the same, different, and specific views of these sources.

Data analysis in this study analyzed the subject’s answer data from the question sheets, and interviews carried out with the data reduction step. Presentation of data is done descriptively, which means digging in-depth information and describing it according to the results obtained. The data is described sequentially and clearly so that a conclusion can be drawn. Then draw conclusions intended to explain the meaning of the data presented and follow the formulation of this research problem.

C. RESULTS AND DISCUSSION

1. Results

Using the data obtained in this study, researchers can analyze students’ intuitive thinking through written tests and compare them with interview results to strengthen their credibility. Following are the analysis results of four subjects with different levels at Van Hiele’s.

a. Subject V1

Based on the research results with a series of tests that aim to analyze the intuitive thinking picture generated by the subject, as shown in Figure 1.
The picture above shows that in understanding V1’s task directly when looking at the picture. The subject identified the shapes contained in the problem by calculating the total length of the ribs in each shape. Where the first form is \(12 \times 8 = 96\), the second \((4 \times 15) + (4 \times 9) + (4 \times 8) = 128\), and the third \((3 \times 10) + (6 \times 9) = 84\). This understanding shows that the problem has three shapes, so the subject with this shortcut appears by counting the ribs first. V1 understands problems by intuitively identifying shapes from shapes think by calculating the rib length using its way without prior knowledge. The interview results confirmed this.

Researchers: Try to explain how you would answer the question.
V1: Please read the question first; then, I will immediately calculate the side length of each wake.
Researchers: What did you think about when calculating the rib length?
V1: I quickly added up all the rib lengths of the three shapes.
Researchers: Have you ever worked on questions like this?
V1: I do not think so.
Researchers: Then how do you answer the question so that you get an answer like that?
V1: Yes, without thinking much, I just answered as I thought.

However, V1 uses a shortcut to solve the problem. It was found that V1, in understanding the questions. He did not begin by writing down important things, such as what was known and what was asked in the questions on the answer sheet. This description shows that in completing the V1 task, using intuitive thinking on the character of catalytic inference, with the subject indicator answering questions directly, suddenly using shortcuts, not going into detail, and not giving reasons for answering questions. So V1, in solving the problem in this problem, has fulfilled all the indicators of intuitive thinking based on catalytic inference.

b. V2 subject

The research results about geometry are to obtain information about the intuitive picture of the subject’s thoughts. The process of completing tasks will be different at each level in understanding and fulfilling learning, as shown in Figure 2.
Based on the picture above, it can be seen that in understanding question V2 directly, what comes to mind when looking at the picture, the subject identifies the shapes in the problem by calculating the total length of the ribs in each shape. Each shape has side length $s^I = 12 \times 8 = 96$, $s^II = (4 \times 15) + (4 \times 9) + (4 \times 8) = 128$, and $s^III = (3 \times 10) + (6 \times 9) = 84$. Understanding the problem in this way shows that questions have three different forms, then questions with this shortcut appear by first calculating the ribs directly on the answer sheet. The results of the subject analysis show that the results of solving V2 use problem-solving with short answers. V2 solves this problem directly without using any prior information or knowledge. Intuitive thinking V2 appears after reading the questions and briefly writing down the answers. In this case, it is the power required by the interview.

Researcher: Explain how you can do the problem!
V2: I read the questions first and immediately calculated the length of the edges of each shape because what I was thinking about was how to calculate the edges first.

Researcher: What did you think about when calculating the rib length?
V2: I will add up all the side lengths of the three shapes and calculate the remaining wire.

Researcher: Have you ever worked on a problem similar to this one?
V2: I do not think so.

Researcher: However, how come you can do it right away?
V2: Yes, that is what was on my mind right then and there.

Based on the explanation above, it shows that the results of completing task V2 use an intuitive way of thinking in the character of catalytic inference, with indicators that the subject answers questions directly, suddenly uses shortcuts, does not go into detail, and does not give reasons to answer questions. Answers to questions. So, V2, in solving this problem, has fulfilled all indicators of intuitive thinking based on catalytic inference.

c. A1 subject
The results of subject A1 can be obtained to illustrate the subject's intuitive thinking in solving problems, as shown in Figure 3.
In this understanding, subject A1 knows the things contained in the problem and can write them sequentially. Subjects immediately get a solution to how to solve existing problems. The results of the picture show that A1 in understanding straightforward questions when mentioning the important thing, namely the cube’s side length is 14 cm and the height of the cone is 18 cm, and what is asked is what is the volume of the shape? Subjects can identify essential things in the picture of waking up suddenly by referring to their knowledge by writing down the results of their thoughts on the answer sheet so that the subject has included fulfilling indicators of intuitive thinking, namely understanding questions directly by involving the knowledge and experience they have, apart from using a buffer the steps in the completion process in the proper order. Live interviews with the subject reinforce this.

Researcher : What do you understand from these questions?
A1 : There are images of overlapping shapes, and the side lengths and heights are known

Researcher : What's the problem in the problem?
A1 : It asks for the volume of two forms

Researcher : Is there any other information you got?
A1 : There calculates the volume of each shape to get the final answer.

Researcher : Have you studied a problem similar to this one?
A1 : Yes, I have, so it immediately occurred to me to use the same method I used when working on the problems.

Subject A1 obtained problem-solving solutions based on previous experience and knowledge. Based on the picture, the subject can explain that the problem is the wake volume. The problem explains that there are known to be two shapes, namely a cube and a cone, whose respective books must be calculated so that the final answer is the sum of the volumes of the two shapes. Subject A1 solves the problem directly and suddenly in intuitive thinking by using steps based on his knowledge. So in solving the problem, the subject has met the indicators of intuitive thinking in common sense.

d. Subject A2

The results obtained by subject A2 in this study were from written tests and interviews. The results will describe the intuitive way of thinking produced by the subject, as shown in Figure 4.
The process of understanding subject A2 shows that they can understand the problem nicely by writing down what is known (side length 14 cm, cone height 18 cm) and what is asked (what is the volume of the shape? Moreover, the subject can correctly determine things to solve the problem. There is direct A2 understanding when mentioning the essential things contained in the problem previously owned by writing answers directly on the worksheet.

Researcher : What understanding did you gain from this question?
A2 : It is known that there are two essential things, namely, two figures marked by the side’s length and the cone’s height.

Researcher : Can you tell what the problem is in the problem above?
A2 : Find the volume

Researcher : Is there any other information you can get from this question?
A2 : Yes, you can find the cone’s diameter first, then calculate the volume of the cone.

Researcher : Have you studied issues similar to this one?
A2 : Yes, so I can go straight to it like before

Based on the results obtained, subject A2 can find solutions to problems that the steps and results of the calculations have obtained. Subject A2 gets solutions in solving problems based on the knowledge they already have. Based on the picture above, the subject explained that the problem was finding the volume of the shape. When asked for an explanation, the subject indicated that it was known that two spatial figures had rib length and height. Then the final answer is the sum of the volumes of the two figures. A2 solves solutions instantly and spontaneously in intuitive thinking, using steps based on prior knowledge. This course has met common sense intuitive thinking indicators in solving problems.

2. Discussion

When faced with a mathematical problem, they will most likely try to think about solving it or relax in dealing with it. Thinking is a mental activity that involves knowledge or experience in one’s mind. At the same time, thinking is to find a solution to a problem. Based on Schafer (2018), The characteristics of intuitive thinking are that there are several characteristics of mental activity that are considered a form of intuition, namely direct and self-evident, feel no
need to give a formal explanation, and think that other people can understand it (Schafer, 2018).

The way of thinking of students at level 0 (visualization) requires a relatively long time because students can understand the question sentences well, know what is being asked, and understand the shape of the picture in the problem. However, in this case, students are not perfect in determining the steps to solving existing problems. This is shown in Hiele’s theory, namely, at level 0, the visual object of students’ visual thinking is still dominated by shapes and what that shape looks like (Yudianto et al., 2018). According to the opinion of Sofyana & Budiarto (2013), students at level 0 can name and recognize shapes with a profile view but do not specifically identify the properties of shapes (Sofyana & Budiarto, 2013).

Students do intuitive thinking at level 0 on written answers. Then these students are included in the component indicators of intuitive thinking character components in catalytic inference, namely understanding problems globally and using shortcuts. Meanwhile, Sa’o (2016) then understands the problem in the form of sudden ideas as a decision-making strategy. It shows that the process of intuitive thinking on the character of catalytic inference is suitable and appropriate when associated with the level of Van Hiele’s geometric thinking at level 0 (Sa’o, 2016).

While the way of thinking of level 1 students (analysis) in understanding and solving questions has been done properly and correctly, so this subject is included in the Van Hiele geometric thinking level at level 1. It is shown in Hiele’s theory that students begin to recognize and apply geometric ideas, adequately describe various properties, and can identify images as part of a larger picture (Abdullah & Zakaria, 2013; Clements et al., 2018; Yi et al., 2020). Opinion from Musa (2016), namely the thinking process of students at level 1 begin to understand geometric shapes based on their characteristics and can mention the properties of geometric shapes, even though they do not know other parts of the shapes (Musa, 2016).

Think intuitively in solving the questions that the student does from the written answers. Then the student is included in the indicator of the character component of common sense intuitive thinking; that is, students can sort solutions according to their experience and knowledge and can understand some of the statements in the problem at hand. Suppose it is related to Fischbein’s study than in solving problems. In that case, the intuition that arises spontaneously can generate spontaneous ideas and is obtained from previously owned knowledge constructs, while those related to Calabretta et al. (2016) in solving intuition problems, his thinking is more oriented toward a strategy for determining the steps for solving it. Shows that the intuitive way of thinking of the components of the common sense character follows the Van Hiele geometric thinking level at level 1. Where the character of intuitive thinking is common sense, students can solve problems neatly and neatly and use their knowledge (Calabretta et al., 2016). It shows that the intuitive way of thinking of the components of common sense character follows the Van Hiele geometric thinking level.

**D. CONCLUSIONS AND SUGGESTIONS**

Based on the results obtained in this study that students at level 0 (visualization) use intuitive thinking by utilizing catalytic inference. Intuitive thinking is accepted spontaneously and suddenly in solving problems, not using sequential steps and without basing the knowledge
possessed by writing answers directly on the existing answer sheet. Where intuitive comes up globally and uses shortcuts. Meanwhile, students at level 1 (analysis) use intuitive thinking in common sense. Intuitive thinking is obtained directly and immediately by using neat problem-solving steps and based on knowledge by writing the results of spontaneous thinking on the answer sheet provided. The advice that can be given in this study is that future research should use sequential intuitive thinking steps and learning media in the form of Geogebra software or software that supports learning geometry on Van Hiele's theory.

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