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ANALYTICAL CALCULATION FOR LAMINAR AIR FLOW MOMENTUM TRANSPORT IN UNDERGROUND MINE TUNNEL USING NEWTONIAN EQUATIONS OF MOTION

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ABSTRAK

Dalam kasus tertentu terdapat terowongan percabangan tambang bawah tanah yang tidak teralirkan udara secara mekanis tetapi udara tetap mengalir secara alami teridentifikasi beraliran laminar. Studi ini memodelkan perpindahan momentum aliran laminar udara terowongan dengan dimensi rectangular atau Cartesian untuk memvisualisasikan profil distribusi kecepatan linier fluida dan profil tegangan geser fluida. Metode pemodelan menggunakan perhitungan analitik matematika dari persamaan gerak aliran fluida Newtonian untuk rejim laminar Navier-Stokes. Beberapa data yang dikumpulkan adalah dimensi terowongan, kecepatan linier udara, dan temperatur. Beberapa sifat fisik udara dikutip dari literatur. Hasil pengukuran kecepatan udara rata-rata diinputkan ke dalam model empiris untuk mendapatkan profil koordinat dua dimensi (x,z) untuk distribusi tegangan geser dan kecepatan udara yang mendekati keadaan sebenarnya untuk aliran laminar pada dimensi Cartesian. Hasil penelitian didapatkan pressure drop aliran sebesar 2,2 x 10^{-5} Pa, aliran mengandalkan perpindahan molekular bukan perpindahan secara konveksi paksa karena dilihat dari profil sebaran tegangan geser yang terlalu kecil dan laju alir sebesar 0,32 m^3/s tidak memenuhi fungsi ventilasi tambang bawah.

Kata kunci: perpindahan momentum; aliran laminar; fluida newtonian; ventilasi tambang.

ABSTRACT

In certain case there is a mine tunnel branch that is not mechanically ventilated but air still flow naturally identified as laminar flow. This study modelled the momentum transport of the tunnel air laminar flow with rectangular or Cartesian dimensions to visualize the linear velocity distribution profile of the fluid and the fluid shear stress profile. The modelling method uses analytical mathematical calculations of the Newtonian fluid flow equations of motion for the laminar regime of Navier-Stokes. Some of the data collected are tunnel dimensions, air linear velocity, and temperature. Some physical properties of air are cited from literature. The results of the average air velocity measurement are input into the empirical model to obtain two-dimensional coordinate profiles (x,z) for the shear stress distribution and air velocity that are close to the actual situation for laminar flow in the Cartesian dimension. The results of the study obtained a pressure drop of the flow was 2,2 x 10^{-5} Pa, the flow using molecular transport instead of forced convection transport because seen from the shear stress distribution profile was too small and the flow rate was 0,32 m^3/s did not fulfill the underground mine ventilation function.

Keywords: momentum transport; laminar flow; newtonian fluid; mine ventilation.

INTRODUCTION

Mine ventilation function is to circulate air to underground mining (Widiatmojo, Widodo, & Sasaki, 2021) for the purposes of breathing for worker and to create a comfortable and productive work atmosphere (Kanam & Ahmed, 2021), Ventilation is a vital requirement in underground mining activities for air supply which is distributed through the access tunnel using compressor or blower equipment. There are three air transport systems through the tunnel (Obracaj, Korzec, & Deszcz, 2021). First, the exhaust system causes the air in the mine to be sucked in from one of the openings, as a result, the air enters the other openings so that air exchange occurs. Second, the forcing system, air is pushed using a blower from the surface into the mine. Third, the overlapping system or a combination between forcing and exhaust systems. The success of the mine ventilation system is that air needs are met, air exchange in the mine is able to create a comfortable work environment.

This study empirically models the phenomenon of air momentum transfer for the case of mine ventilation air flow with a laminar regime on rectangular flow dimensions according to Cartesian dimensions. The aim of the modelling is to get an overview of the profile of air velocity and shear stress due to air transport and to prove that laminar fluid flow does not fulfill the ventilation function of underground mines when it is examined for several parameters in the model. Momentum transfer modelling uses analytical calculations of the Newtonian equations of motion (Laloui & Rotta Loria, 2020; Rapp, 2017) from Navier-Stokes equation. The equation corresponds to the flow conditions for the laminar regime (Imberger, 2013) and symmetrical Cartesian dimensions (R. Byron Bird, Stewart, & Edwin N. Lightfoot, 2002). The amount of air flow rate that enters the mine is a form of fluid transfer rate. The object of this case is the mine ventilation flow suction system, in a certain section of the tunnel, air flows at a small velocity. Although in practice in the mine ventilation field turbulent flow like the simulation by (Obracaj et al., 2021; Sasmito, Birgersson, Ly, & Mujumdar, 2013) and the research of (Widiatmojo et al., 2021).

Identification of laminar flow in tunnel sections where the air delivery sub-system is no longer functional, such as in the branching of former excavation pits where air is deliberately not flowed mechanically but air is still moving from the main tunnel to the branching tunnel by diffusion as shown in Figure 1 is a sketch of the J2 branching of an underground coal mine. Actually, if there are activities carried out in the excavation front, of course, the air distribution to the branch tunnel is assisted by an additional air blower. However, from the concept of underground mine ventilation, that an air flow rate that is too small is not capable of carrying out mass transfer quickly, meaning that it is not suitable for the function of mine ventilation.

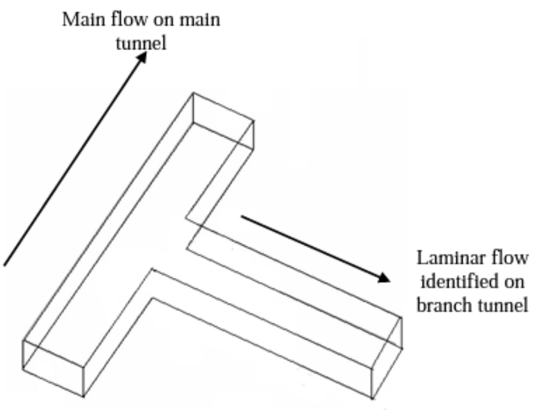

Figure 1. Branch tunnel sketch, modified sketch from (Sasmito et al., 2013)

Newtonian fluid is a form of viscous fluid through the plane of flow that gives rise to shear stresses in the direction of flow (Laloui & Rotta Loria, 2020; Tiwow, Fisika, & Makassar, 2015). The shear stress expression is proportional to the viscosity gradient with flow velocity over the path as in equation 20. The fluid shear stress and the fluid velocity produce a straight line function (Zołek-Tryznowska, 2015) as seen on Figure 2. Examples of Newtonian fluids are gas, water, and air (Franco & Partal, 2008). The dynamic viscosity coefficient of a Newtonian fluid depends on temperature and pressure and does not depend on the velocity gradient (Wicaksono, Subekti, & Indriyanto, 2021). In contrast to dilatant and pseudoplastic fluids, the stress curve and flow velocity of a Newtonian fluid form a straight line gradient.

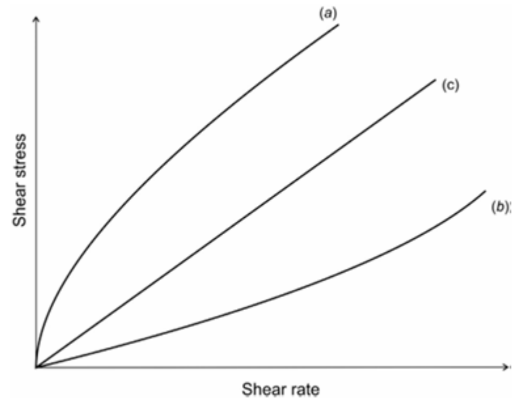

Figure 2. Graphical comparison of the relationship between flow velocity and shear stress of Newtonian (c), pseudoplastic (a), and dilatant (b) fluids (Zołek-Tryznowska, 2015)

METHOD

The basis of the research method is case studies, in general case studies are included in qualitative research examining phenomena that occur in real life with interpretation and natural approaches. (Patnaik & Pandey, 2019). The case of laminar flow of underground mines is not fully studied with a qualitative approach. Achievement of research objectives accompanied by quantitative analysis. Based on the modelling of the basic concepts of Newtonian fluids and suitability with the flow regime. If these conditions are met, the momentum balance equation calculation or the derivation of the Newtonian Navier-Stokes equation of motion is carried out to obtain a model equation that describes the phenomenon being studied. The equation analysis method uses a modified numerical-analytical mixed calculation method (Li et al., 2016). Derivation of the Newtonian equation of motion for the momentum balance can use analytical calculations, the method for calculating differentiation and integration of equations with simple mathematical solutions while numerical methods are used to solve partial differential equations which cannot be integrated with simple mathematical calculations but are assisted by programs (Denis, 2020), this is in accordance with the numerical modeling by (Khan & Yovanovich, 2014). Interpretation of equation models and flow profiles using qualitative and quantitative approaches. This means that there is a combination of descriptive data analysis by presenting numerical data which is evaluated by comparing it with pre-existing models.

Research Object

The research activity was carried out in an underground coal mine which was used as a mine education facility in Sawahlunto, West Sumatra. One of the branch tunnels in J2 according to the sketch in Figure 1 is connected directly to the main tunnel which is fed by exhaust system air flow. This branch is not ventilated mechanically but still flows naturally from the main tunnel.

Instrument

Field equipment is an anemometer which functions to measure the linear velocity of the air flow. This anemometer is equipped with a temperature sensor so that temperature data is also measured using this tool. Supporting equipment used is a meter to measure the dimensions of the tunnel. The data processing instrument is Microsoft Excel.

Research Procedure

The flow of research activities starts from (1) identifying the main tunnel branches that do not use additional blowers (2) Measuring linear air velocity, the anemometer is positioned perpendicular to the wind direction or parallel to the tunnel cross section. The purpose of the measurement is to identify airflow classified as laminar or turbulent based on the average linear velocity data and tunnel dimensions. The measurement method uses fixed mode traversing for rectangular cross sections at point c Figure 3. (3) Measurement of air quality, refers to measurement of air temperature. The temperature function is used to determine the value of the physical property of air, namely the viscosity changes in proportion to the change in temperature. (4) Calculation of momentum transfer, the basis for modeling fluid momentum transfer is the Newtonian Navier-Stokes equation of motion and the concept of plane momentum balance. This means that the incoming momentum flux is equal to the outcoming momentum flux. The integration of the momentum balance equation obtained the empirical equation of the fluid velocity and shear stress profile then plotted onto the graph of the fluid velocity profile and side view stress using Microsoft Excel in Cartesian coordinates adjusting to the rectangular tunnel dimensions. Coordinate point 0 is in the middle of the tunnel axis. The length of the track L is in the direction of the x-axis and the height of the tunnel δ is in the direction of the z-axis, while the width of the tunnel is in the direction of the y-axis. Model constraints from $z = \delta$ to $z = -\delta$ on a tunnel path along L as shown in Figure 4. (5) Data interpretation, presentation of the profile equation of the momentum equation model on the parameters of linear velocity and pressure difference to prove the hypothesis. Some unknown parameters during measurement are taken from literature data. The momentum transfer equation which contains velocity and fluid tension profiles gives rise to the calculated velocity based on the model as a result of the combination with the measured average air velocity.

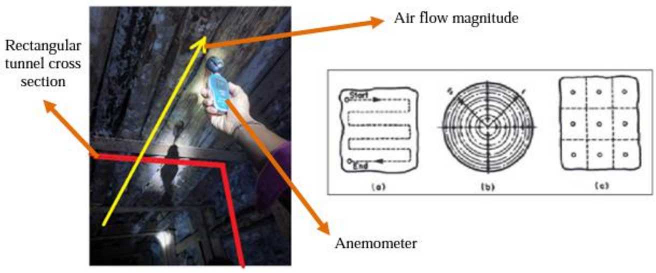

Figure 3. Measuring for linear air velocity and methods for measuring liner velocity: (a) continuous traversing; (b) fixed point traversing for circular; (c) fixed point for rectangular (Kanam & Ahmed, 2021)

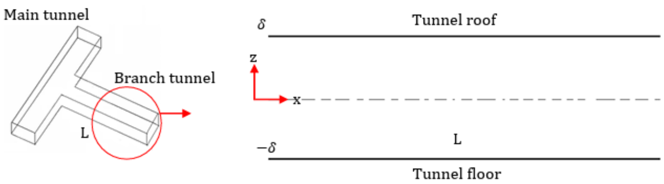

Figure 4. The boundaries of the Cartesian coordinates are shown on the side according to the dimensions of the tunnel

Data Collection and Analysis Techniques

Primary data consists of air quality and quantity measured directly using a measuring instrument. Each measurement was repeated 9 times according to the fixed point for rectangular method to obtain an average that represents the actual linear velocity. Secondary data consists of the density and viscosity of air obtained from the literature. Data analysis using analytical calculations. Data interpretation is done qualitatively and quantitatively. The data presented in the form of a profile picture is analyzed through a description. Data in the form of numbers in the form of transport equations were analyzed with a quantitative approach.

Interpretation and Conclusion of Research Results

The output of the research is in the form of empirical equations and fluid momentum transfer models consisting of linear velocity equations and profiles and fluid shear stress. This model is only valid for Newtonian fluid laminar flow. The way of interpreting the flow regime is determined through an empirical approach to the Reynolds equation. The next research goal is to prove that the laminar flow regime of air entering the ventilation system does not fulfill the ventilation function in terms of linear velocity and pressure drop.

RESULT AND DISCUSSION

Fluid Flow Regime

Both of laminar and turbulent flow is distinguished based on the Reynolds number, which is proportional to the linear velocity. Dynamic viscosity, density, shear stress, flux, and paths affect the Reynolds number. Rule of thumb for laminar flow if the Reynolds number is 0-2000, transitional flow Re: 2000-4000, and turbulent flow Re: >4000 (Widiatmojo et al., 2021). Equation 1 presents the empirical equation of the Reynolds number through a cross section with diameter D, if the cross section is not cylindrical then the equivalent diameter is used. Laminar flow forms a regular profile depicting a neat pattern of movement to form layers, but turbulent flow does not have a definite pattern of movement, as a result a large momentum occurs from between the fluid sections so that an even shear stress is formed. (Maria Ulfah Handayani, Zahrani Dalimunthe, Rika Sari Indah, 2016), as a result of mass dispersion between layers so that the linear velocity is almost uniform at each coordinate, which is indicated by the ratio of the average velocity/velocity at each coordinate close to 1.

$$Re = \frac{\rho . u . D}{\mu} \tag{1}$$

The dynamic viscosity μ per fluid mass density can be replaced by the kinematic viscosity v (Maria Ulfah Handayani, Zahrani Dalimunthe, Rika Sari Indah, 2016), then equation 3 obtained.

$$v = \frac{\mu}{\rho} \tag{2}$$

$$Re = \frac{u . D}{v} \tag{3}$$

Air Qualities and Quantities

Air quality consists of temperature and air composition. The quantity of air transport to underground mine is flow rate on volume of air per time (Q). Measurement of air flow rate is done by measuring the linear velocity of air (u). The ideal gas concept affects the amount of air that moves if the difference in temperature and pressure is significant, the air flow rate is calculated by multiplying the cross-sectional area (A) that the fluid passes by with the average linear velocity of the fluid flow. Based on the ideal gas principle, the value of Q or V/t is significantly affected by temperature and air pressure.

$$\frac{V}{t} = Q = u . A \tag{4}$$

$$\frac{V}{t} = \frac{n . R . T}{P . t} \tag{5}$$

Accumulation of fluid physical property measurement data from the literature can be seen in Table 1 that airflow is estimated in the laminar regime range.

Table 1. Data for momentum transport modelling

Parameter	value	Reference
Air dynamic viscosity at 26°C (interpolation)	0,018415 x 10^{-3} Pa.s	(R. Byron Bird et al., 2002)
Air kinematic viscosity at 26°C (interpolation)	0,90002 x 10^{-4} m^2/s	(R. Byron Bird et al., 2002)
Air temperature	26°C	Measurement
Reynolds number	1777,7	Calculation
Tunnel length limit	10 m	Measurement
Tunnel height	2 m	Measurement
Tunnel width	4 m	Measurement
Average linear velocity of air	0,04 m/s	Measurement
Air density	1,15 kg/m^3	(Prakash, 2014)

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Momentum Balance

A particle with a certain mass m moving with velocity u is the basis of the concept of the particle's momentum. As the particle size changes, m is replaced by the particle density.

$$p = \rho . u \tag{5}$$

Momentum applies the law of conservation of momentum, namely momentum equilibrium, meaning that the rate of the moment moving into a plane and the rate of the moment, leaving after passing through the plane are the same. The convective movement of particles through the plane represents the convective momentum flux tensor, which is the multiplication of the momentum and the velocity of the particle, a vector quantity over a difference in position and time (Lue, 2014). The total momentum flux in equation 7 is the accumulation of convective and molecular transport flux in equation 6.

$$\pi = p\delta_{ij} + \tau_{ij} \tag{6}$$

$$\phi_{ij} = \rho . u_i u_j + p\delta_{ij} + \tau_{ij} \tag{7}$$

The momentum transfer model is derived from equations derived from momentum balance. The momentum balance is the sum of the momentum fluxes entering the layer plane or often called the shell will be equal to the sum of the momentum fluxes passing through the planes. The direction of the momentum flux is perpendicular to the plane through which it passes. The shape of the field adjusts the space through which the fluid passes. Flow in pipes uses cylindrical coordinates, flow in rectangular space uses Cartesian coordinates, and flow in spheres uses spherical coordinates. (R. Byron Bird et al., 2002). Figure 5 is an illustration of applying momentum to a Cartesian coordinate plane with dimensions w, L, and δ.

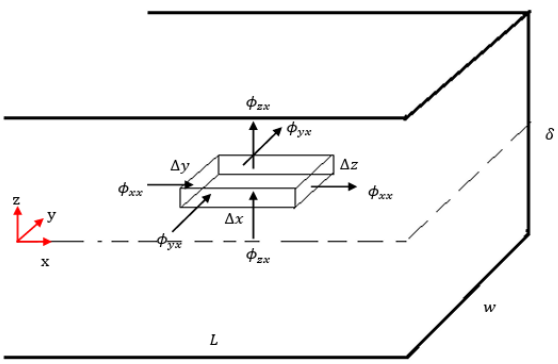

Figure 5. Shell Momentum balance through which the particle travels in the Cartesian coordinate system, modified of (Lue, 2014; R. Byron Bird et al., 2002)

The formulation of momentum balance is guided by the Cartesian plane momentum balance Figure 5 obtains the form of equation 8 for the momentum balance. Contrary to equation 7, the total momentum flux is the accumulation of the convective transport momentum flux $\rho . u . u$ and the molecular transport flux $\pi = p\delta_{ij} + \tau_{ij}$. The direction of transport passes through the tunnel plane. w is the tunnel width, δ is the tunnel height, L is the tunnel length. The form of momentum balance is as follows.

$$\left\{\begin{matrix} Convective \\ momentum \\ transport \\ rate\ in \end{matrix}\right\} - \left\{\begin{matrix} Convective \\ momentum \\ transport \\ rate\ in \end{matrix}\right\} + \left\{\begin{matrix} Molecular \\ transport \\ momentum \\ rate\ in \end{matrix}\right\} - \left\{\begin{matrix} Molecular \\ transport \\ momentum \\ rate\ in \end{matrix}\right\} + \left\{\begin{matrix} \text{Gravity force} \\ \text{influece} \end{matrix}\right\} = 0 \tag{8}$$

The momentum rate corresponds to the accumulated momentum flux in Figure 4, so the formulation of equation 9 is obtained. The momentum rate in the y direction is ignored because the model is limited to 2D, x and z dimensions.

$$\left(|\phi_{zx}.w.L|_z - |\phi_{zx}.w.L|_{z+\Delta z\}}\right) + \left(|\phi_{xx}.w.\Delta z|_{x=0} - |\phi_{xx}.w.\Delta z|_{x=L\}}\right) + (\rho.g.cos\alpha.w.\Delta z.L + P_O.w.\Delta z - P_L.w.\Delta z) = 0 \tag{9}$$

w, L, and ∆z can be eliminated, the equation is divided by the variables w, l, and ∆z. For the acceleration of gravity does not affect the Cartesian dimension in the direction of the x axis, so the following equation is obtained.

$$\left(\left|\frac{\phi_{zx}}{\Delta z}\right|_z - \left|\frac{\phi_{zx}}{\Delta z}\right|_{z+\Delta z}\right) + \left(\left|\frac{\phi_{xx}}{L}\right|_{x=0} - \left|\frac{\phi_{xx}}{L}\right|_{x=L}\right) + \left(\frac{P_O}{L} - \frac{P_L}{L}\right) = 0 \tag{10}$$

$$\lim_{\Delta z = 0}\left(\frac{|\phi_{zx}|_z - |\phi_{zx}|_{z+\Delta z}}{\Delta z}\right) + \left(\left|\frac{\phi_{xx}}{L}\right|_{x=0} - \left|\frac{\phi_{xx}}{L}\right|_{x=L}\right) + \left(\frac{P_O}{L} - \frac{P_L}{L}\right) = 0 \tag{11}$$

ϕ_{zx} is the accumulated momentum flux in the z direction along the x-axis with the value

$\rho.u_z u_x + p\delta_{zx} + \tau_{zx}$. Then the following equation is obtained.

$$\left(\left|\frac{\rho.u_z u_x + p\delta_{zx} + \tau_{zx}}{\Delta z}\right|_z - \left|\frac{\rho.u_x u_x + p\delta_{zx} + \tau_{zx}}{\Delta z}\right|_{z+\Delta z\}}\right) + \left(\left|\frac{\rho.u_x u_x + p\delta_{xx} + \tau_{xx}}{L}\right|_{x=0} - \left|\frac{\rho.u_x u_x + p\delta_{xx} + \tau_{xx}}{L}\right|_{x=L}\right) + \left(\frac{P_O}{L} - \frac{P_L}{L}\right) = 0 \quad (12)$$

u_i represents the fluid's linear velocity and τ_{ij} represents the fluid's shear stress. The laminar flow has a convective momentum transfer flux $\rho.u_z u_x$ in the z direction but it is not significant which means it is close to 0. The molecular transport flux values molecular $p\delta_{zx}$, $p\delta_{xx}$, and τ_{xx} are close to 0 so they are ignored. convective momentum flux transport $\rho.u_x u_x$ at x = 0 and x = L are the same then the elimination value becomes 0. So,

$$\lim_{\Delta z=0}\left(\frac{|\tau_{zx}|_z - |\tau_{zx}|_{z+\Delta z}}{\Delta z}\right) + \left(\frac{P_O}{L} - \frac{P_L}{L}\right) = 0 \quad (13)$$

Then the differential equation of the momentum balance:

$$\frac{d\tau_{zx}}{dz} = \frac{P_0}{L} - \frac{P_L}{L} \quad (14)$$

$$\int d\tau_{zx} = \left(\frac{P_O}{L} - \frac{P_L}{L}\right)\int dz \quad (15)$$

$$\tau_{zx} = \left(\frac{P_O}{L} - \frac{P_L}{L}\right) z + c \quad (16)$$

The boundary condition for the coordinate is z = 0, so the shear stress value τ_{zx} is close to the minimum or is considered to zero, so the value c = 0.

$$0 = \left(\frac{P_O}{L} - \frac{P_L}{L}\right) 0 + c \quad (17)$$

$$\tau_{zx} = \left(\frac{P_O}{L} - \frac{P_L}{L}\right) z \quad (18)$$

$$\tau_{zx} = \frac{\Delta P}{L} z \quad (19)$$

Equation 19 is an empirical model of interfacial shear stress or shear stress between the fluid and the path. Equation 18 generates to a differential equation according to the linear velocity relationship shear stress in equation 20 that the value of τ is:

$$\tau = -\mu\frac{du}{dz} \quad (20)$$

Substitution of equation 20 into equation 18 obtains the following equation.

$$-\mu\frac{du_x}{dz} = \left(\frac{P_O}{L} - \frac{P_L}{L}\right) z \quad (21)$$

$$\frac{du_x}{dz} = \frac{1}{-\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right) z \quad (22)$$

$$\int du_x = \frac{1}{-\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\int z\, dz \quad (23)$$

$$u_x = \frac{1}{-\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{1}{2}z^2 + c_1 \quad (24)$$

The boundary condition for coordinates z = 0 means that the value of u_x is close to the maximum. Coordinate z =δ, the value of u_x has a minimum or is considered to zero, then the value of C_1 = equation 26.

$$0 = \frac{1}{-\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{1}{2}\delta^2 + c_1 \quad (25)$$

$$c_1 = \frac{1}{\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{1}{2}\delta^2 \quad (26)$$

Equation 26 is substituted into equation 24, so the following equation is obtained.

$$u_x = \frac{1}{-\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{1}{2}z^2 + \frac{1}{\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{1}{2}\delta^2 \quad (27)$$

$$u_x = \frac{\delta^2 - z^2}{2\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right) \quad (28)$$

$$u_x = \frac{\Delta P}{L}\left(\frac{\delta^2 - z^2}{2\mu}\right) \quad (29)$$

Equation 29 is an empirical model of the linear velocity of the fluid along a path. The equation for the maximum velocity at z = 0 is as follows.

$$u_{max} = \frac{\delta^2}{2\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right) \quad (30)$$

Determination of the model of the interfacial tension equation and linear velocity can use the Newtonian Navier-Stokes equation of motion for the cartesian coordinates of the direction of motion on the x axis as follows.

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \rho g_x \quad (31)$$

Some of the neglected expressions are time variables at $\frac{\partial u_x}{\partial t}$ not in a transient state or the flow is considered steady. $\frac{\partial u_x}{\partial x}$ is ignored because the rate equation expression u_x with toward to x is minimum. The linear flow rate is regular in the direction of the x axis for laminar flow, meaning that the dispersion is only in the linear direction and there is no dispersion at y and z directions as occurs in turbulent flow, so u_y and u_z are considered to zero. $\frac{\partial^2 u_x}{dy^2}$ can be determined for the velocity profile u_x with respect to y, but is ignored for the 2-dimensional velocity profile u_x toward to z. Gravity doesn't affect u_x in the x-axis direction, so ρg_x is zero. So the remaining equation expression is as follows.

$$-\mu\frac{\partial^2 u_x}{dz^2} = \frac{\partial P}{\partial x} \quad (32)$$

$$\frac{\partial}{\partial z}\left(-\mu\frac{\partial u_x}{\partial z}\right) = \frac{\partial P}{\partial x} \quad (33)$$

$$\frac{\partial}{\partial z}(\tau) = \frac{\partial P}{\partial x} \quad (34)$$

$\frac{\partial P}{\partial x}$ is an expression of $\frac{P_O}{L} - \frac{P_L}{L}$ or $\frac{\Delta P}{L}$. The integration of differential equation 21 is continuation of equation 14 for the expression of the value of τ using equation 20.

The average velocity value from equation 39 is obtained from the comparison between the integration of the linear velocity equation with the boundary conditions w and δ to the integration of the cross section of the track with the boundary conditions w and δ.

$$\overline{u_x} = \frac{\int_0^w\int_0^\delta u_x dzdy}{\int_0^w\int_0^\delta dzdy} \quad (35)$$

$$\overline{u_x} = \frac{\int_0^w\int_0^\delta \left(\frac{P_O}{L} - \frac{P_L}{L}\right)\frac{\delta^2 - z^2}{2\mu}dzdy}{\int_0^w\int_0^\delta dzdy} \quad (36)$$

$$\overline{u_x} = \frac{\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\int_0^w\int_0^\delta \frac{\delta^2 - z^2}{2\mu}dzdy}{\int_0^w\int_0^\delta dzdy} \quad (37)$$

$$\overline{u_x} = \frac{w.\left(\frac{P_O}{L} - \frac{P_L}{L}\right)\int_0^\delta \frac{\delta^2 - z^2}{2\mu}dz}{w.\delta} \quad (38)$$

$$\overline{u_x} = \frac{\delta^2}{3\mu}\left(\frac{P_O}{L} - \frac{P_L}{L}\right) \quad (39)$$

Profile of Linear Velocity Distribution and Fluid Shear Stress

The laminar flow momentum transfer model for steady state condition, in line with Newton's law II when the steady state there is no acceleration until the linear velocity is constant (Tiwow et al., 2015). The profile gave rise to the interfacial shear stress equation model in equation 19 and the linear velocity equation in equation 29, to describe the stress profile and linear velocity it is necessary to interpret the model that $P_o - P_l$ or ΔP towards L is pressure drop, tends to occur due to forced convection as a result of encouragement from mechanical equipment (Faruk & Kamiran, 2017) (Utami & Azhar, 2017) (Nasution et al., 2014). The pressure drop is obtained from equation 39, if the measured average air velocity is 0,04 m/s and substituted for the value δ, μ, L, the air pressure drop due to laminar transport is 2,2 x 10^{-5} Pa. This proves that laminar flow with a flow rate of 0,32 m^3/s with a small pressure drop indicates no forced convection, only molecular transport so that it does not match at the ventilation function.

The next step is to estimate the shear stress and linear velocity for each coordinate (x,z) in stress distribution profile and linear velocity in Figure 6 and Figure 7. The transport profile in shows that the movement of the fluid and the shear stress are unidirectional and form a regular pattern. The maximum velocity value is located in the middle of the x-axis and is proportional to the stress value which is close to 0. However, it is best at the position z = δ indicating that there is friction between the fluid interface that is in direct contact with the track wall so that the shear stress is maximum and the linear velocity is minimum at these coordinates.

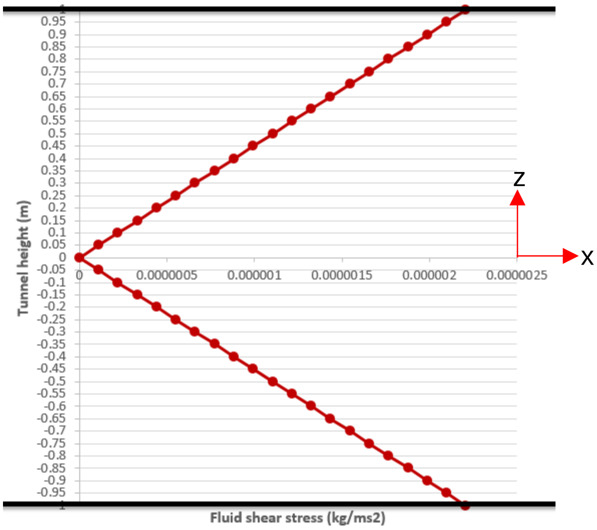

Figure 6. Fluid shear stress distribution profile

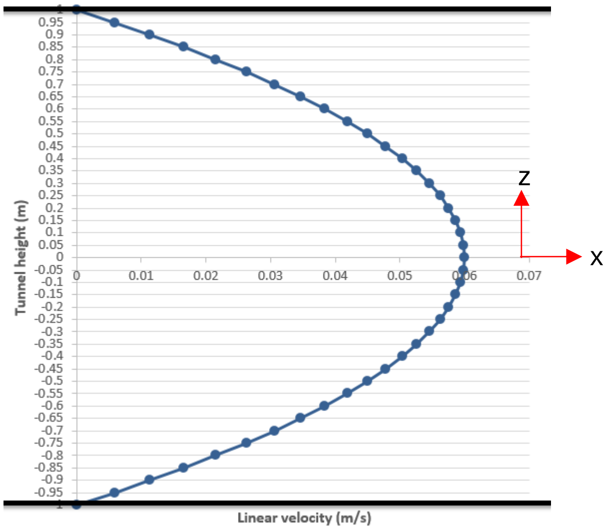

Figure 7. Fluid linear velocity distribution profile

Laminar fluid flow is rare in mine ventilation systems (Widiatmojo et al., 2021). Underground mines with a long reach from the surface to the excavation area require a turbulent fluid boost. Therefore, this transport model is only valid for certain conditions which represent a laminar flow profile. The disadvantage of laminar flow is that it is not able to move the mass of the particles following it as well as turbulent flow. The turbulent regime is able to move the mass of particles due to the momentum that occurs compared to the movement of molecular diffusion (Faruk & Kamiran, 2017), while laminar flow tends to be dominated by molecular motion, using equations 19 and 29 in Microsoft Excel to obtain profiles of the distribution of shear stress and linear velocity of the fluid as shown in Figures 6 and Figure 7.

As a comparison with the distribution profile carried out by (Li et al., 2016) that the minimum velocity is in coordinates close to the contact between the interface or between the wall and the fluid, the maximum velocity is in the middle of the flow axis. The situation is reversed for the shear stress distribution. The maximum value of interfacial friction is right on the track wall, the fluid rubs directly with the track plane (Fukuchi & Fukuchi, 2014), although basically the viscosity of air is very small so that the shear stress is also very small.

CONCLUSION AND SUGGESTIONS

Analytical calculations for the momentum transfer of laminar flow can be concluded that (1) Air flow is modelled according to the Newtonian fluid properties using the Navier-Stokes equations of motion at Cartesian coordinates adjusting to the dimensions of the tunnel, the shear stress and linear velocity equation models are obtained according to the laminar flow concept (2) Laminar flow tends towards molecular displacement and is weak against momentum so that air movement is slow, as evidenced by the small pressure drop and air flow rate. Turbulent flow has these two aspects which laminar flow does not. Some suggestions from the results of research and suggestions for further research are (1) The researcher tried to discuss the topic of laminar airflow which is rare in mine ventilation systems. In this case, the findings of this discussion do not look for loopholes that have a negative impact on the real object of mine ventilation in the future. Therefore, it is necessary to have further discussion studies for this research topic (2) Perform mass transfer and heat transfer modelling for mine ventilation airflow cases.

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