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Dissecting Students' Strategies: How Do Polya's Steps Reveal Problem-Solving Abilities?

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ABSTRACT

This study aims to analyze junior high school students' mathematical problem-solving abilities on Geometric Sequences and Series based on Polya's problem-solving steps. A descriptive qualitative approach was employed involving 27 eighth-grade students. The research stages included administering a contextual problem-solving test, selecting subjects through purposive sampling, conducting semi-structured interviews, and analyzing data using the Miles and Huberman model (data reduction, data display, and conclusion drawing). Four students representing high, medium, and low ability levels were selected as the main subjects. Data were collected using contextual problem-solving tests and supported by semi-structured interviews. The analysis focused on students' performance across Polya's four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. Students' problem-solving performance showed clear differences across Polya's four stages. High-ability students applied all stages systematically, with only minor notation errors. Medium-ability students demonstrated sufficient conceptual understanding but struggled with planning strategies, maintaining procedural consistency, and interpreting problems, often leading to incorrect formula selection and ratio miscalculations. In contrast, low-ability students faced difficulties from the initial stage, particularly in identifying given information, distinguishing arithmetic and geometric concepts, and selecting appropriate formulas. Their errors commonly included using incorrect models, incomplete procedures, and lack of verification. These findings indicate that successful mathematical problem solving depends not only on formula mastery but also on conceptual understanding, strategic planning, and verification skills. Therefore, instruction should emphasize structured problem-solving processes and strengthen students' conceptual and metacognitive abilities.

A. INTRODUCTION

Mathematical problem-solving skills are an important competency in mathematics learning because they reflect how students understand and process problems systematically (Faradillah & Fadhilah, 2021; Hutajulu et al., 2019; Kusuma et al., 2024; Nugroho et al., 2025; Nugroho & Septianisha, 2025). This ability is evident in students' ability to identify known information, determine completion strategies, carry out completion steps, and re-examine the results obtained (Polya, 1977). However, students often have difficulties in carrying out these stages sequentially and appropriately (Gunawan et al., 2023; Kosim et al., 2020; Kriswanto et al., 2020; Mustofa et al., 2020; Nugroho et al., 2024; Rahmah et al., 2019; Ukobizaba et al., 2021). Therefore, an in-depth analysis of students' strategies and thinking processes in solving mathematical problems is needed to comprehensively understand students' problem-solving skills.

Problem-solving skills in the junior high school curriculum can be effectively developed through algebra topics, particularly Geometric Sequences and Series. This material requires conceptual understanding, the ability to recognize patterns, and the application of formulas and generalizations in various contexts (Erni, 2021; Irawati et al., 2022). The relevance of materials on Geometric Series and Sequences is further strengthened by their real-life applications, such as population growth, compound interest, loan installments, savings, and their use in physics and computer science (Maryati & Fadhilah, 2021). However, empirical studies indicate that students still experience considerable difficulties in this topic. For instance, Erni (2021) found that a significant proportion of students struggled to distinguish between arithmetic and geometric patterns, while Amalia & Nuriadin (2023) reported that many students relied on memorizing formulas without understanding underlying concepts. Furthermore, Maryati & Fadhilah (2021) showed that students often failed to interpret contextual problems and select appropriate solution strategies. These difficulties indicate significant obstacles to solving problems involving Geometric Series and Sequences, which require deeper study.

To outline a student's cognitive process in solving mathematical problems, Polya's steps have long been recognized as a systematic and effective heuristic framework. Polya identified four main steps in problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1973, 1977). This framework enables a structured analysis of each stage of student thinking, allowing the specific identification of difficulties and errors.

Several studies have shown that the Polya step is an effective problem-solving analysis tool. However, there is still a research gap in studies that specifically examine junior high school students' mathematical problem-solving abilities in Geometric Series and Sequences using Polya's stages (Priyani & Ekawati, 2018; Siskawati et al., 2021; Winarso et al., 2022). This gap indicates that the application of Polya's framework in this topic remains limited. Such analysis is important to better understand students' thinking patterns, difficulties, and factors influencing their success in solving mathematical problems.

Therefore, this study aims to comprehensively analyze the mathematical problem-solving ability of junior high school students in the Geometric Series and Sequences materials using Polya's steps.

B. METHODS

This study employs a qualitative descriptive approach, with the research steps presented in Figure 1, to analyze students' mathematical problem-solving abilities in solving contextual problems on Geometric Series and Sequences. This approach was chosen because it allows researchers to examine in depth the thinking processes and strategies students use when solving mathematical problems at specific stages (Emylia & Setyaningsih, 2025).

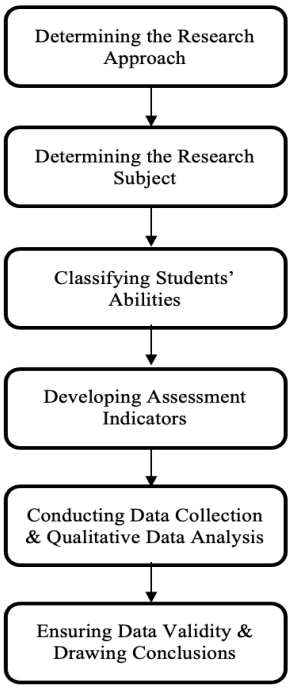


Figure 1. Research Steps in the Qualitative Descriptive Approach

The participants were 27 Grade VIII Science 1 students from a state school in Surakarta, selected through purposive sampling to represent varying ability levels and who had studied Geometric Series and Sequences. Based on the results of the mathematical problem-solving test, three students were selected as the main subjects, representing high, medium, and low ability categories. The grouping of ability categories is determined using the criteria proposed by Arikunto (2013), based on the average score ($\bar{x}$) and standard deviation (s), as presented in Table 1.

Table 1. Criteria for Grouping Students' Mathematical Problem-Solving Abilities

Criteria	Category
$skor > \bar{x} + s$	High
$\bar{x} - s \leq skor \leq \bar{x} + s$	Medium
$skor < \bar{x} - s$	Low

The researchers acted as the primary instrument, supported by a mathematical problem-solving test and interview guidelines based on Polya's problem-solving stages. Each question item has a maximum score of 25. Data analysis was conducted using the qualitative data analysis model proposed by Miles & Huberman (1994), which includes data reduction, data display, and conclusion drawing. The data were derived from students' mathematical problem-solving tests and in-depth interviews. The interview procedures were conducted semi-structurally, focusing on exploring students' reasoning and strategies at each stage of Polya's problem-solving process. Prior to implementation, the research instruments were validated by two expert lecturers to ensure content

validity and clarity. Data validity was ensured through method triangulation by comparing written test results with interview data to obtain a comprehensive understanding of students' mathematical problem-solving abilities in Geometric Series and Sequences.

C. RESULT AND DISCUSSION

1. Categories of Students' Mathematical Problem-Solving Ability

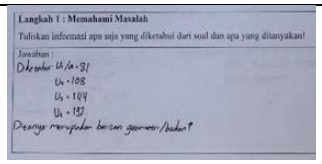
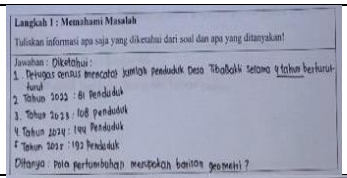
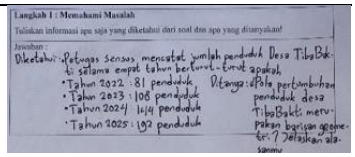
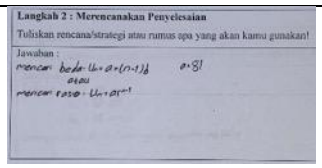
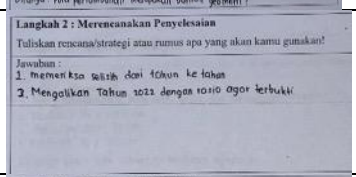
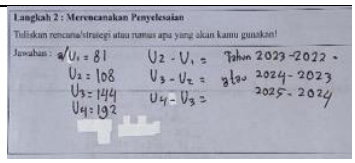
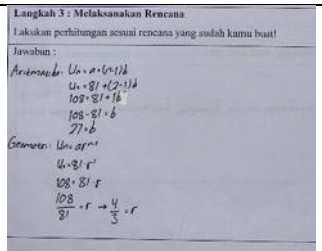
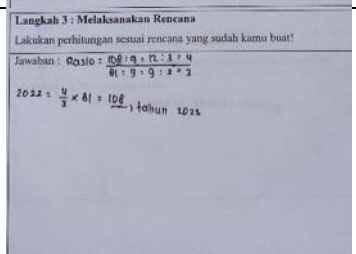
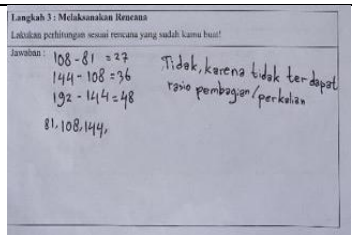
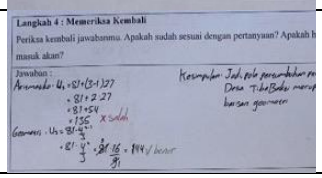
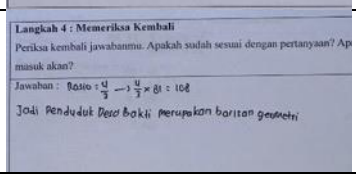
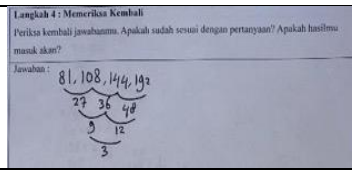
The categorization of students' mathematical problem-solving abilities on Geometric Series and Sequences indicates that most students were classified in the medium category, with 17 students scoring between 30.71 and 75.21. Meanwhile, five students demonstrated high problem-solving ability, with scores above 75.21, and five were categorized as having low ability, with scores below 30.71. This distribution indicates that students' mathematical problem-solving abilities vary across categories and are not evenly distributed, underscoring the need for further analysis to characterize each ability level according to Polya's problem-solving steps, namely: Understanding the Problem (A1); Planning the Solution (A2), Carrying Out the Plan (A3), and Looking Back (A4).

2. Analysis of Mathematical Problem-Solving Ability Based on Polya's Steps

Problem 1

> *A census officer recorded the population of TibaBakti Village over four consecutive years. In 2022, the population was 81; in 2023, it increased to 108; in 2024, it reached 144; and in 2025, it reached 192. Based on these data, does the population growth pattern of TibaBakti Village form a geometric sequence? Explain your answer.*

Table 1. Responses of Students in the High Category for Problem 1

Indicators	High	Medium	Low
	LM	KY	UM
A1			
A2			
A3			
A4			

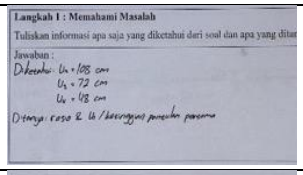
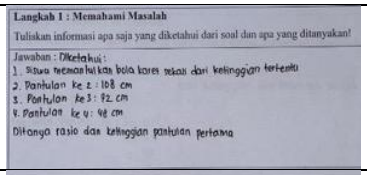
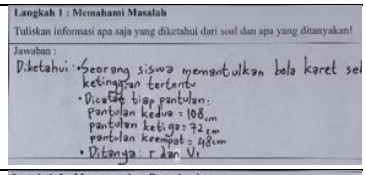
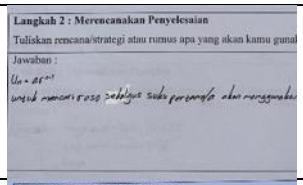
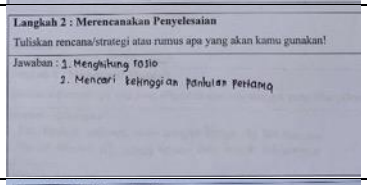
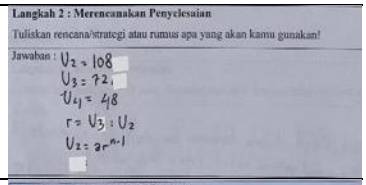
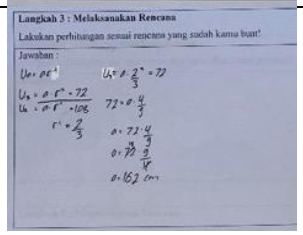
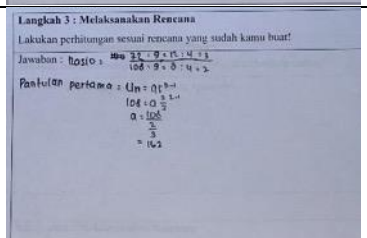
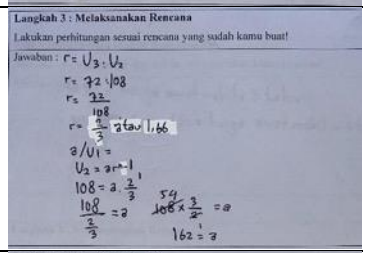
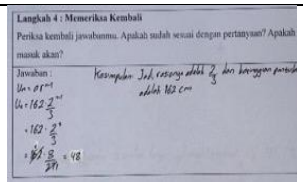
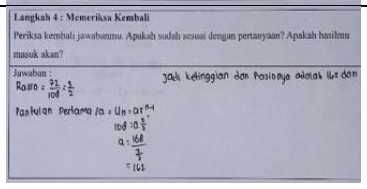
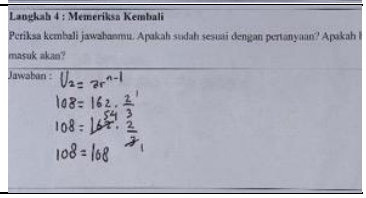
Based on LM's solution (Table 1), during the Understanding the Problem stage, LM identified all known information but showed uncertainty about whether the sequence

was arithmetic or geometric, indicating weak conceptual understanding (Ramadhani et al., 2024; Winarso et al., 2022). This uncertainty persisted during the Planning the Solution stage, in which both formulas were written without selecting one. In the Carrying Out the Plan stage, LM correctly calculated both sequences but only tested the initial terms. In the Looking Back stage, LM showed that the sequence was not arithmetic due to an incorrect value $U_3 = 135$, while successfully proving it was geometric by correctly obtaining= $U_5 = 144$. Thus, LM concluded that the population was growing geometrically. Based on KY's solution (Table 1), KY correctly identified the given data and focused on ratio analysis as a proof strategy. Despite minor errors in algebraic notation, KY correctly calculated and verified the ratio and used it to determine subsequent terms. Based on UM's solution (Table 1), UM identified the given information but incorrectly focused on differences instead of ratios, leading to an arithmetic interpretation. This misconception, consistent with prior studies (Emylia & Setyaningsih, 2025; Kalyuga et al., 2010; Priyani & Ekawati, 2018; Siskawati et al., 2021; Yapatang & Polyiem, 2022) persisted through all stages, resulting in an incorrect conclusion that the sequence was not geometric.

Problem 2

During a physical education class, a student bounced a rubber ball from a certain height. The recorded heights of the ball were 108 cm for the second bounce, 72 cm for the third bounce, and 48 cm for the fourth bounce. Based on these data, determine the ratio and the height of the first bounce.

Table 2. Students' Responses in the High Category for Problem 2

Indicators	High	Medium	Low
	LM	KY	UM
A1			
A2			
A3			
A4			

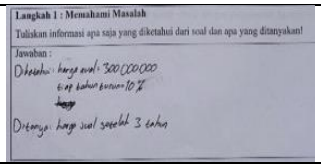
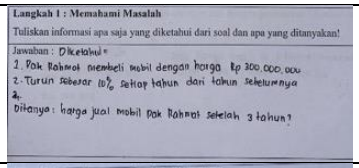
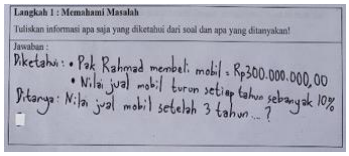
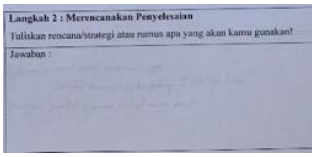
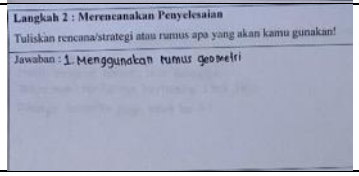
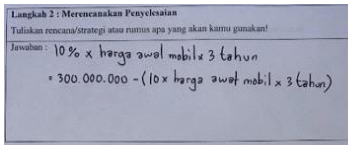
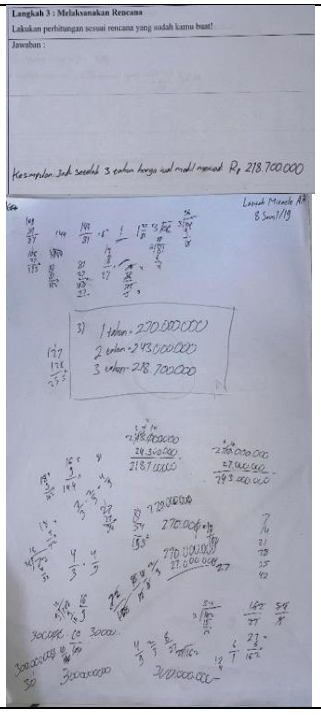
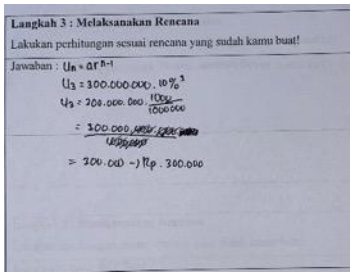
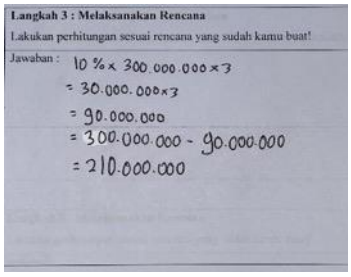
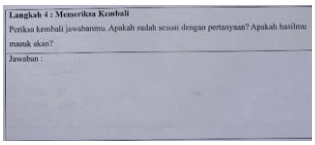
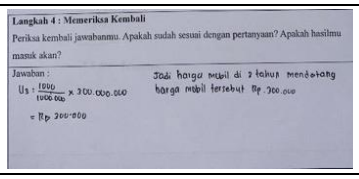
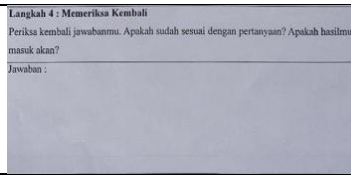
Based on LM's solution (Table 2), LM correctly identified the given information and question. However, during the planning and execution stages, although the intended strategy using the Geometric Sequence formula was appropriate, unconventional notation, unclear procedures, and errors in writing the ratio formula were observed. These findings support previous studies (Khasawneh et al., 2023; Nugroho & Septianisha,

2025), underscoring the need to improve students' mathematical communication. Despite these issues, LM successfully verified the result and presented a correct final conclusion.. Based on KY's solution (Table 2), KY identified the data and applied an appropriate strategy; however, errors in ratio calculation and inconsistent notation persisted throughout the execution and verification stages, resulting in critical inaccuracies. Based on UM's solution (Table 2), UM demonstrated strong conceptual understanding by selecting appropriate formulas, accurately calculating the ratio and first term, and verifying the solution through substitution, resulting in a correct and well-justified conclusion.

Problem 3

Mr. Rahmad purchased a car for Rp 300,000,000. Based on market data, the car's resale value is estimated to decrease by 10% per year relative to the previous year's price. What is the estimated resale value of Mr. Rahmad's car after 3 years?

Table 3. Students' Responses in the High Category for Problem 3

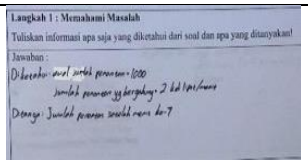
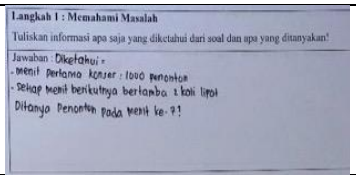
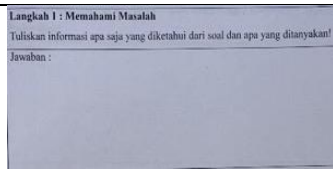
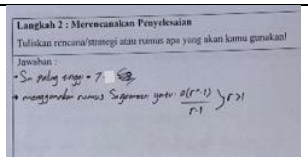
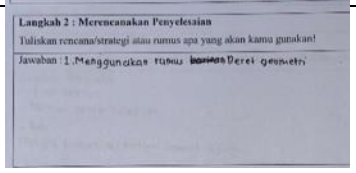
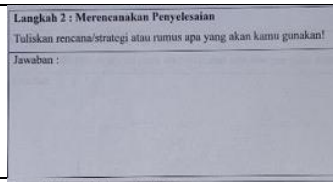
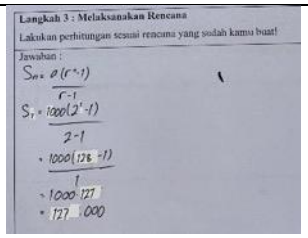
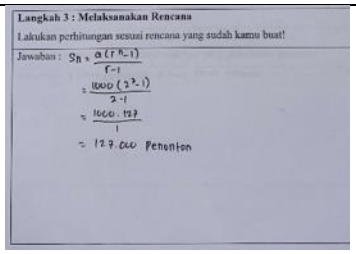
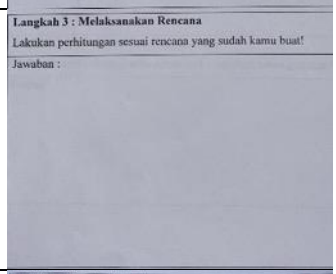
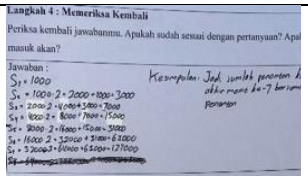
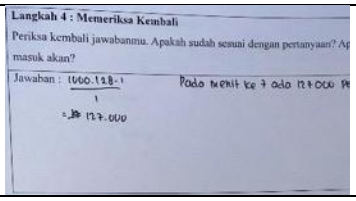
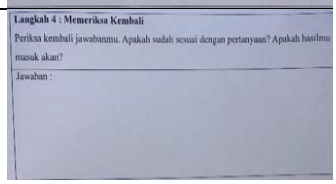
Indicators	High	Medium	Low
	LM	KY	UM
A1			
A2			
A3			
A4			

Based on LM's solution (Table 3), LM correctly identified the given information and the required quantity. Although no explicit strategy or formula was provided, LM obtained the correct result and drew an accurate conclusion, indicating successful problem solving despite the absence of formal mathematical expressions, consistent with Irawati et al. (2022) and Ramadhani et al. (2024). Based on KY's solution (Table 3), KY identified the given data and selected the Geometric Sequence formula; however, incorrect application of the formula and conceptual misunderstanding of depreciation led to an unrealistic and incorrect final answer. Based on UM's solution (Table 3), UM correctly identified the data but applied an inappropriate linear depreciation strategy instead of a Geometric model. Although the calculations were consistent, the incorrect model resulted in an inaccurate conclusion, consistent with prior findings on students' difficulties with Geometric depreciation.

Problem 4

A K-pop group held a live-streamed Comeback Showcase. In the first minute of the concert, 1,000 viewers joined. In each subsequent minute, the number of viewers joining doubled compared to the previous minute. What is the total number of viewers who have watched the concert by the end of the 7th minute?

Table 4. Students' Responses in the High Category for Problem 4

Indicator	High	Medium	Low
	LM	KY	UM
A1			
A2			
A3			
A4			

Based on LM's solution (Table 4), LM correctly identified the given information and recognized that the problem required finding the sum of a Geometric Series. LM appropriately applied the Sn. formula, verified the result by manual summation, and stated an accurate conclusion. Based on KY's solution (Table 4), KY similarly misinterpreted the question as asking for a total rather than a specific term. This misunderstanding led to the use of an inappropriate formula and persisted through all stages, resulting in an incorrect conclusion despite correct procedural calculations.

Meanwhile, UM (Table 4) did not provide any written response to the problem. The absence of an answer indicates that UM was unable to demonstrate understanding at any stage of Polya's problem-solving process.

Students' mathematical problem-solving abilities on geometric sequences and series vary across ability levels. High-ability students generally applied all Polya's steps systematically, with minor notation errors. Medium-ability students showed adequate conceptual understanding but struggled in planning and execution, particularly in maintaining procedural consistency and interpreting problem contexts. Low-ability students faced difficulties from the initial stage, especially in understanding problems, distinguishing between arithmetic and geometric sequences, and selecting appropriate formulas.

This study has several limitations. First, the small number of participants limits the generalizability of the findings. Second, the analysis relied mainly on written tests, which may not fully capture students' thinking processes. Third, the study focused only on geometric sequences and series, so other mathematical topics were not examined.

D. CONCLUSION AND SUGGESTIONS

Based on the findings of this study, students' mathematical problem-solving abilities in Geometric Sequences and Series vary across ability levels. High-ability students tend to apply Polya's problem-solving steps systematically, whereas medium-ability students continue to experience difficulties in the planning and execution stages, particularly in maintaining strategy accuracy and interpreting problem contexts. Low-ability students face challenges at the problem-comprehension stage, particularly in identifying the given and required information and distinguishing between arithmetic and geometric sequences. These findings indicate that mathematical problem-solving ability depends not only on formula mastery but also on conceptual understanding, strategic accuracy, and logical verification, consistent with Polya's steps.

Future research should involve larger samples, employ additional methods to better capture students' thinking processes, and examine other mathematical topics to achieve a more comprehensive understanding of problem-solving abilities.

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